Anisotropic Hydrodynamics

Theory and Phenomenology

Michael Strickland
Kent State University

Peking University – Institute of Theoretical Physics

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KENT STATE UNIVERSITY

U.S. DEPARTMENT OF ENERGY

- At the Relativistic Heavy Ion Collider (**RHIC**) @ Brookhaven National Lab (**BNL**) scientists concluded that the **quark-gluon plasma** (QGP) behaves like a "nearly perfect fluid"

- Experiments continue to this day at RHIC and started in 2010 at even higher energies at the Large Hadron Collider (**LHC**) @ **CERN**.

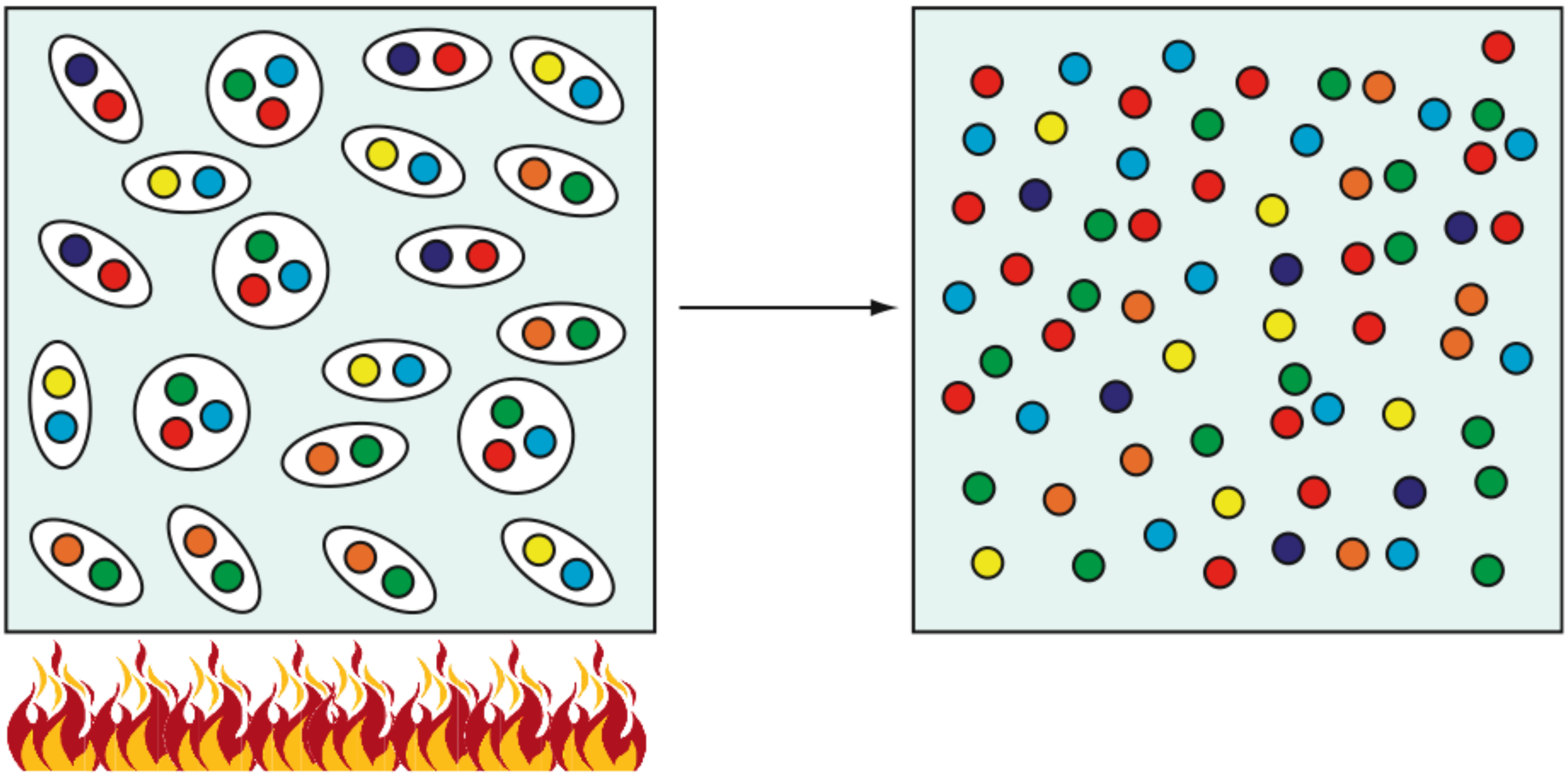
QGP thermodynamics

$T \lesssim 10^{12}$ Kelvin

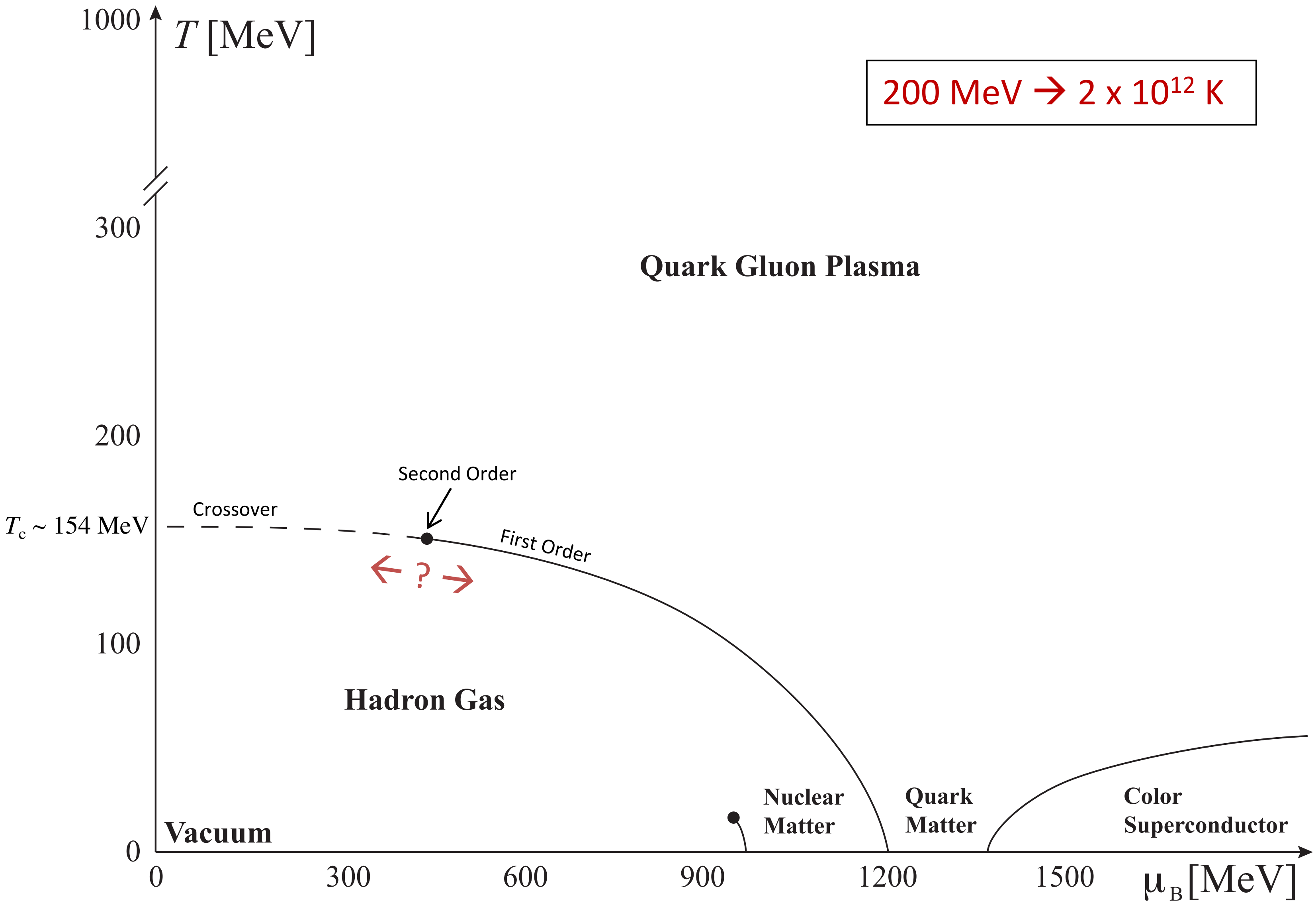
Protons, Neutrons, Pions, etc.

$T \gtrsim 10^{12}$ Kelvin

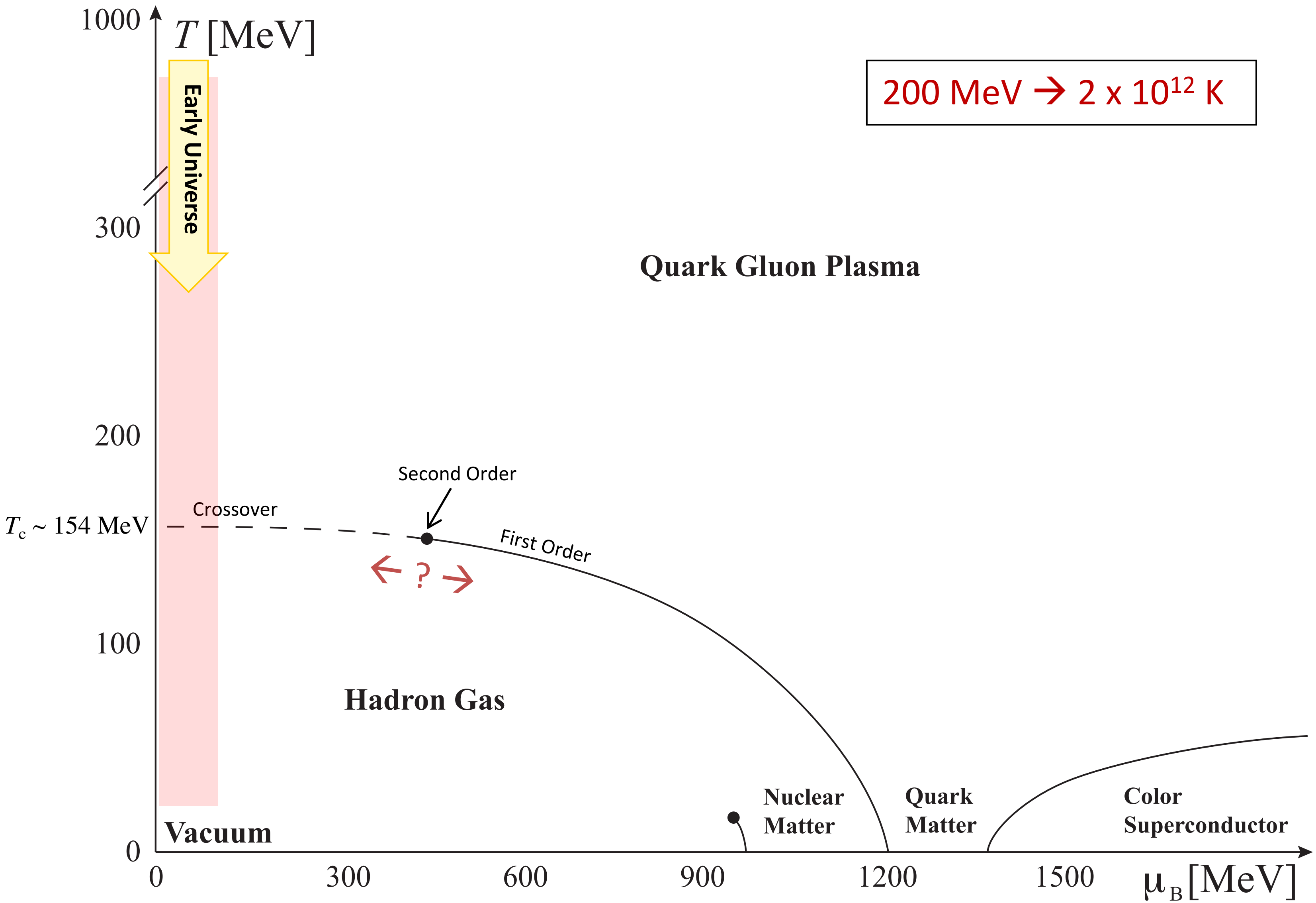
Quark Gluon Plasma (QGP)



