Patterns of Strong Coupling for LHC Searches

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Precision measurement at the LHC

A general $2 \rightarrow 2$ scattering at low energy:

$$\mathcal{A}(\phi \phi \rightarrow \phi \phi) \sim g_{SM}^{2} \left(1 + \frac{g_{*}^{n} E^{2}}{g_{SM}^{n} m_{*}^{2}} + \cdots \right)$$

where $n \leq 2$, for weakly coupled theory $g_{*} \sim g_{SM}$:

$$\frac{\delta \sigma}{\sigma} < 1$$

for the expansion to make sense.

But the present several searches (VH, VV) at the LHC sensitive to $O(1)$ effects.
Precision measurement at the LHC

One can thinking of the LHC open a new door to strong coupling!
Power counting of $\hbar$

Natural units:

$$\hbar = c = 1$$

Let’s restore $\hbar$ in our action for the path-integration:

$$e^{iS/\hbar} = e^{i \int d^4x \mathcal{L}/\hbar}$$

For the non-canonically normalized fields:

$$\mathcal{L}/\hbar = \frac{1}{g_\phi^2 \hbar} \left( \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} \phi^2 + \cdots \right)$$

So that:

$$[g_\phi] = \hbar^{-1/2}, \quad A_n \propto g_\phi^{n-2}$$
SILH scenario

SILH can be thinking of as a set of power-counting rules associated with following considerations:

- Two sectors: the elementary sector (including SM gauge bosons and fermions), the composite (strong) sector (including the Higgs).
- Higgs are further assumed as pseudo-Goldstone bosons for naturalness consideration.
- The physics of the new sector is broadly characterized by one scale $m_*$ and one coupling $g_*$.  
- The elementary fields are assumed linearly coupled to the strong sector according to the hypothesis of partial compositeness.
Partial Compositeness

The mixing Lagrangian in the UV:

$$\mathcal{L}_{\text{mix}} = \epsilon_A A_\mu J^\mu + \epsilon_\psi \psi \mathcal{O}_\psi + \text{h.c.},$$

leading to the effective Lagrangian below the scale $m_*$:

$$\mathcal{L}_{\text{eff}} = \frac{1}{g_*^2} \left\{ m_*^4 L \left( \frac{\Phi}{m_*}, \frac{D_\mu}{m_*}, \frac{\epsilon_A \hat{F}^i_{\mu\nu}}{m_*^2}, \frac{\epsilon_\psi \hat{\psi}}{m_*^{3/2}} \right) - \frac{1}{4}(\hat{F}^i_{\mu\nu})^2 + i \bar{\psi} \gamma^\mu D_\mu \psi \right\},$$

$$D_\mu \equiv \partial_\mu + i \epsilon_A T_i \hat{A}_\mu^i,$$

$$\hat{F}^i_{\mu\nu} \equiv \partial_\mu \hat{A}_\nu^i - \partial_\nu \hat{A}_\mu^i - \epsilon_A f^{ijk} \hat{A}_\mu^j \hat{A}_\nu^k.$$
Partial Compositeness

- To go to canonically normalized fields: $\hat{A}_\mu = g_* A_\mu, \hat{\psi} = g_* \psi$
- The gauge coupling: $g \equiv g_* \epsilon_A$
- $\epsilon$ measures the degree of the compositeness of the SM fields.
- $\epsilon \sim 1$ means fully composite, which can be achieved for the right-handed top quark, if $3/2 < \text{dim}\mathcal{O}_{t_R} < 5/2$.

Partial Compositeness

The composite fields can be continuously deformed to the elementary fields.
\[ \mathcal{L}_6 = \frac{1}{m^2} \sum_i c_i \mathcal{O}_i. \]

| \( \mathcal{O}_H \) | \( \frac{1}{2} (\partial^\mu |H|^2)^2 \) |
|---|---|
| \( \mathcal{O}_T \) | \( \frac{1}{2} \left( \left( H_\dagger D_\mu H \right) \right)^2 \) |
| \( \mathcal{O}_6 \) | \( |H|^6 \) |
| \( \mathcal{O}_W \) | \( \frac{i}{2} \left( H_\dagger \sigma^a D_\mu H \right) \) \( D^\nu W^a_\mu\nu \) |
| \( \mathcal{O}_B \) | \( \frac{i}{2} \left( H_\dagger D_\mu H \right) \) \( \partial^\nu B_\mu\nu \) |
| \( \mathcal{O}_{HW} \) | \( i \left( D_\mu H \right)^\dagger \sigma^a \left( D^\nu H \right) W^a_\mu\nu \) |
| \( \mathcal{O}_{HB} \) | \( i \left( D_\mu H \right)^\dagger \left( D^\nu H \right) B_\mu\nu \) |
| \( \mathcal{O}_{BB} \) | \( |H|^2 B_\mu\nu B^{\mu\nu} \) |
| \( \mathcal{O}_{GG} \) | \( |H|^2 G_\mu\nu G^{A\mu\nu} \) |