QCD phase diagram

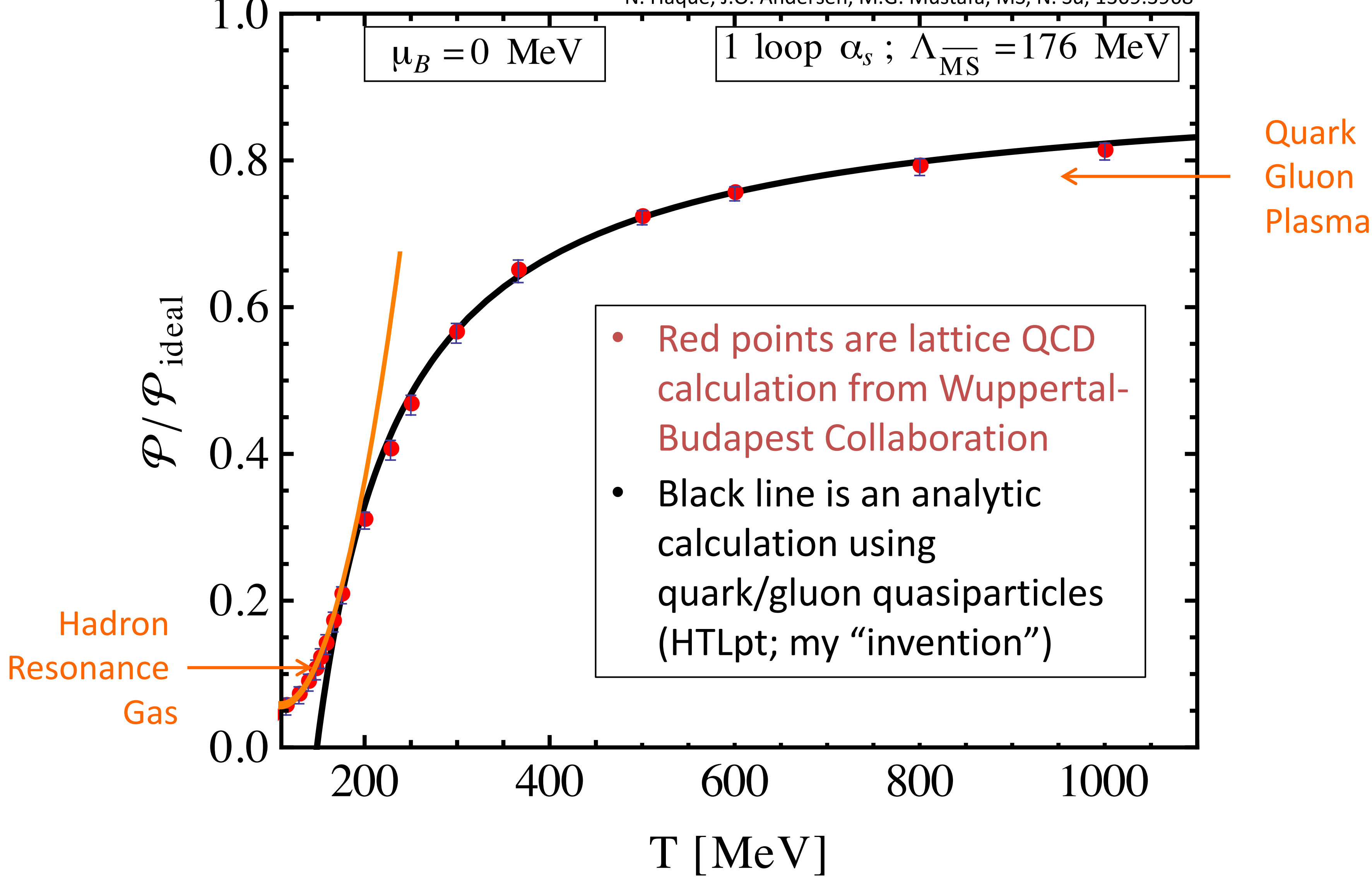


QCD phase diagram



Pressure vs temperature – μ_B = 0 MeV

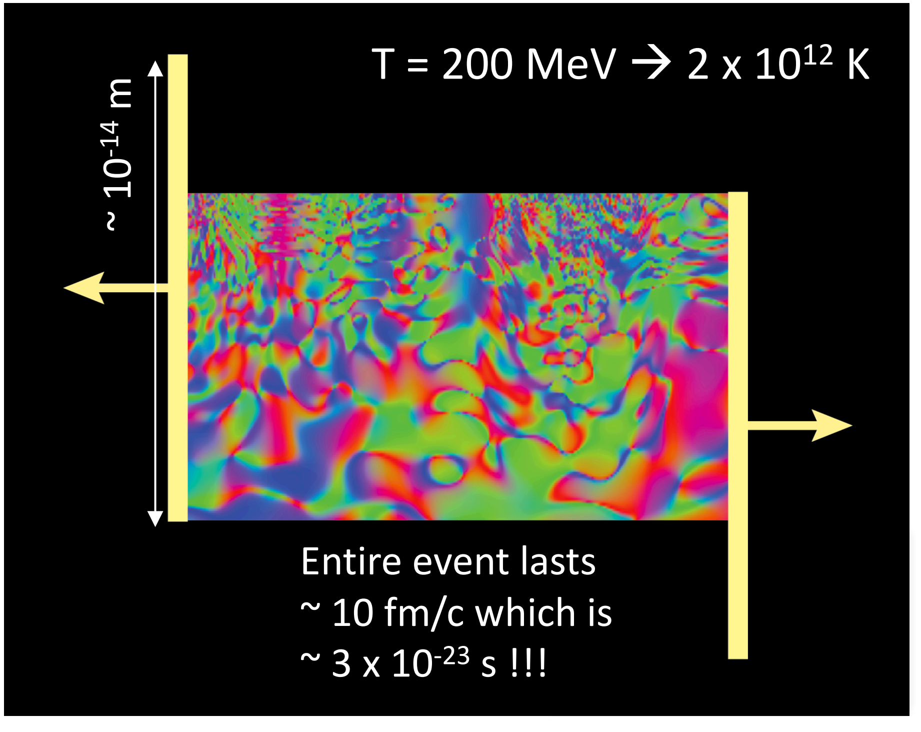
Andersen, Leganger, Su, and MS 1009.4644, 1103.2528
N. Haque, J.O. Andersen, M.G. Mustafa, MS, N. Su, 1309.3968

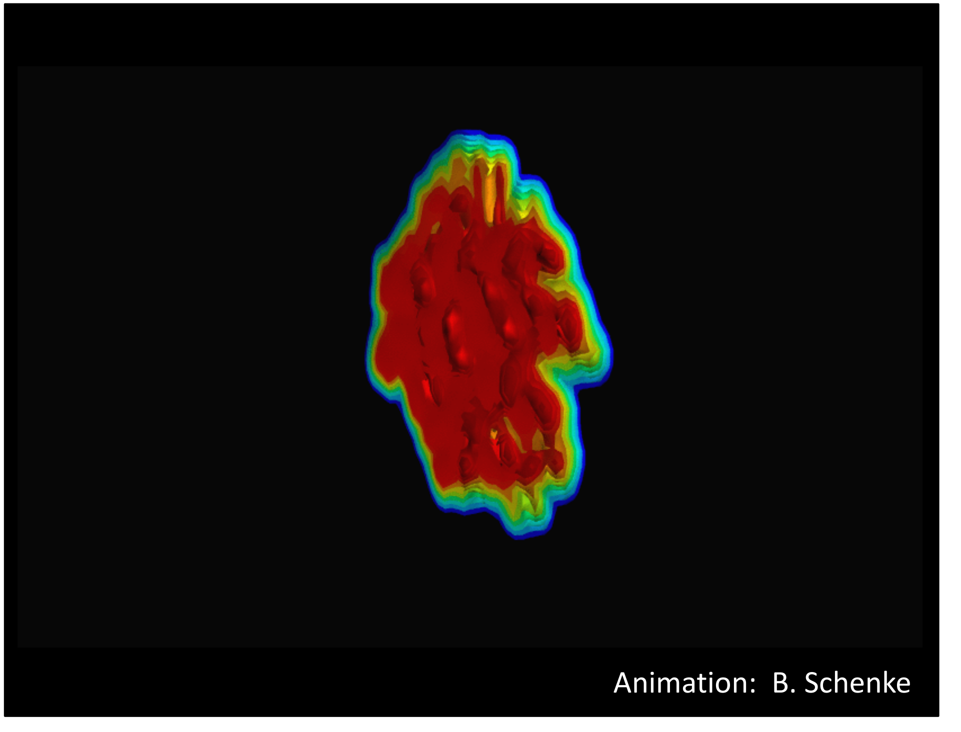


- Red points are lattice QCD calculation from Wuppertal-Budapest Collaboration
- Black line is an analytic calculation using quark/gluon quasiparticles (HTLpt; my "invention")

High-energy ultrarelativistic heavy-ion collisions

- **RHIC**, BNL – Au-Au @ 200 GeV/nucleon (highest energy) → T_0 ~ 400 MeV
- **LHC**, CERN – Pb-Pb @ 2.76 TeV → T_0 ~ 600 MeV
- **LHC**, CERN – Pb-Pb @ 5.02 TeV → T_0 ~ 700 MeV
- **RHIC**, BNL **BES** – Au-Au @ 7.7 - 39 GeV → T_0 ~ 30-100 MeV [+finite density]
- **FAiR** (GSI), **NICA** (Dubna) – U-U @ 35 GeV -> T_0 ~ 100 MeV [+finite density]





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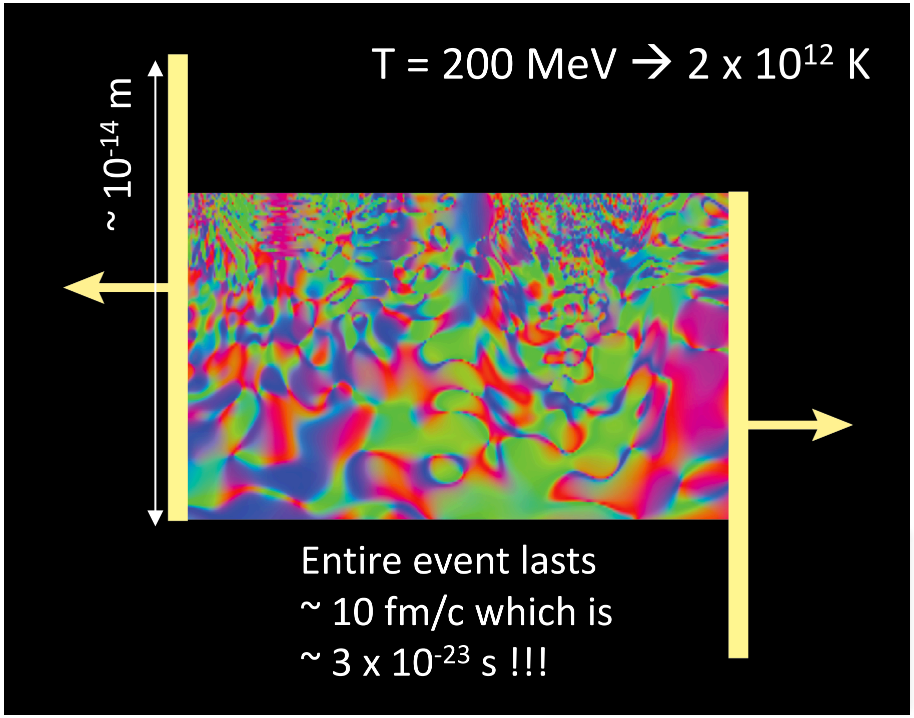

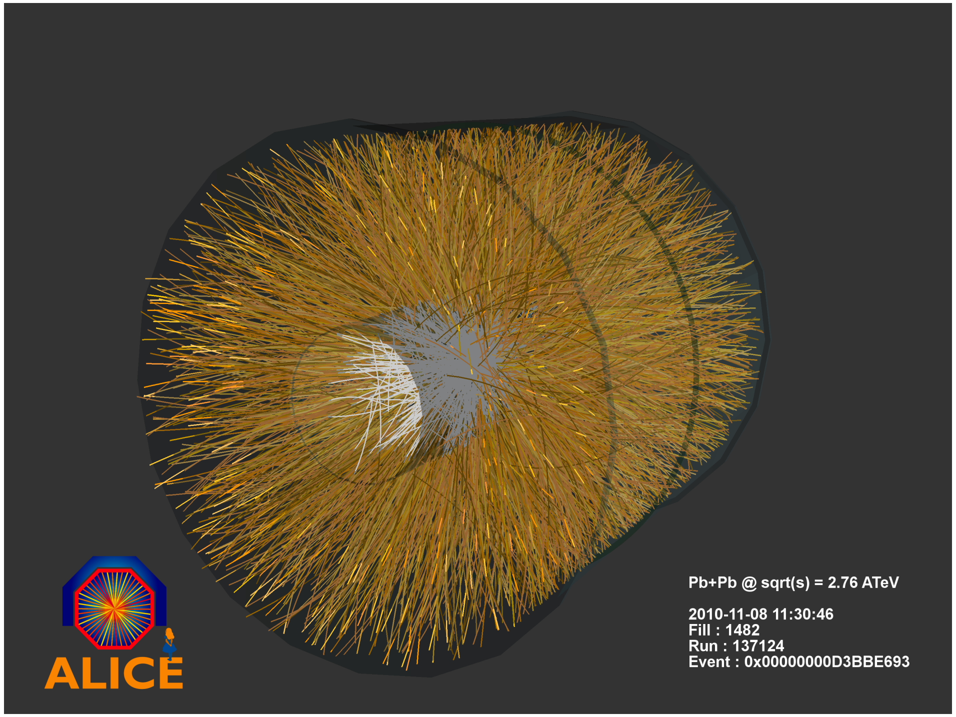

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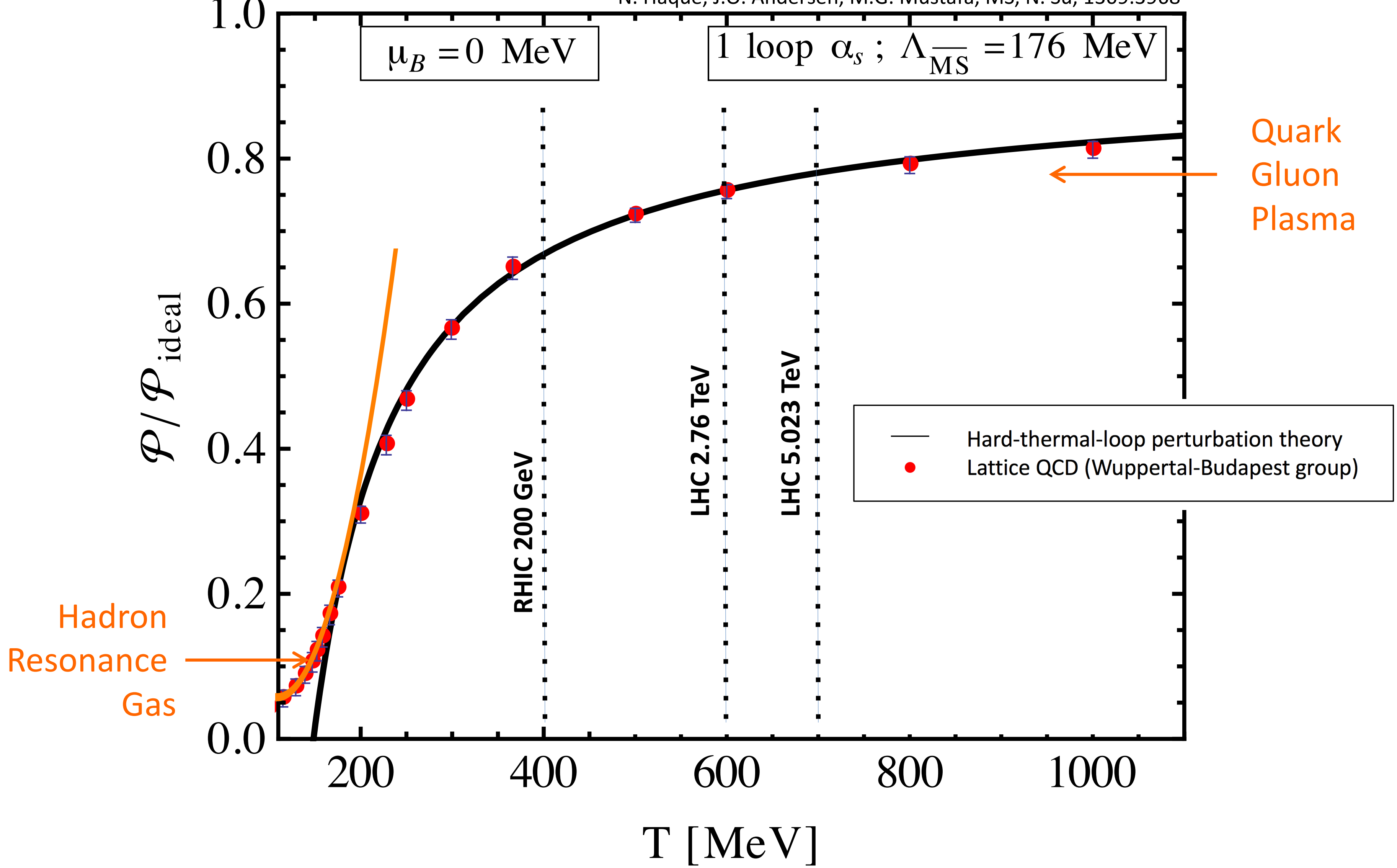
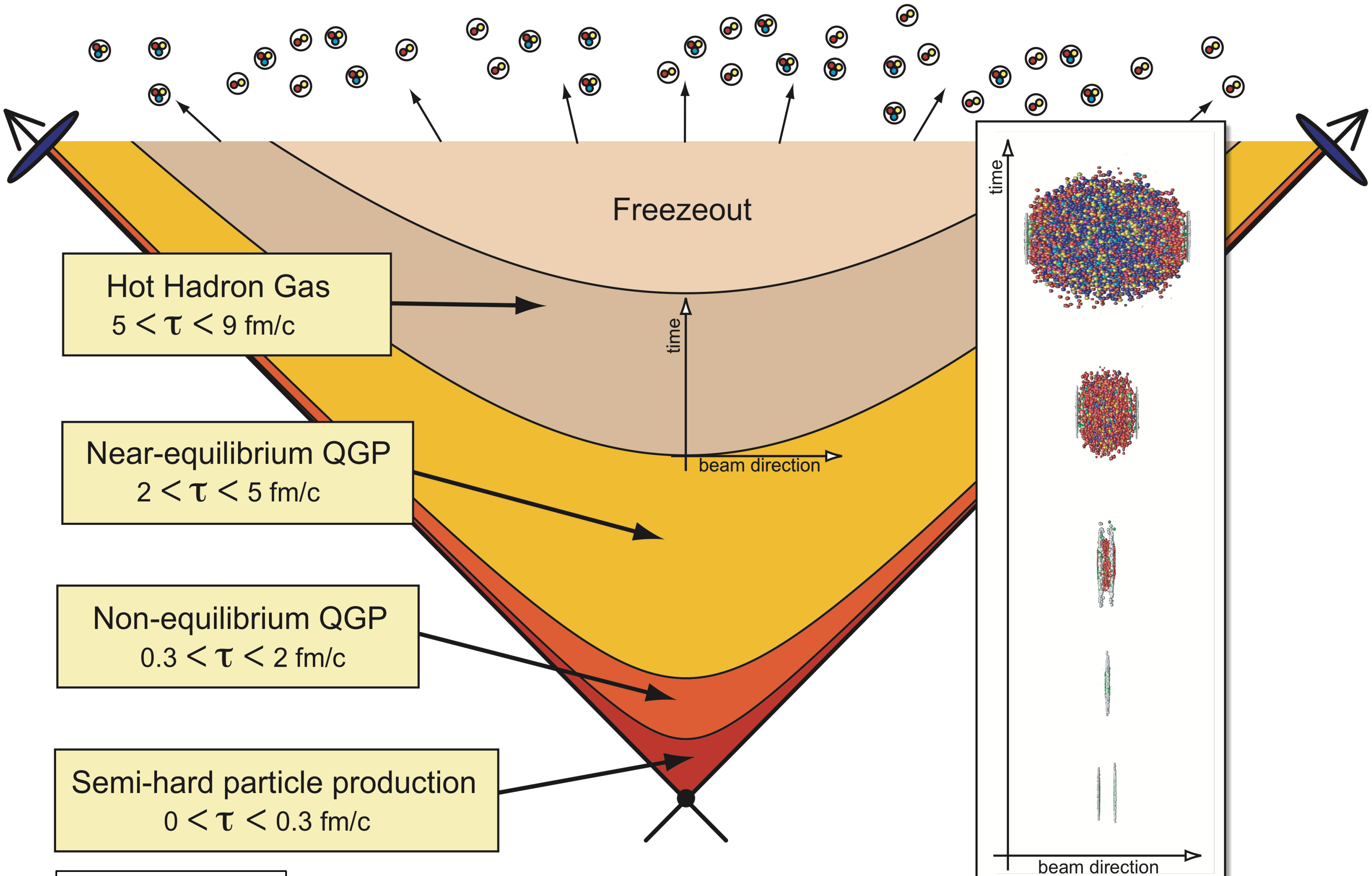


QGP dynamics

RHIC heavy-ion collision timescales

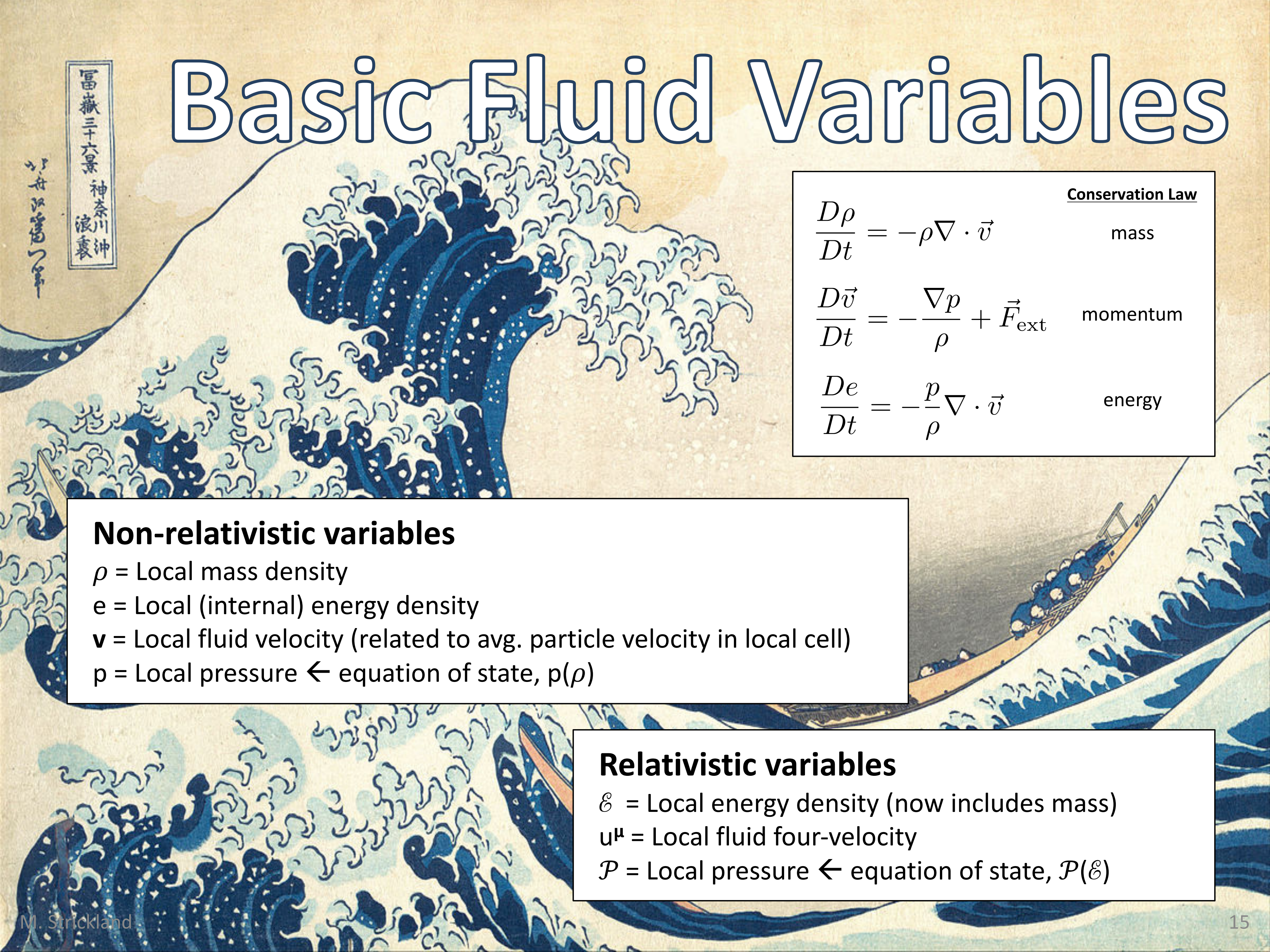


1 fm/c ≅ 3 x 10^{-24} s

How can we understand QGP "fluidity"?

- The statement that the QGP behaves like a nearly perfect fluid comes from the success of "**hydrodynamical models**" in describing experimental observables.
- One way to view this is that **hydrodynamics is a kind of universal effective theory that describes the long wavelength dynamics** of any system.
- The catch, however, is that traditional hydrodynamics equations are derived in the context of a **near-equilibrium** system.
- Today, I would like to present a different view: That hydrodynamics emerges as an efficient approximation to the full kinetic theory of the QGP which can be applied **far from equilibrium**.
- The goal of the **anisotropic hydrodynamics (aHydro)** program is to provide an optimized framework that is **more accurate out of equilibrium** and optimized for heavy-ion collisions.

Need to be careful how we define fluid-like behavior!



Basic Fluid Variables

	Conservation Law
$\frac{D\rho}{Dt} = -\rho\nabla\cdot\vec{v}$	mass
$\frac{D\vec{v}}{Dt} = -\frac{\nabla p}{\rho} + \vec{F}_{\rm ext}$	momentum
$\frac{De}{Dt} = -\frac{p}{\rho}\nabla\cdot\vec{v}$	energy

Non-relativistic variables

ρ = Local mass density
e = Local (internal) energy density
v = Local fluid velocity (related to avg. particle velocity in local cell)
p = Local pressure ← equation of state, p(ρ)

Relativistic variables

$\mathcal{E}$ = Local energy density (now includes mass)
u^μ = Local fluid four-velocity
$\mathcal{P}$ = Local pressure ← equation of state, $\mathcal{P}(\mathcal{E})$

The "ideal" energy-momentum tensor

The energy-momentum tensor describes the density and flux of energy and momentum in space time. It generalizes the stress tensor of Newtonian physics. **For a system that is in isotropic equilibrium, one has**

$$T^{\mu\nu}_{\text{ideal}} = (\mathcal{E} + \mathcal{P})u^\mu u^\nu - \mathcal{P} g^{\mu\nu}$$

- pressure: $\mathcal{P}$
- metric tensor: $g^{\mu\nu} = \underbrace{\text{diag}(1,-1,-1,-1)}_{\text{Flat Minkowski Space}}$
- energy density: $\mathcal{E}$
- fluid four-velocity, which satisfies $u^\mu u_\mu = 1$

In the local rest frame (LRF) $u^\mu = (1,0,0,0)$ and one has:

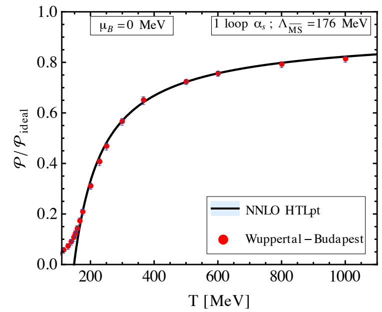
$$\longrightarrow \quad T^{\mu\nu}_{\text{ideal,LRF}} = \begin{pmatrix} \mathcal{E} & 0 & 0 & 0 \\ 0 & \mathcal{P} & 0 & 0 \\ 0 & 0 & \mathcal{P} & 0 \\ 0 & 0 & 0 & \mathcal{P} \end{pmatrix}$$

Ideal hydrodynamics – Equations of motion

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} = (\mathcal{E} + \mathcal{P}) u^\mu u^\nu - \mathcal{P} g^{\mu\nu}$$

- In ideal hydrodynamics, one assumes that the energy-momentum tensor is always in its ideal form.
- In this case, the equations of motion results from the requirement of energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$



Degrees of Freedom
1 : Energy Density
1 : Pressure
3 : Independent components of u^μ
5 : Total

Equations
4: ν = 0,1,2,3
1 : EQUATION OF STATE $T^\mu{}_\mu = \#$ → $\mathcal{P}(\mathcal{E})$
5 : Total

Viscous hydrodynamics

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \Pi^{\mu\nu}$$

viscous stress tensor

- Viscous stress tensor encodes corrections to ideal hydrodynamics.
- Non-equilibrium corrections can make the pressures (defined via T^{xx}, T^{yy}, and T^{zz}) anisotropic, i.e P_x != P_y != P_z.