**Table 1:** Dimension-6 operators used in our analysis.
SILH operators

- For purely bosonic operators, only $O_W, O_B, O_{2V}$ can be generated at tree level by exchange massive vectors in minimally coupled theory (Holographic composite Higgs model and little Higgs model).

- $O_{GG}, O_{BB}$ subject to the same selection rule for the Higgs potential, will have extra suppression $y_t^2/g_*^2$.

- In the general case $GSILH$, the minimal coupling condition is relaxed.
## SILH operators

|       | $|H|^2$          | $|H|^4$          | $O_H$  | $O_6$  | $O_V$ | $O_{2V}$ | $O_{3V}$ |
|-------|----------------|----------------|--------|--------|-------|----------|----------|
| ALH   | $m_*^2$        | $g_*^2$        | $g_*^2$| $g_*^4$| $g_V$ | $g_V^2$  | $g_V^2$  |
| GSILH | $\frac{y_t^2}{16\pi^2} m_*^2$ | $\frac{y_t^2}{16\pi^2} g_*^2$ | $g_*^2$ | $\frac{y_t^2}{16\pi^2} g_*^4$ | $g_V$ | $g_V^2$  | $g_V^2$  |
| SILH  | $\frac{y_t^2}{16\pi^2} m_*^2$ | $\frac{y_t^2}{16\pi^2} g_*^2$ | $g_*^2$ | $\frac{y_t^2}{16\pi^2} g_*^4$ | $g_V$ | $g_V^2$  | $\frac{g_V^2}{16\pi^2} g_V$ |

<table>
<thead>
<tr>
<th></th>
<th>$O_{HV}$</th>
<th>$O_{VV}$</th>
<th>$O_{y_\psi}$</th>
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<td>$g_V$</td>
<td>$g_V^2$</td>
<td>$y_\psi g_*^2$</td>
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<td>GSILH</td>
<td>$g_V$</td>
<td>$\frac{y_t^2}{16\pi^2} g_V^2$</td>
<td>$y_\psi g_*^2$</td>
</tr>
<tr>
<td>SILH</td>
<td>$\frac{g_*^2}{16\pi^2} g_V$</td>
<td>$\frac{y_t^2}{16\pi^2} g_V^2$</td>
<td>$y_\psi g_*^2$</td>
</tr>
</tbody>
</table>
Strong multi-polar interactions

An object can have large multi-pole and small monopole:

\[ q = 1, \quad |\vec{d}| \sim 50R \]
Strong multi-polar interactions

- Two couplings involved: gauge coupling (monopole) $g$ and the strong coupling $g_*$ controlling the multipole interactions of the resonances.

- The small parameter $\epsilon = g/g_*$ is technically natural, since $\epsilon = 0$ is a stable point by deformed symmetry (not enhanced).

Abelian case

$$
\mathcal{L}_{\text{eff}} = \frac{m^4}{g_2^2} L \left( \frac{\hat{F}_{\mu\nu}}{m_*}, \frac{\partial_\mu}{m_*}, \frac{\hat{\Phi}}{m_*} \right),
$$

The effective Lagrangian can be deformed by including small charges:

$$
\partial_\mu \Phi \rightarrow (\partial_\mu + i\epsilon q_\Phi A_\mu) \Phi,
$$
Strong multi-polar interactions

The situation can be generalized to non-Abelian cases by requiring:

- There are $N_A$ composite $U(1)^{N_A}$ gauge bosons.
- The $U(1)^{N_A}$ photons transform in the adjoint under the global symmetry $G$ of the strong sector.