Approximation: 1st order in gradients of u^ν → Relativistic Navier-Stokes Theory

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu}\Phi$$

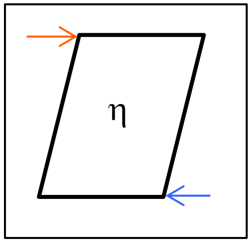
$$\pi^{\mu\nu} = \eta\nabla^{\langle\mu}u^{\nu\rangle}$$

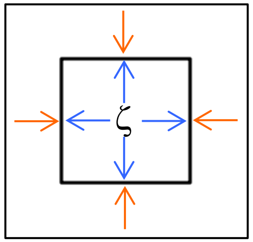
$$\Phi = \zeta\nabla_\alpha u^\alpha$$

*Angle brackets project out traceless symmetric part

η = Shear Viscosity

ζ = Bulk Viscosity





Relativistic Navier-Stokes theory is sick: Violates causality!!! To fix this problem, one must go to second order in gradients → **second-order viscous hydrodynamics**

Connection to kinetic theory

For small departures from equilibrium, we can linearize

$$f(x,p) = f_{\rm eq}\left(\frac{p^\mu u_\mu}{T}\right)(1+\delta f(x,p))$$

$$T^{\mu\nu}(x) = \int dP\, p^\mu p^\nu f(x,p)$$

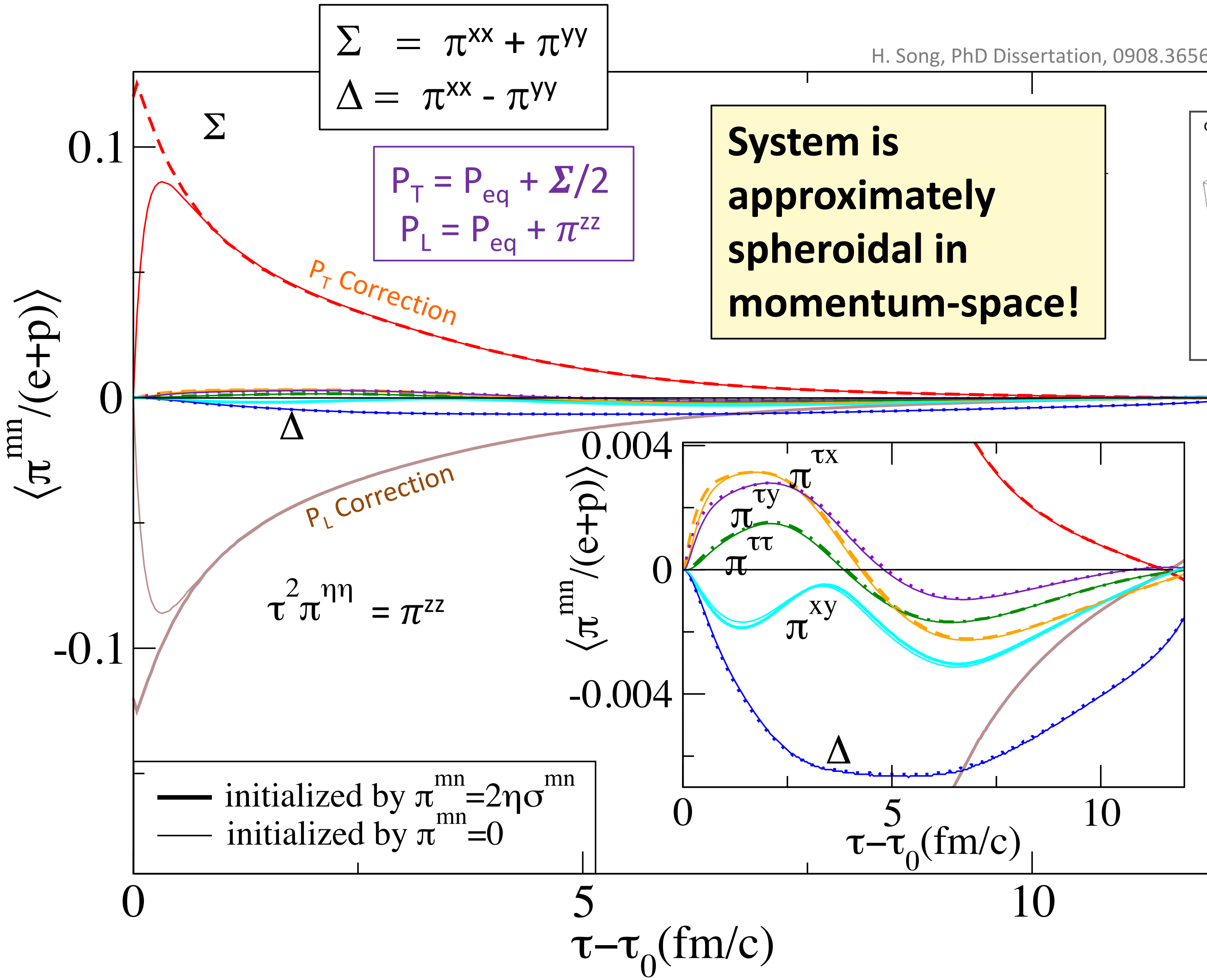
$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \int dP\; p^\mu p^\nu f_{\rm eq}\,\delta f \equiv T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu}$$

$$\longrightarrow \quad \Pi^{\mu\nu} = \int dP\; p^\mu p^\nu f_{\rm eq}\,\delta f$$

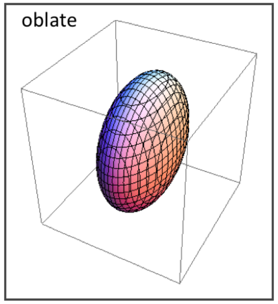
In standard viscous hydro, one expands δf in a **gradient expansion**:
n^th order in gradients → "n^th-order viscous hydrodynamics"

- 1st order Hydro : Relativistic Navier-Stokes (parabolic diff eqs → **acausal**)
  [e.g. Eckart and Landau-Lifshitz]
- **2nd order Hydro** : Including quadratic gradients **fixes causality problem**; hyperbolic diff eqs
  [e.g. Israel-Stewart, Chapman-Enskog, DNMR, etc.]
- …

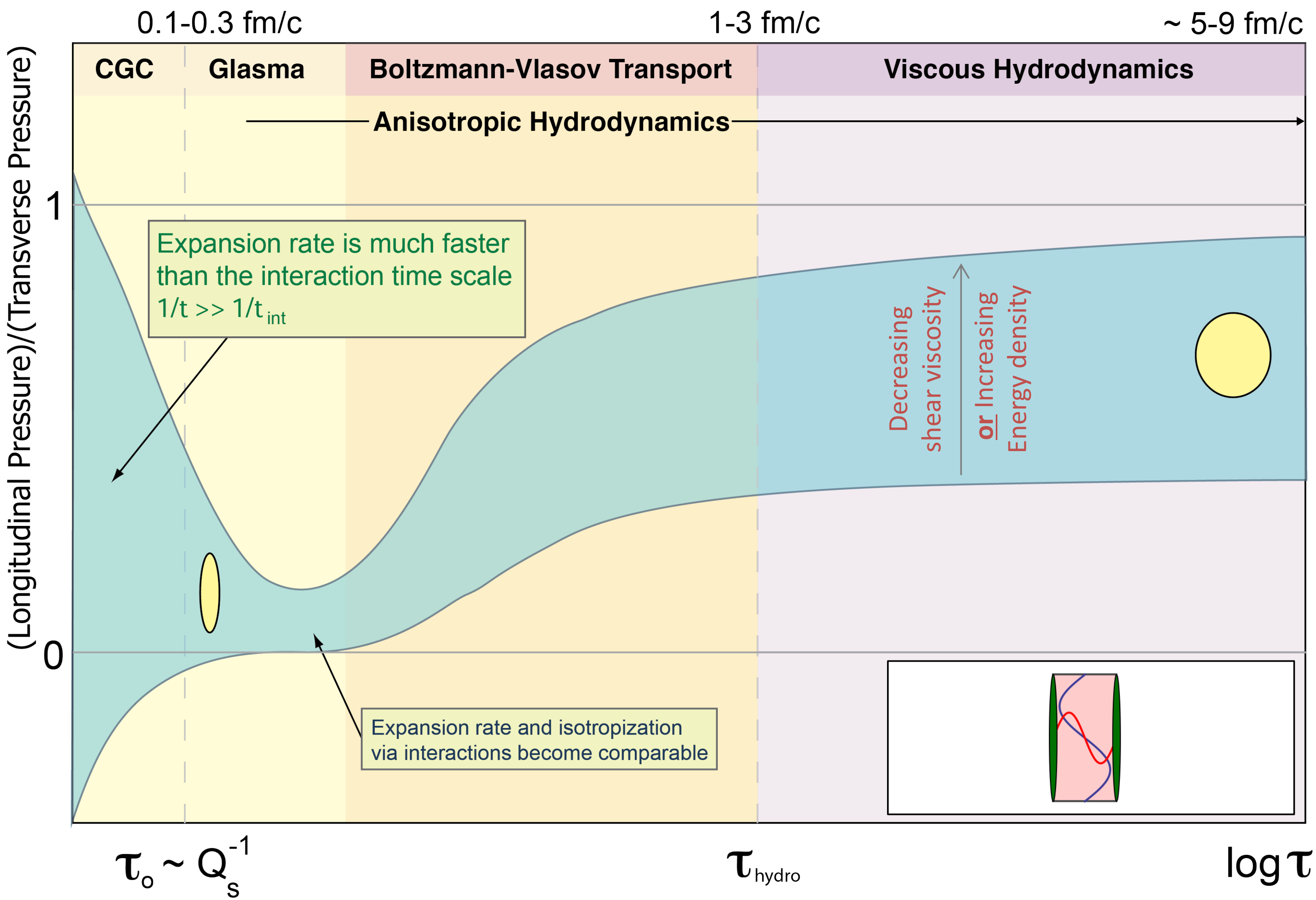
What are the largest viscous corrections?



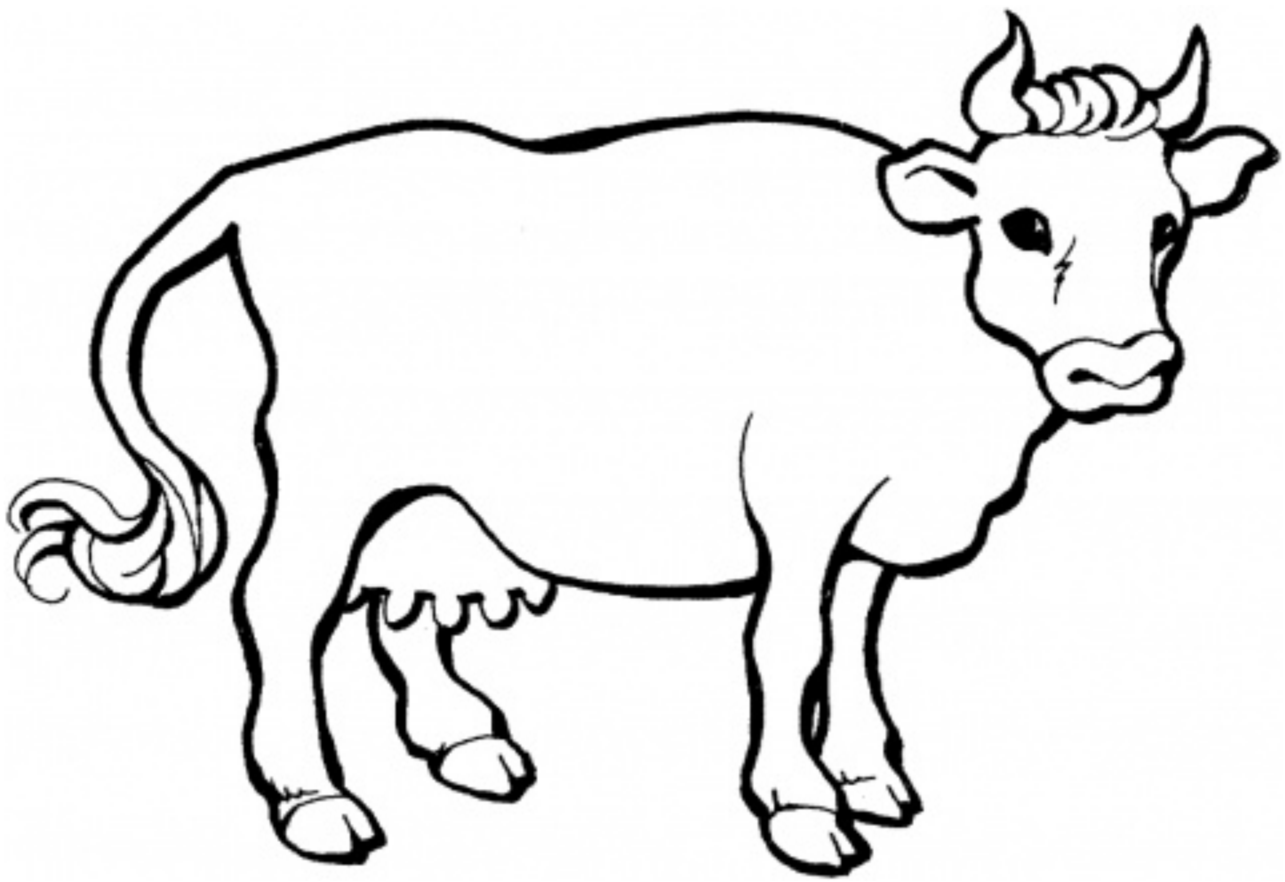
System is approximately spheroidal in momentum-space!



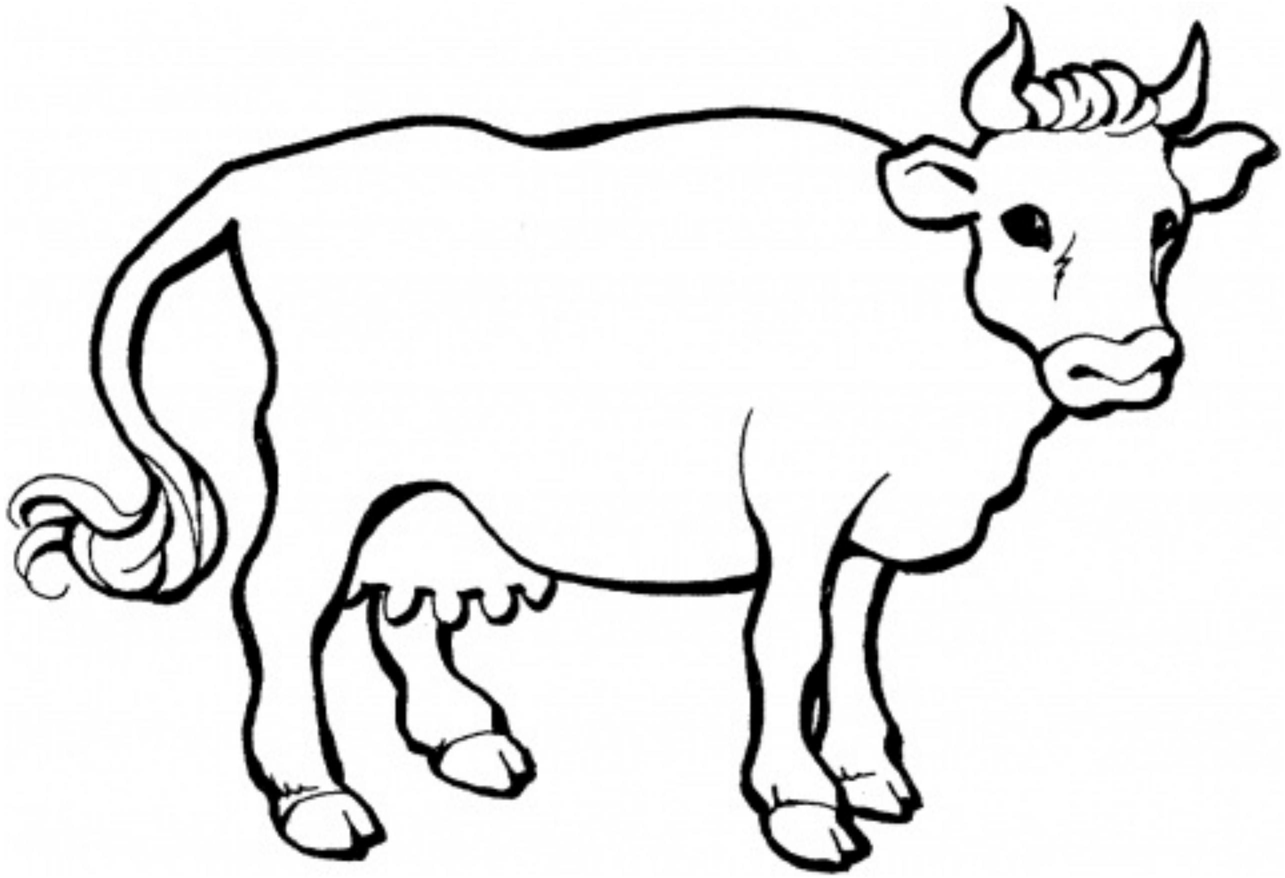
QGP momentum anisotropy cartoon



Physics 101



Cows are spheres?



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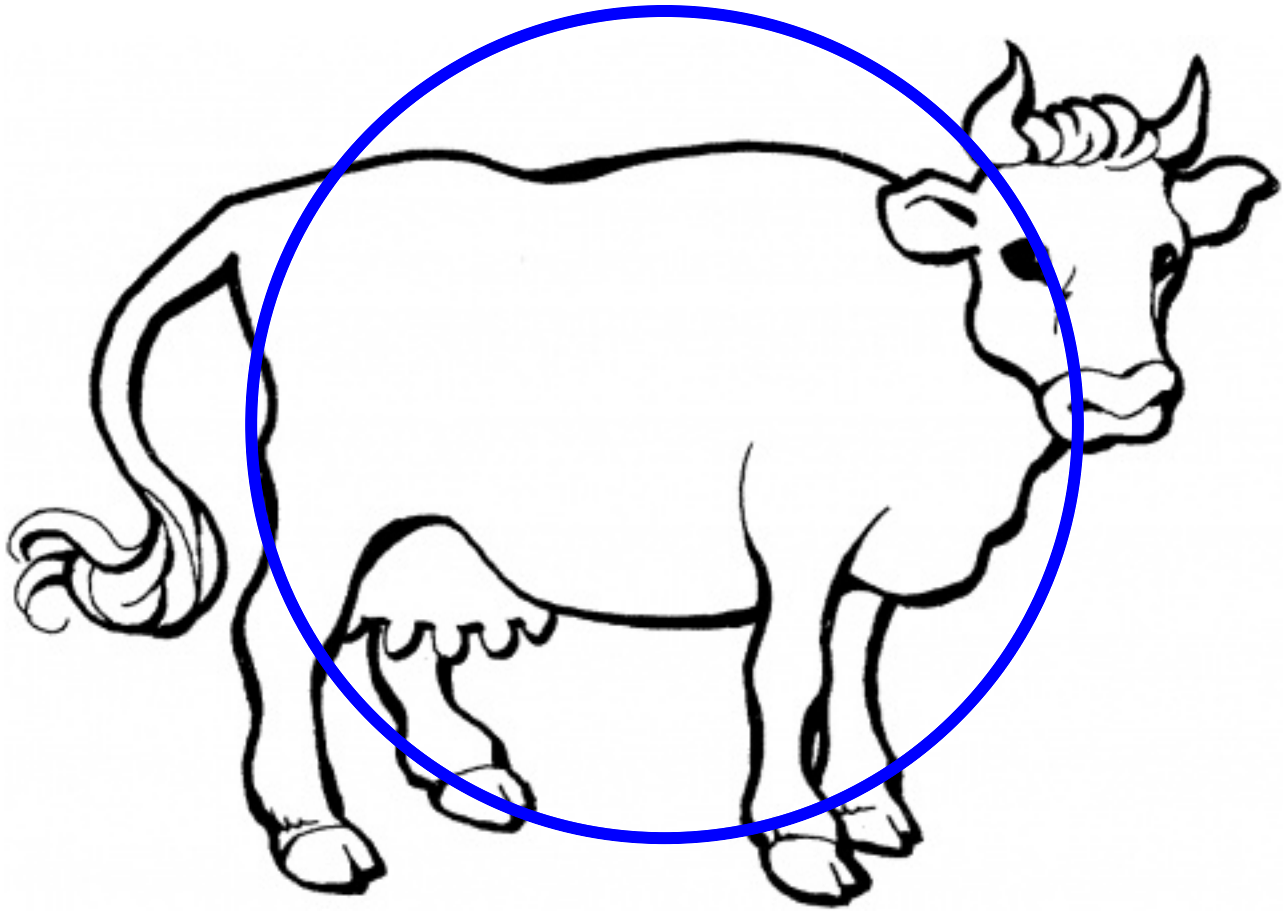
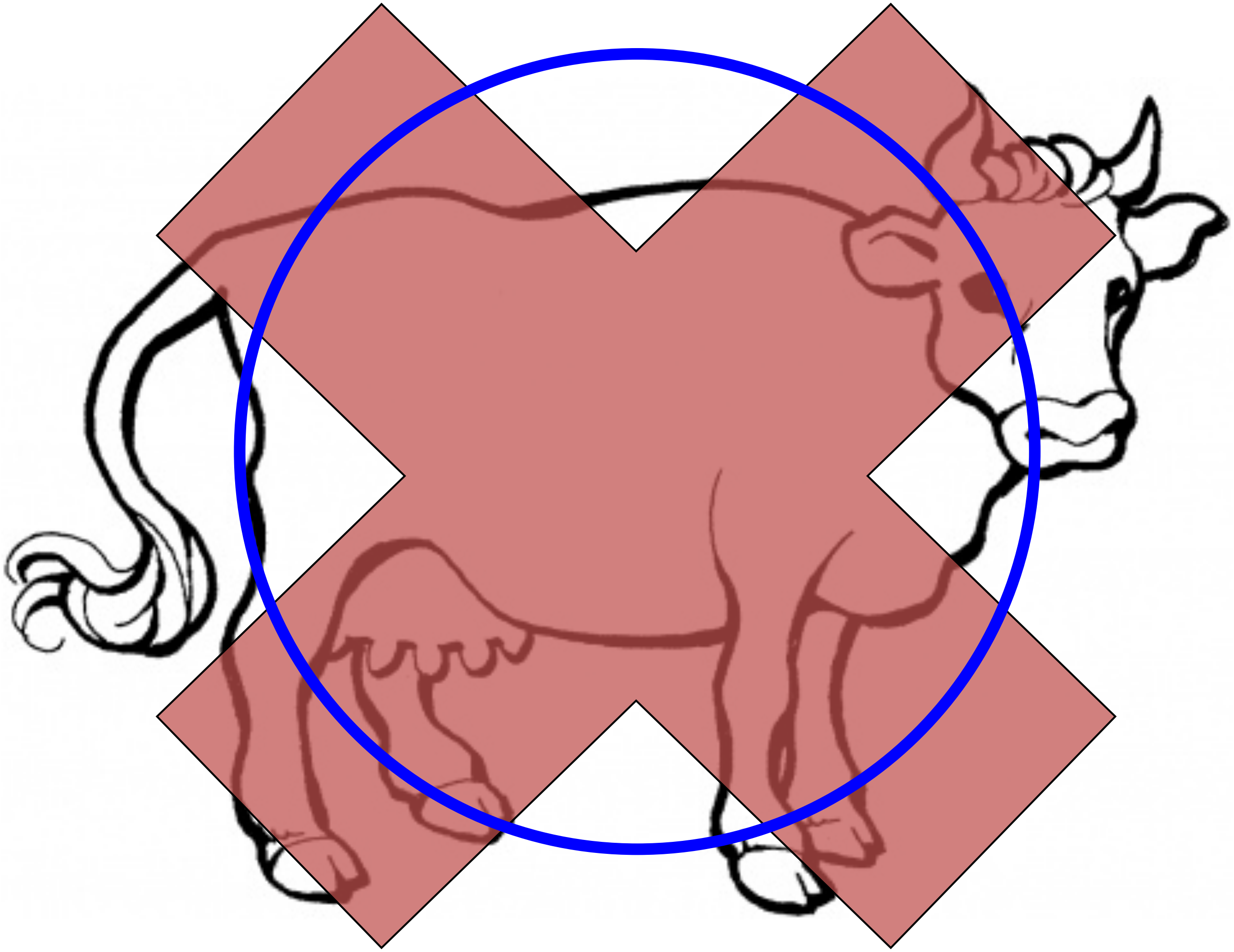
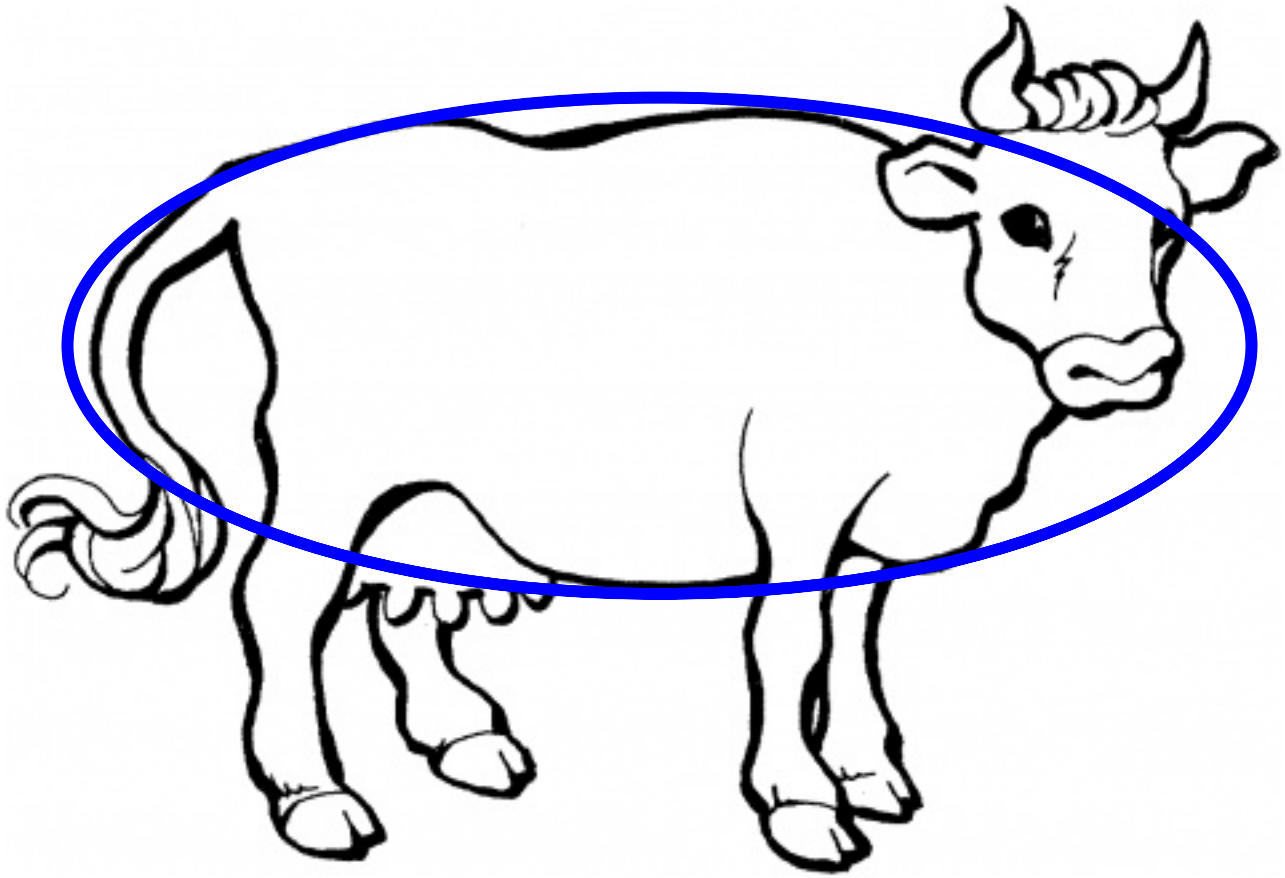


Cows are <u>not</u> spheres!



Cows are more like ellipsoids!