Inonu-Wigner (IW) contraction

Small charges are included by deforming the symmetry:

$$[G]_{global} \rtimes [U(1)^{N_A}]_{local} \rightarrow [G]_{local}.$$

leading to the following effective Lagrangian:

$$\mathcal{L}_{eff} = \frac{m^4}{g^2_*} L \left( \frac{\hat{F}_{\mu\nu}}{m_*^2}, \frac{D_\mu}{m_*} \right),$$
Can we obtain Strong multi-pole interactions from Partial Compositeness?

\[ \Delta \mathcal{L}_{\text{mix}} = \epsilon F F_{\mu \nu} \mathcal{O}^{\mu \nu}, \]

\[ L \rightarrow L \left( \frac{\hat{\Phi}}{m_*}, \frac{D_\mu}{m_*}, \frac{\epsilon F \hat{F}_{\mu \nu}}{m_*^2}, \frac{\epsilon \psi \hat{\psi}}{m_*^{3/2}} \right). \]

We can define a effective coupling:

\[ g_{\text{eff}} \sim \epsilon F g_* \frac{E}{m_*}. \]

However,

Unitarity of CFT require \( \text{dim } \mathcal{O}^{\mu \nu} \geq 2 \), the mixing is irrelevant except \( \mathcal{O}^{\mu \nu} \) is a free field.
If only the gauge bosons are involved in the strong dynamics, the following operators are enhanced:

\[ c_3W, c_3G \sim g_*, \quad c_2W, c_2B, c_2G \sim 1. \]

The phenomenological consequences:

\[ c_3W \sim g_* \quad \Rightarrow \quad \delta A(\bar{\psi}\psi \rightarrow V_T V_T) \sim g g_* \frac{E^2}{m_*^2}, \]

\[ \delta A(V_T V_T \rightarrow V_T V_T) \sim g g_* \frac{E^2}{m_*^2}, \frac{g_*^2 E^4}{m_*^4}, \]

\[ c_2W, c_2B \sim 1 \quad \Rightarrow \quad \delta A(\psi \bar{\psi} \rightarrow V_T^* \rightarrow \psi \bar{\psi}) \sim g^2 \frac{E^2}{m_*^2}. \]
Note that,

- As long as \( g_* \frac{E^2}{m_*^2} > g \), dimension-8 operators are needed for consistent analysis of \( WW \) scattering.

- The anomalous TGC:

\[
\lambda_\gamma \equiv \frac{c_{3W}}{g} \frac{m_W^2}{m_*^2} \sim \frac{g_*}{g} \frac{m_W^2}{m_*^2},
\]

- The high precision of LEP makes \( c_{2W,2B} \) more relevant.
The modification of the gauge propagator can be traded as the $W, Y$ parameters:

$$W, Y \equiv c_{2W,2B} \frac{m_W^2}{m^*_2} \sim \frac{m_W^2}{m^*_2}.$$  

$$W, Y \lesssim 10^{-3} \Rightarrow m^*_1 \gtrsim 3\text{TeV}$$

Compared with:

$$\lambda_\gamma \lesssim 10^{-2} \Rightarrow m^*_1 \gtrsim 1.5 \sqrt{\frac{g^*_4}{4\pi}} \text{ TeV}$$
It is more motivated to include the Higgs as Pseudo-Goldstone bosons of the strong sector:

$$G = [SO(5) \times \tilde{SU}(2) \times U(1) \times]_{global} \times [U(1)^4]_{local}$$

An extra global $\tilde{SU}(2)$ is needed to make the Higgs mass stable.

The effective Lagrangian

$$L_{eff} = \frac{m^4_*}{g^2_*} L \left( U, \frac{\hat{F}_{\mu\nu}}{m^2_*}, \frac{D_\mu}{m_*} \right)$$
In the limit \( g = g' = 0 \), the extra \( \tilde{SU}(2) \) forbids the operators involving both gauge fields and the Higgs bosons \( O_{W,HW} \)

- \( B_{\mu\nu} \) is a singlet of the global symmetry
- \( SO(4) \) symmetry further kills \( O_{B,HB} \)

One extra operator

\[
O_H \sim g_*^2
\]

Dimension-8 operators enhanced by \( g_*^2 \)

\[
8O_{HW\bar{W}} = D_\mu H^\dagger D_\nu H W_\rho^{a\mu} W^{av\rho}, \quad 8O_{HBB} = D_\mu H^\dagger D_\nu H B_\rho^{\mu} B^{\nu\rho}
\]
If we give up UV completion within QFT, the non-compact group can be considered:

\[ \mathcal{G} = [ISO(4)]_{global} \rtimes [U(1)^4]_{local}, \]

The Higgs are living in the flat coset \( ISO(4)/SO(4) \):

\[ H \rightarrow H + c, \quad H \rightarrow RH \]

which kills \( O_H \).