Spheroidal expansion method

See e.g.
- M. Martinez and MS, 1007.0889
- W. Florkowski and R. Ryblewski, 1007.0130
- D. Bazow, U. Heinz, and MS, 1311.6720
- D. Bazow, U. Heinz, and M. Martinez, 1503.07443
- E. Molnar, H. Niemi, and D. Rischke, 1602.00573; 1606.09019

Viscous Hydrodynamics Expansion

$$f(\tau,\mathbf{x},\mathbf{p}) = \underline{f_{\rm eq}(\mathbf{p},T(\tau,\mathbf{x}))} + \delta f$$

Isotropic in momentum space

Anisotropic Hydrodynamics (aHydro) Expansion

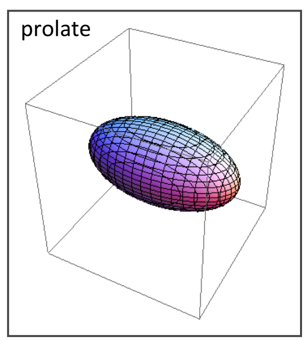
$$f(\tau,\mathbf{x},\mathbf{p}) = f_{\rm aniso}(\mathbf{p},\underbrace{\Lambda(\tau,\mathbf{x})}_{T_\perp},\underbrace{\xi(\tau,\mathbf{x})}_{\rm anisotropy}\,) + \delta\tilde{f}$$

Treat this term perturbatively
→ "NLO aHydro"

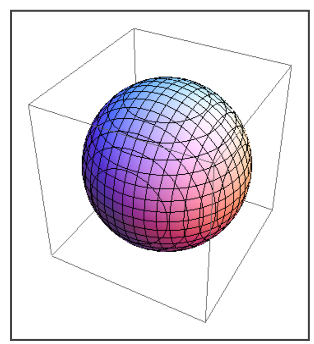
→ "Romatschke-Strickland" form in LRF

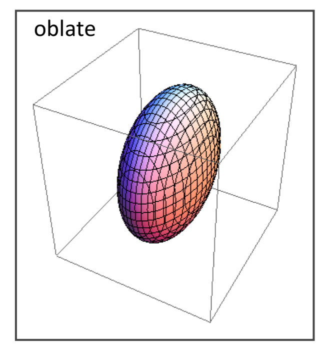
$$f^{LRF}_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2+\xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

$$\xi = \frac{\langle p_T^2\rangle}{2\langle p_L^2\rangle} - 1$$

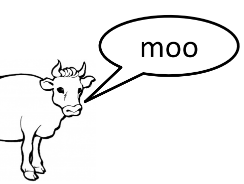


$-1 < \xi < 0$


$\xi = 0$



$\xi > 0$



Generalized aHydro formalism

In generalized aHydro, one assumes that the distribution function is of the form

$$f(x,p) = f_{\rm eq}\left(\frac{\sqrt{p^\mu \Xi_{\mu\nu}(x) p^\nu}}{\lambda(x)}, \frac{\mu(x)}{\lambda(x)}\right) + \delta\tilde{f}(x,p)$$

$$\Xi^{\mu\nu} = \underbrace{u^\mu u^\nu}_{\text{LRF four velocity}} + \underbrace{\xi^{\mu\nu}}_{\text{Traceless symmetric anisotropy tensor}} - \underbrace{\Delta^{\mu\nu}}_{\text{Transverse projector}}\underbrace{\Phi}_{\text{"Bulk"}}$$

$$u^\mu u_\mu = 1$$
$$\xi^\mu{}_\mu = 0$$
$$\Delta^\mu{}_\mu = 3$$
$$u_\mu \xi^{\mu\nu} = u_\mu \Delta^{\mu\nu} = 0$$

- 3 degrees of freedom in u^μ
- 5 degrees of freedom in $\xi^{\mu\nu}$
- 1 degree of freedom in Φ
- 1 degree of freedom in λ
- 1 degree of freedom in μ

→ 11 DOFs

See e.g.
- M. Martinez, R. Ryblewski, and MS, 1204.1473
- L. Tinti and W. Florkowski, 1312.6614
- M. Nopoush, R. Ryblewski, and MS, 1405.1355

Equations of motion

- The EOM are **obtained from moments of the Boltzmann equation** including a temperature-dependent quasiparticle mass which is fit to reproduce the **lattice equation of state**. Today, we work at zero net baryon density ($\mu=0$).

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

- 4 equations from the **1st moment** [energy-momentum conservation]
- 6 equations from the **2nd moment** [dissipative dynamics]
- Automatically includes effects of shear and bulk viscosity plus an infinite number of higher order transport coefficients!

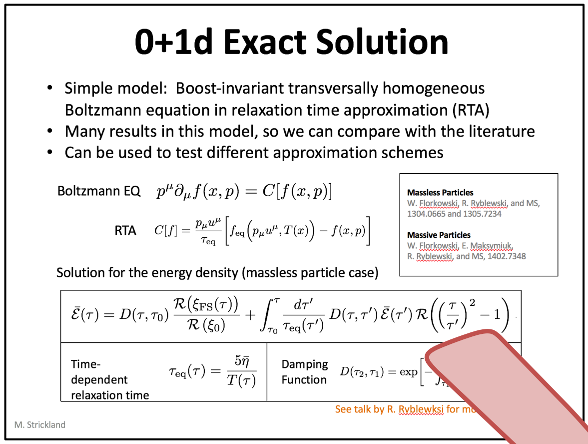
$$\partial_\mu T^{\mu\nu} = 0$$

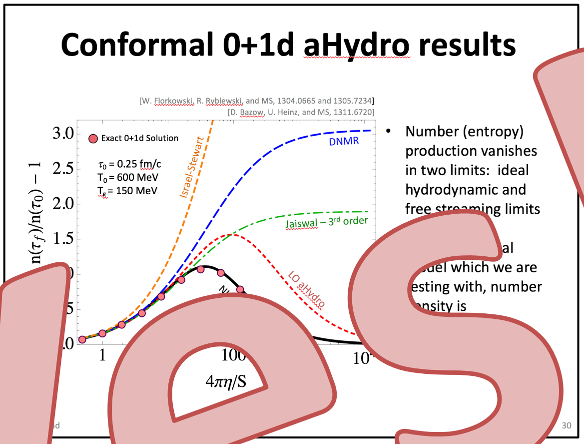
$$\partial_\mu \mathcal{I}^{\mu\nu\lambda} = \frac{1}{\tau_{\rm eq}}\left(u_\mu \mathcal{I}^{\mu\nu\lambda}_{\rm eq} - u_\mu \mathcal{I}^{\mu\nu\lambda}\right)$$

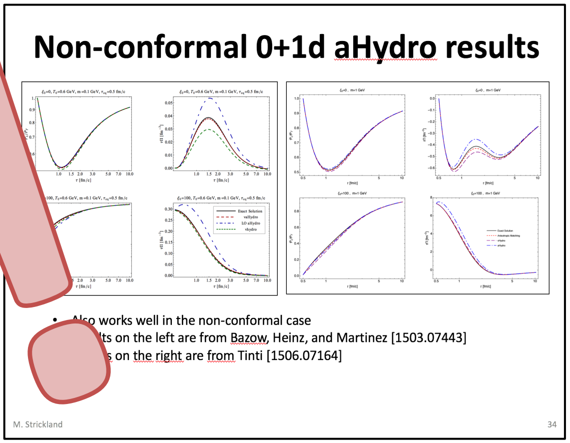
$$\mathcal{I}^{\mu\nu\lambda} \equiv \int dP\, p^\mu p^\nu p^\lambda f(x,p)\,.$$

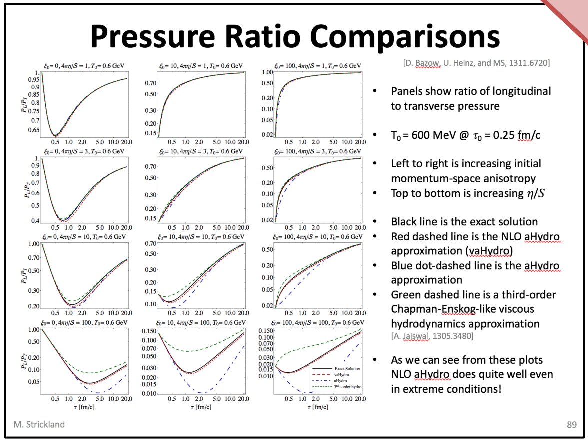
Is it really better?

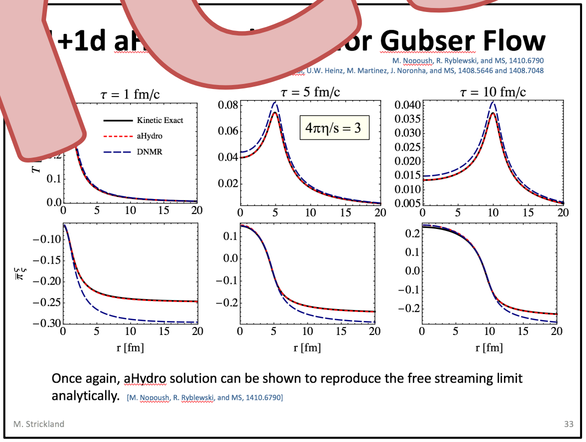
aHydro reproduces exact solutions to the Boltzmann equation in a variety of expanding backgrounds better than standard viscous hydrodynamics.

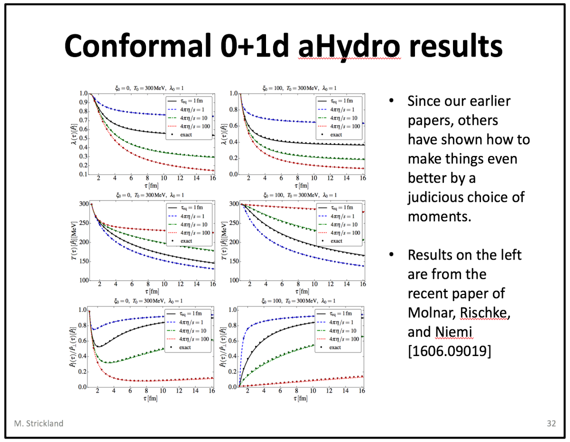






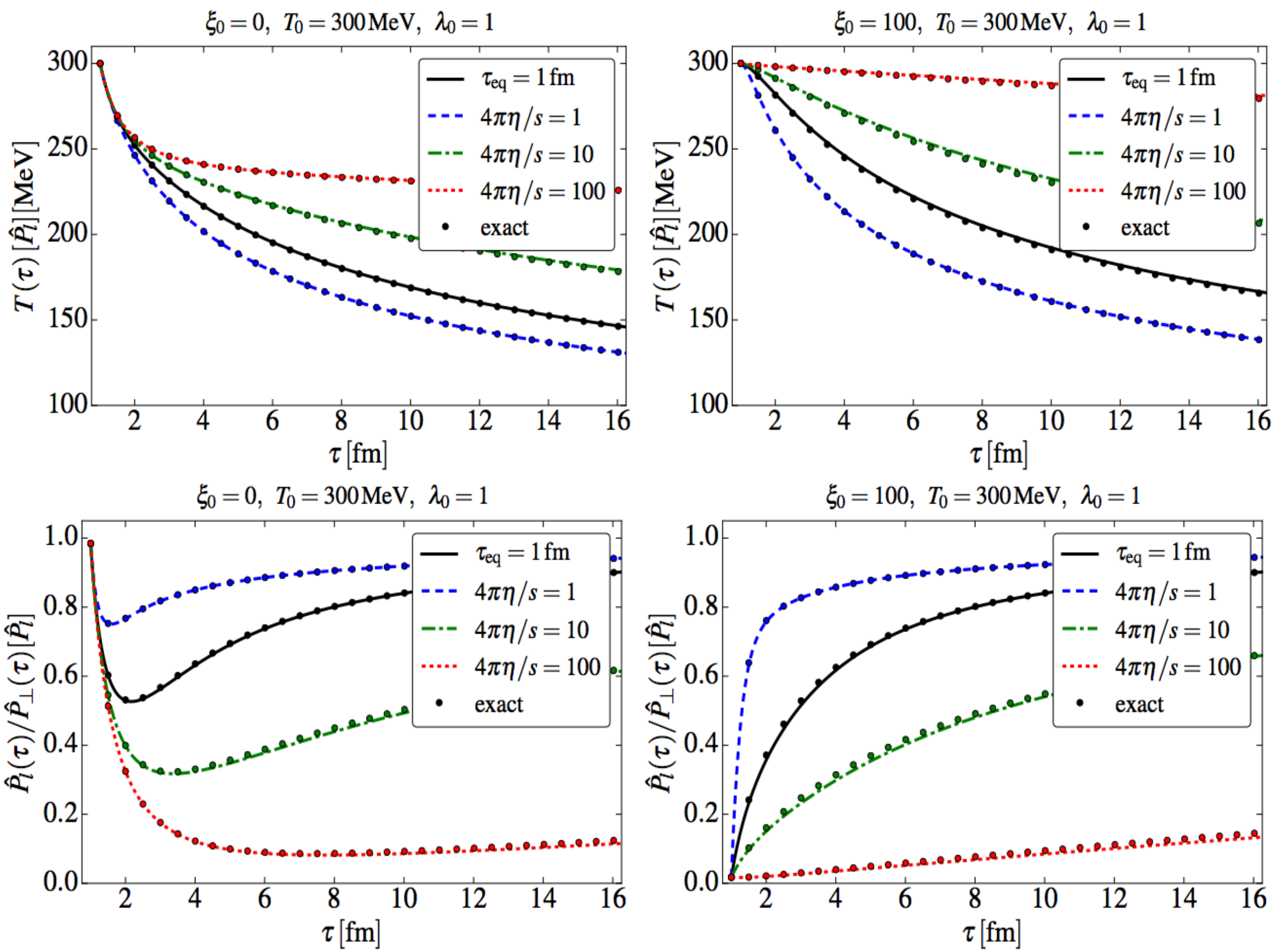




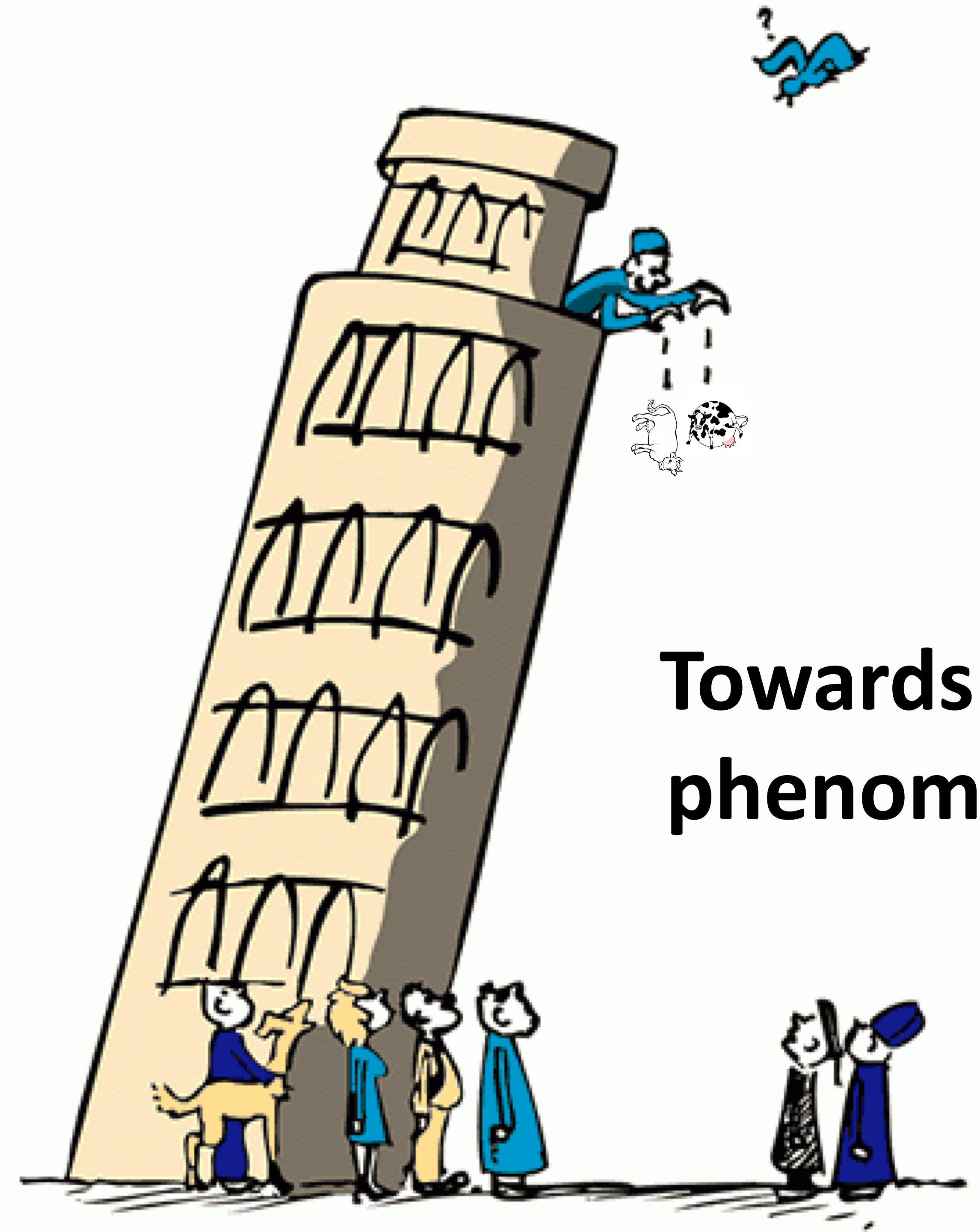


Yes!

Example: Conformal 0+1d aHydro results



- aHydro results (lines) on the left are from the recent paper of Molnar, Rischke, and Niemi [1606.09019]
- Exact solution is shown by dots [W. Florkowski, R. Ryblewski, and MS, 1304.0665 and 1305.7234]



Towards realistic phenomenology

3+1d aHydro Equations of Motion

- Assuming an ellipsoidal form for the anisotropy tensor (ignoring off-diagonal components for now), one has seven degrees of freedom ξ_x, ξ_y, ξ_z, u_x, u_y, u_z, and λ which are all fields of space and time.
- Ignore $\delta\tilde{f}$ for now

First Moment

$$D_u\mathcal{E} + \mathcal{E}\theta_u + \mathcal{P}_x u_\mu D_x X^\mu + \mathcal{P}_y u_\mu D_y Y^\mu + \mathcal{P}_z u_\mu D_z Z^\mu = 0\,,$$

$$D_x\mathcal{P}_x + \mathcal{P}_x\theta_x - \mathcal{E} X_\mu D_u u^\mu - \mathcal{P}_y X_\mu D_y Y^\mu - \mathcal{P}_z X_\mu D_z Z^\mu = 0\,,$$

$$D_y\mathcal{P}_y + \mathcal{P}_y\theta_y - \mathcal{E} Y_\mu D_u u^\mu - \mathcal{P}_x Y_\mu D_x X^\mu - \mathcal{P}_z Y_\mu D_z Z^\mu = 0\,,$$

$$D_z\mathcal{P}_z + \mathcal{P}_z\theta_z - \mathcal{E} Z_\mu D_u u^\mu - \mathcal{P}_x Z_\mu D_x X^\mu - \mathcal{P}_y Z_\mu D_y Y^\mu = 0\,.$$

$$\mathcal{I}^{\mu\nu\lambda} \equiv \int dP\, p^\mu p^\nu p^\lambda f(x,p)\,.$$

$$\mathcal{I}_i = \alpha\, \alpha_i^2\, \mathcal{I}_{\rm eq}(\lambda, m)\,,$$

$$\mathcal{I}_{\rm eq}(\lambda, m) = 4\pi \tilde{N} \lambda^5 \hat{m}^3 K_3(\hat{m})\,,$$

Second Moment

$$D_u\mathcal{I}_x + \mathcal{I}_x(\theta_u + 2u_\mu D_x X^\mu) = \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_x)\,,$$

$$D_u\mathcal{I}_y + \mathcal{I}_y(\theta_u + 2u_\mu D_y Y^\mu) = \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_y)\,,$$

$$D_u\mathcal{I}_z + \mathcal{I}_z(\theta_u + 2u_\mu D_z Z^\mu) = \frac{1}{\tau_{\rm eq}}(\mathcal{I}_{\rm eq} - \mathcal{I}_z)\,.$$

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

Implementing the equation of state

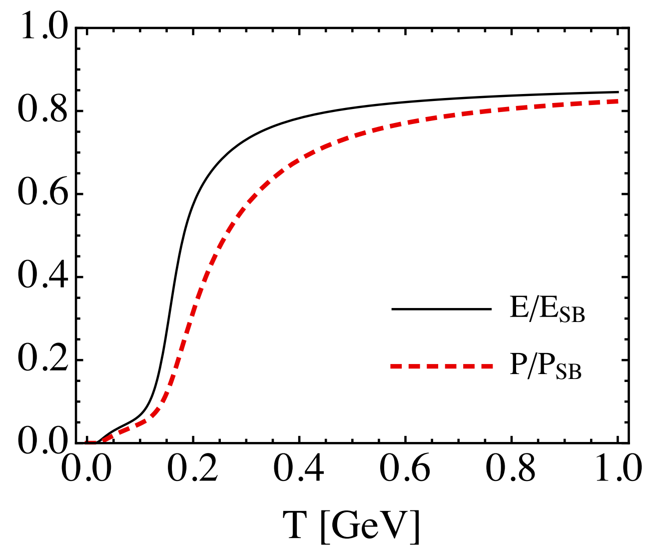
M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

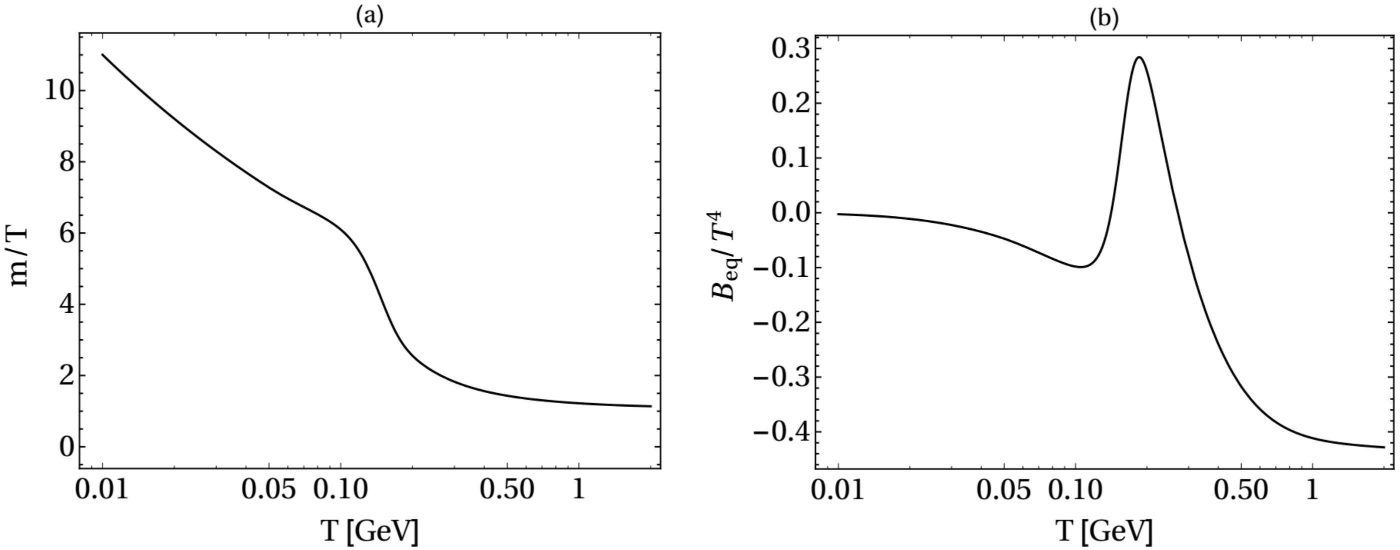
Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\text{kinetic}} + Bg^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$





Implementing the equation of state

M. Alqahtani, M. Nopoush, and MS, 1509.02913; 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

Quasiparticle Method

$$T^{\mu\nu} = T^{\mu\nu}_{\rm kinetic} + B g^{\mu\nu}$$

$$p^\mu \partial_\mu f + \frac{1}{2}\partial_i m^2 \partial^i_{(p)} f = -\mathcal{C}[f]$$

$$\partial_\mu B = -\frac{1}{2}\partial_\mu m^2 \int dP\, f(x,p)$$

Shear viscosity

Fix relaxation time as a function of the energy density by requiring fixed shear viscosity to entry density ratio.

$$\frac{\eta}{\tau_{\rm eq}} = \frac{1}{T} I_{3,2}(\hat{m}_{\rm eq})$$

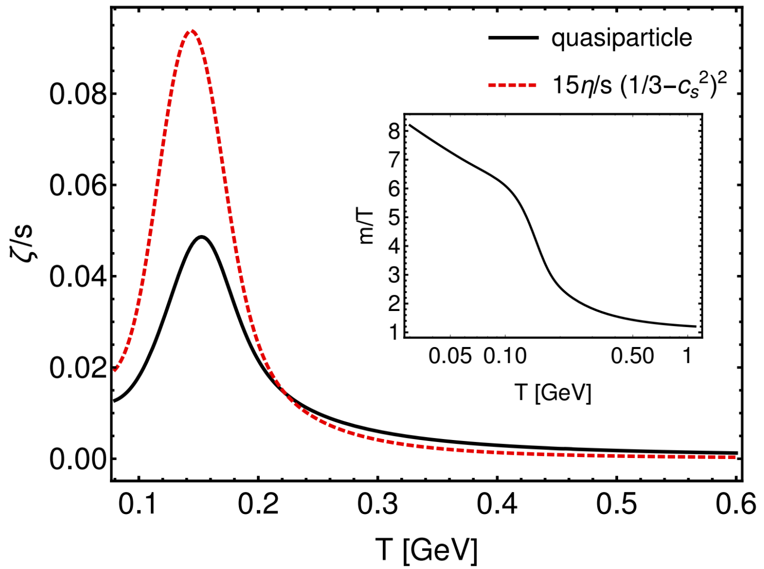
Bulk viscosity

$$\frac{\zeta}{\tau_{\rm eq}} = \frac{5}{3T} I_{3,2} - c_s^2(\mathcal{E}+\mathcal{P}) + T\hat{m}^3 \frac{dm}{dT} I_{1,1}$$

$$I_{3,2}(x) = \frac{N_{\rm dof} T^5 x^5}{30\pi^2}\left[\frac{1}{16}\Big(K_5(x) - 7K_3(x) + 22K_1(x)\Big) - K_{i,1}(x)\right],$$

$$K_{i,1}(x) = \frac{\pi}{2}\left[1 - xK_0(x)\mathcal{S}_{-1}(x) - xK_1(x)\mathcal{S}_0(x)\right],$$

$$I_{1,1} = \frac{g\, m^3}{6\pi^2}\left[\frac{1}{4}(K_3 - 5K_1) + K_{i,1}\right]$$