\((3, 1)\) is an irreducible representation of \( SO(4) \)

\[ O_{HW} \sim g_*^2 \]
Remedios + ISO(4)

The phenomenology:

\[
\delta g_{1}^{Z} = \frac{\delta \kappa_{\gamma}}{\cos^{2} \theta_{W}} = \frac{\delta g_{hZ\gamma}}{\sin \theta_{W} \cos \theta_{W}} = -\frac{m_{Z}^{2}}{m_{*}^{2}} \frac{c_{HW}}{g} \sim \frac{m_{Z}^{2}}{m_{*}^{2}} \frac{g_{*}}{g}
\]

\[
\lambda_{\gamma} = \frac{m_{W}^{2}}{m_{*}^{2}} \frac{c_{3W}}{g} \sim \frac{m_{W}^{2}}{m_{*}^{2}} \frac{g_{*}}{g}
\]

where our convention

\[
\delta \mathcal{L}_{hZ\gamma} = \delta g_{hZ\gamma} \frac{h}{\sqrt{2}} Z_{\mu \nu} A^{\mu \nu}
\]
\( \mathcal{G} \) breaking effects

The source of breaking:

\[
\mathcal{L}_{\text{break}} = -\epsilon_t g_* \left[ \bar{Q}_L \hat{H} t_R + \ldots \right] + \epsilon_2 m_*^2 \left[ |H|^2 + \ldots \right] - \epsilon_4 \frac{g_*^2}{2} \left[ |H|^4 + \ldots \right]
\]

with the following identification:

\[
y_t \equiv \epsilon_t g_*, \quad \epsilon_2 m_*^2 \equiv m_H^2, \quad \epsilon_4 g_*^2 \equiv \lambda_h
\]

The normalization of the couplings:

\[
\Delta \mathcal{L}_{\psi\psi}^h = \left( \frac{h}{v} \right) \left( \delta g_{h\psi\psi} m_{\psi} \bar{\psi} \psi + \text{h.c.} \right)
\]

\[
\Delta \mathcal{L}_{\gamma\gamma}^h = \left( \frac{h}{v} \right) \delta g_{h\gamma\gamma} F_{\mu\nu} F^{\mu\nu}
\]

\[
\Delta \mathcal{L}_{VV}^h = \left( \frac{h}{v} \right) \delta g_{hVV} m_W^2 \left( W^{+\mu} W^-_{\mu} + \frac{Z_{\mu} Z_{\mu}}{2 \cos^2 \theta_W} \right)
\]
\( \mathcal{G} \) breaking effects

The first class: \( H^\dagger \partial^4 H/m_*^2 \)

- No field strength \(|\Box H|^2/m_*^2\): by field redefinition,

\[
\begin{align*}
    c_6 & \sim \lambda_h^2, \quad c_{4\psi} \sim y_\psi^2 \\
    c_{y_\psi} & \sim y_\psi \lambda_h \quad \Rightarrow \quad \delta g_{h\psi\psi} \sim \frac{m_h^2}{m_*^2}
\end{align*}
\]

- One field strength:

\[
\begin{align*}
    c_B & \sim g', \quad c_W \sim g \quad \Rightarrow \quad \delta \hat{S} \sim \frac{m_W^2}{m_*^2}
\end{align*}
\]

- Two field strengths:

\[
\begin{align*}
    c_{BB} & \sim g''^2 \quad \Rightarrow \quad \delta g_{h\gamma\gamma} \sim \frac{e^2 v^2}{m_*^2}
\end{align*}
\]
$G$ breaking effects

The second class: SM operators + derivatives

$$c_H \sim \lambda_h \implies \delta g_{hVV} \sim \frac{m_h^2}{m_*^2}$$

The third class: Loops of SM fields ($\Delta l_c = 2$)

$$c_T \sim \left(\frac{g_*}{4\pi}\right)^2 \times g'^2 \implies \delta \hat{T} \sim \left(\frac{g_*}{4\pi}\right)^2 \times \tan^2 \theta_W \frac{m_W^2}{m_*^2}$$

$$c_T \sim \frac{y_t^4}{16\pi^2} \implies \delta \hat{T} \sim \left(\frac{y_t}{4\pi}\right)^2 \times \frac{m_t^2}{m_*^2}$$
**Remedios Scenario**

In summary:

<table>
<thead>
<tr>
<th>Model</th>
<th>$O_{2V}$</th>
<th>$O_{3V}$</th>
<th>$O_{HW}$</th>
<th>$O_{HB}$</th>
<th>$O_{V}$</th>
<th>$O_{VV}$</th>
<th>$O_{H}$</th>
<th>$O_{y\psi}$</th>
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<tbody>
<tr>
<td>Remedios</td>
<td>1</td>
<td>$g_*$</td>
<td></td>
<td></td>
<td>$g'$</td>
<td>$g_v$</td>
<td>$g^2_v$</td>
<td>$g_*$</td>
</tr>
<tr>
<td>Remedios+MCHM</td>
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<td>$y_\psi g^2_*$</td>
</tr>
<tr>
<td>Remedios+ISO(4)</td>
<td>1</td>
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<td>$g_*$</td>
<td>$g'$</td>
<td>$g_v$</td>
<td>$g^2_v$</td>
<td>$\lambda_h$</td>
<td>$y_\psi \lambda_h$</td>
</tr>
</tbody>
</table>
Partially Composite Fermions

Assuming the family symmetry, the best way to look at the fermion compositeness is $\psi \psi \rightarrow \psi \psi$:

$$\delta A(\psi \psi \rightarrow \psi \psi) \simeq \epsilon_\psi^4 g_*^2 \frac{E^2}{m_*^2},$$

The bound from LHC Run1 (arXiv:1201.6510):

$$m_* \gtrsim (g_* \epsilon_\psi^2 / 4\pi) \times 60 \text{ TeV}$$

It seems difficult to have fully composite fermions:

$$\epsilon_\psi \sim 1$$
Partially Composite Fermions + Higgs compositeness

If Higgs is also composite, processes like:

$$\bar{\psi}\psi \rightarrow V_L V_T / V_L h$$

are also relevant to probe the scenario.

But, the operators

$$O^{(3)}_L = (iH^\dagger \sigma^a D_\mu H)(\bar{\psi}_L \gamma^\mu \psi_L)$$

are constrained by LEP-I Z-pole physics:

$$m_\ast \gtrsim (g_\ast \epsilon_\psi / 4\pi) \times 40 \text{ TeV}$$
Fermions as composite Pseudo-Goldstini

Can we have soft-IR fermions?

A first attempt:

\[ \psi \rightarrow \psi + \xi \]

The operators starting from dimension-10, the amplitude growing as:

\[ \delta A \propto s^3 \]

disfavored by basic principles (unitarity and analyticity).

A non-linearly realized SUSY can do the job!

\[ \delta \psi = \xi + \frac{i}{2F^2} \partial_\mu \psi (\bar{\psi} \gamma^\mu \xi - \bar{\xi} \gamma^\mu \psi) \]
Fermions as composite Pseudo-Goldstini

The operators starting from dimension-8

\[
\frac{i}{F^2} \bar{\psi} (\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) \psi F_{\mu \rho} F_{\nu \rho}, \quad \frac{i}{F^2} \partial_\mu \phi^\dagger \partial_\nu \phi \bar{\psi} (\gamma^\mu \partial^\nu + \gamma^\nu \partial^\mu) \psi,
\]

\[
\frac{1}{F^2} \bar{\psi}^2 \partial^2 \psi^2, \quad \frac{1}{F^2} \partial_\nu \bar{\psi} \gamma^\mu \psi \bar{\psi} q \gamma_\mu \partial^\nu \psi q, \quad \frac{1}{F^2} \partial_\nu \bar{\psi} q \gamma^\mu \psi \bar{\psi} \gamma_\mu \partial^\nu \psi q.
\]

We can identify:

\[
F \sim m^2_*/g_*
\]

Generations to $\mathcal{N} > 1$ is also possible.
Fermions as composite Pseudo-Goldstini

The phenomenological consequences:

\[
\delta A(\bar{\psi}\psi \rightarrow \psi\psi) \simeq g_*^2 \frac{E^4}{m_*^4},
\]

\[
\delta A(\bar{\psi}\psi \rightarrow V_L V_L) \simeq g_*^2 \frac{E^4}{m_*^4} \left(\frac{g^2 E^2}{m_*^2}\right).
\]

\[
\delta A(\bar{\psi}\psi \rightarrow V_T V_T) \simeq g_*^2 \frac{E^4}{m_*^4} \left(\frac{g g_* E^2}{m_*^2}\right).
\]

The dimension-8 dominates over dimension-6 whenever

\[
E \gtrsim \sqrt{g/g_* m_*}
\]

More importantly, they give sizable contribution to neutral diboson pair production!
Conclusion

- It is still possible to make the SM degrees of freedom emerging from a strong dynamics above the TeV scale.
- We have constructed the effective Lagrangians for the transverse gauge bosons involving in the strong dynamics through multi-pole interactions.
- We also combined the scenario (Remedios) with the composite Higgs models, motivated by naturalness consideration.
- The Fermions can also get involved as pseudo-Goldstini.
- Our scenario motivated several precision measurements (VH,VV) at the LHC, where dimension-8 operators dominates over dimension-6.
Dimension-8 operators

\[(X_{\mu \nu})^4\]

\[\text{SU}(2)_L: \quad 8\, O_{4W} = W^a_{\mu \nu} W^{a\mu \nu} W^b_{\rho \sigma} W^{b\rho \sigma}\]
\[8\, O'_{4W} = W^a_{\mu \nu} W^b_{\mu \nu} W^a_{\rho \sigma} W^{b\rho \sigma}\]
\[8\, O'_{4\tilde{W}} = W^a_{\mu \nu} W^{a\nu \rho} W^b_{\rho \sigma} W^{b\sigma \mu}\]
\[8\, O'_{4\tilde{W}} = W^a_{\mu \nu} W^b_{\nu \rho} W^a_{\rho \sigma} W^{b\sigma \mu}\]

\[\text{U}(1)_Y: \quad 8\, O_{4B} = B_{\mu \nu} B^{\mu \nu} B_{\rho \sigma} B^{\rho \sigma}\]
\[8\, O_{4\tilde{B}} = B_{\mu \nu} B^{\nu \rho} B_{\rho \sigma} B^{\sigma \mu}\]

\[\text{SU}(2)_L \times \text{U}(1)_Y: \quad 8\, O_{2WB} = W^a_{\mu \nu} W^{a\mu \nu} B_{\rho \sigma} B^{\rho \sigma}\]
\[8\, O'_{2WB} = W^a_{\mu \nu} B^{\mu \nu} W^a_{\rho \sigma} B^{\rho \sigma}\]
\[8\, O'_{2\tilde{WB}} = W^a_{\mu \nu} W^{a\nu \rho} B_{\rho \sigma} B^{\sigma \mu}\]
\[8\, O'_{2\tilde{WB}} = W^a_{\mu \nu} B^{\nu \rho} W^a_{\rho \sigma} B^{\sigma \mu}.\]
Dimension-8 operators

\[ D_\psi^2 (X_{\mu \nu})^2 \] Strongly interacting fermions and vectors generate

\[ 8 \mathcal{O}_{TWW} = T^{\mu \nu} W^a_{\mu \rho} W^a_{\nu \rho} \quad 8 \mathcal{O}_{TBB} = T^{\mu \nu} B_{\mu \rho} B_{\nu}^\rho \]
\[ 8 \mathcal{O}_{TWB} = T^a_{\mu \nu} W^a_{\mu \rho} B_{\nu}^\rho \]

where \( T^{\mu \nu} = \frac{i}{4} \bar{\psi} (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi \) and \( T^a_{\mu \nu} = \frac{i}{4} \bar{\psi} (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \sigma^a \psi \) for SU(2)_L doublets.

\[ D^4 H^4 \] In models where the Higgs is composite,

\[ 8 \mathcal{O}_{\{D\}H} = (D_{\{\mu} H^\dagger D_{\nu\}} H)^2 \quad 8 \mathcal{O}_{DH} = (D_\mu H^\dagger D^\mu H)^2 \]
Dimension-8 operators

\[ D^2 H^2 (X_{\mu\nu})^2 \] On the other hand,

\[ 8\mathcal{O}_{HWW} = D_\mu H^\dagger D_\nu H W^a_\mu W^{a\nu\rho} , \quad 8\mathcal{O}_{HBB} = D_\mu H^\dagger D_\nu H B^\mu_\rho B^{\nu\rho} \]
\[ 8\mathcal{O}'_{HWW} = D_\mu H^\dagger \sigma^a D_\nu H W^b_\rho W^{c\nu\rho} \epsilon^{abc} \]
\[ 8\mathcal{O}_{HWB} = D_\mu H^\dagger \sigma^a D_\nu H W^a_\rho B^{\nu\rho} \]

\[ D^3 H^2 \psi^2 \] If the fermions are pseudo-Goldstini,

\[ 8\mathcal{O}_{TH} = T^{\mu\nu} D_\mu H^\dagger D_\nu H \]