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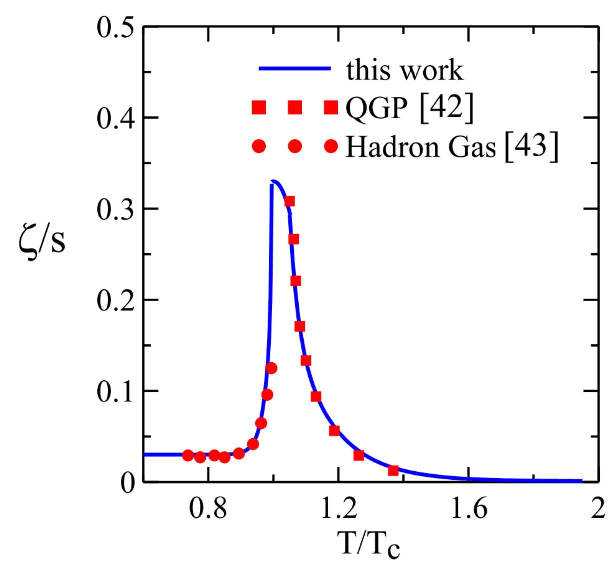
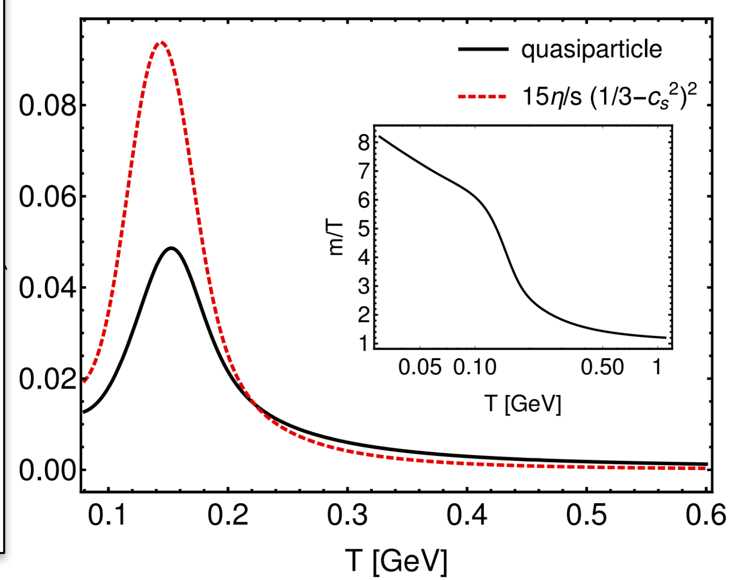
Ryu et al, PRL 115 (2015) no.13, 132301





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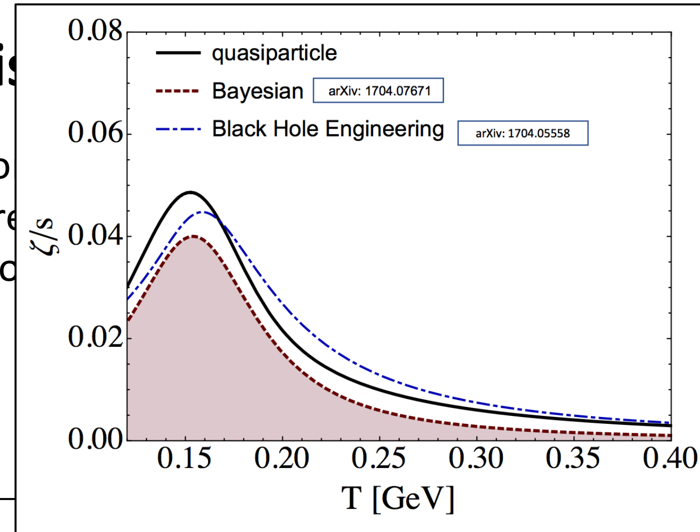

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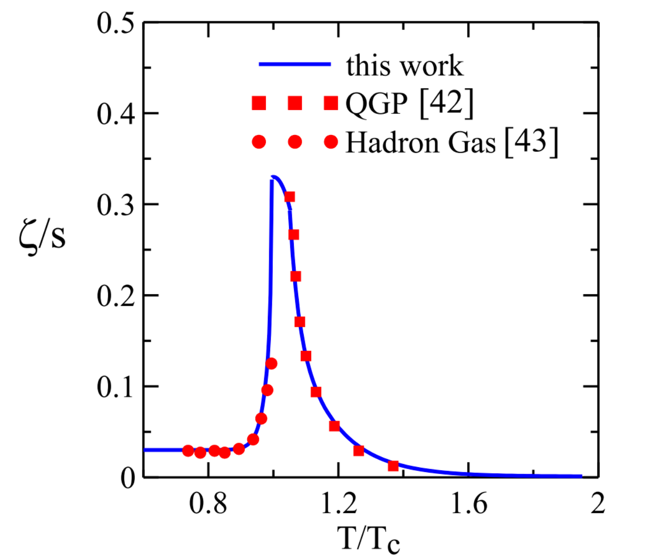
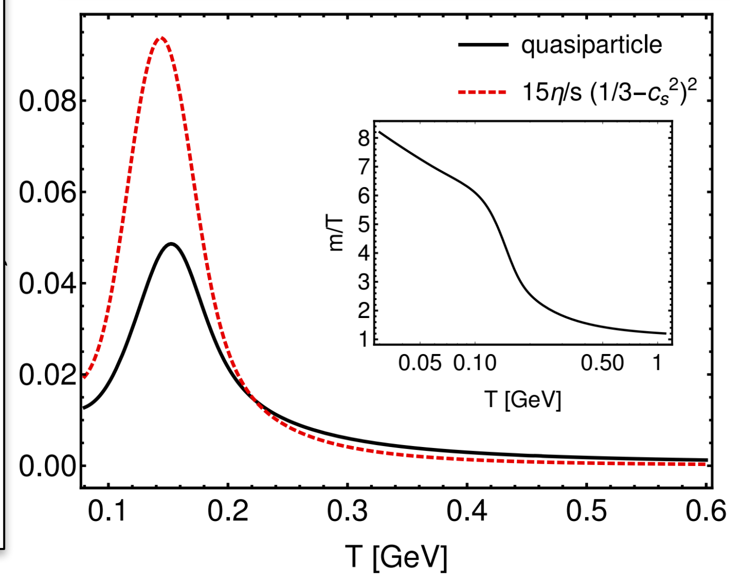




Anisotropic Cooper-Frye Freezeout

M. Alqahtani, M. Nopoush, and MS, 1605.02101
M. Alqahtani, M. Nopoush, R. Ryblewski, and MS, 1703.05808; 1705.10191

- Use same generalized-RS form for "anisotropic freeze-out" at LO
- Form includes both shear and bulk corrections to the distribution function
- Use energy density (scalar) to determine the freeze-out hyper-surface Σ → e.g. $T_{\rm eff,FO}$ = 130 MeV

$$\left(p^0 \frac{dN}{dp^3}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3} \int f_i(x,p)\, p^\mu d\Sigma_\mu \,,$$

$$f(x,p) = f_{\rm iso}\left(\frac{1}{\lambda}\sqrt{p_\mu \Xi^{\mu\nu} p_\nu}\right)$$

$$\Xi^{\mu\nu} = u^\mu u^\nu + \xi^{\mu\nu} - \Phi\Delta^{\mu\nu}$$

isotropic | anisotropy tensor | bulk correction

$$\xi^{\mu\nu}_{\rm LRF} \equiv {\rm diag}(0, \xi_x, \xi_y, \xi_z)$$

$$\xi^\mu{}_\mu = 0 \qquad u_\mu \xi^\mu{}_\nu = 0$$

NOTE: Usual 2nd-order viscous hydro form

$$f(p,x) = f_{\rm eq}\left[1 + (1 - a f_{\rm eq}) \frac{p_\mu p_\nu \Pi^{\mu\nu}}{2(\epsilon + P)T^2}\right]$$

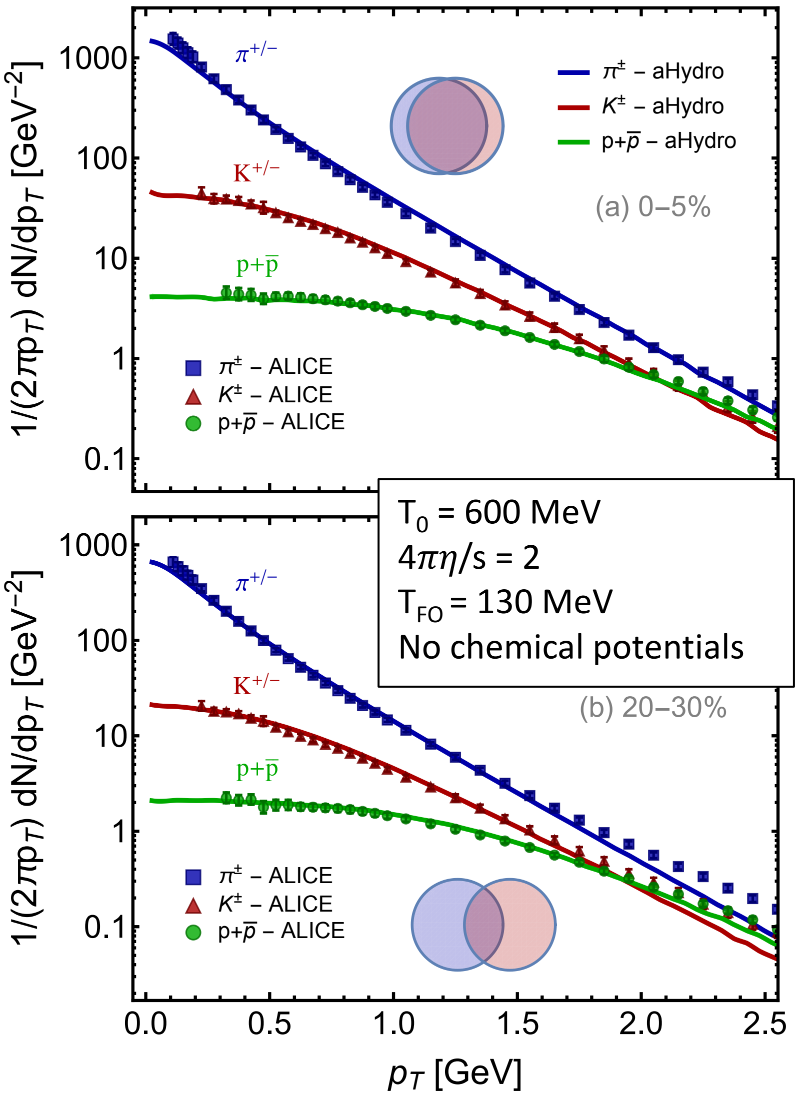
$$f_{\rm eq} = 1/[\exp(p\cdot u/T) + a] \qquad a = \text{-}1,\ +1,\ \text{or } 0$$

- This form suffers from the problem that the distribution function can be negative in some regions of phase space → unphysical
- **Problem becomes worse when including the bulk viscous correction**.

The phenomenological setup

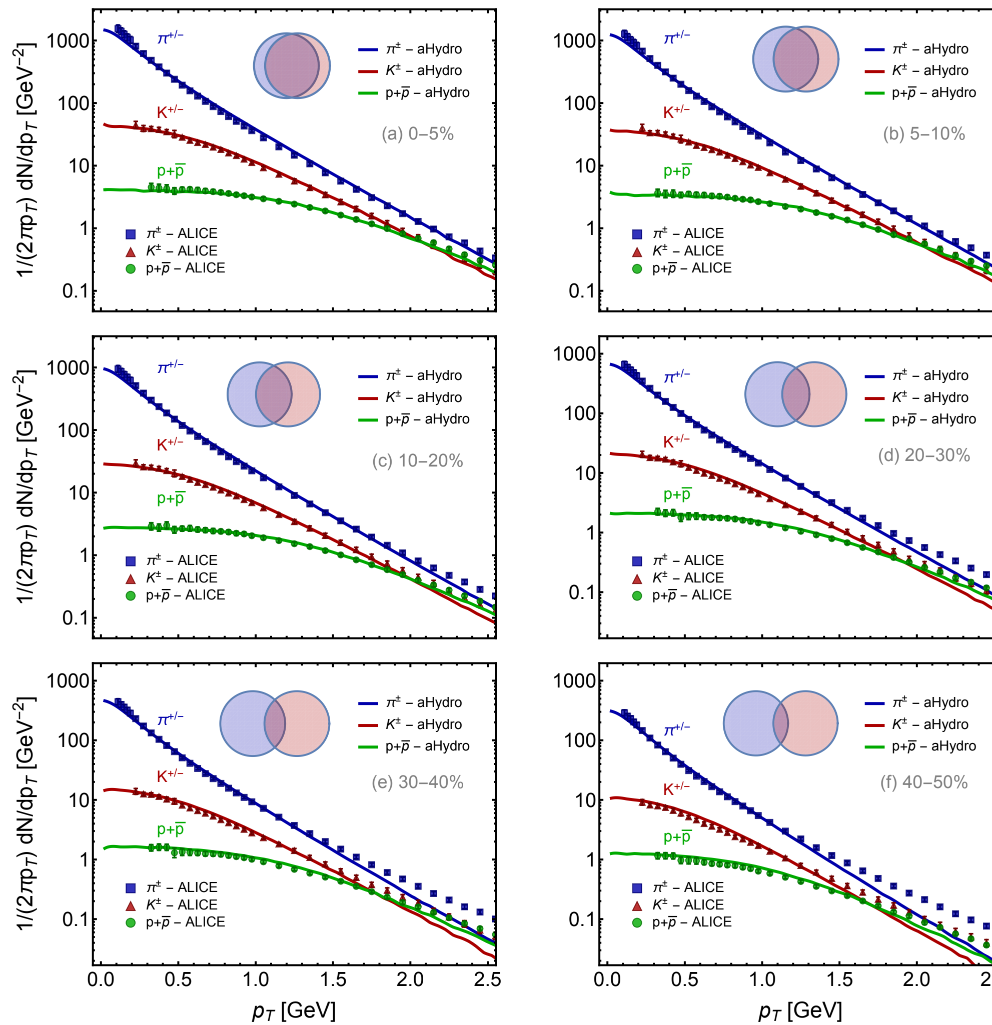
- Keep it simple at first → smooth Glauber initial conditions
- Mixture of wounded nucleon and binary collision profiles with a binary mixing fraction of 0.15 (empirically suggested from prior viscous hydro studies)
- In the rapidity direction, we use a rapidity profile with a “tilted” central plateau and Gaussian “wings”
- We take the system to be initially isotropic in momentum space
- We then run the code and extract the freeze-out hypersurface
- The primordial particle production is then Monte-Carlo sampled using the Therminator 2 [Chojnacki, Kisiel, Florkowski, and Broniowski, arXiv:1102.0273]
- Therminator also takes care of all resonance feed downs
- All data shown are from the **ALICE collaboration**

Identified particle spectra



T_0 = 600 MeV
$4\pi\eta/s$ = 2
T_{FO} = 130 MeV
No chemical potentials

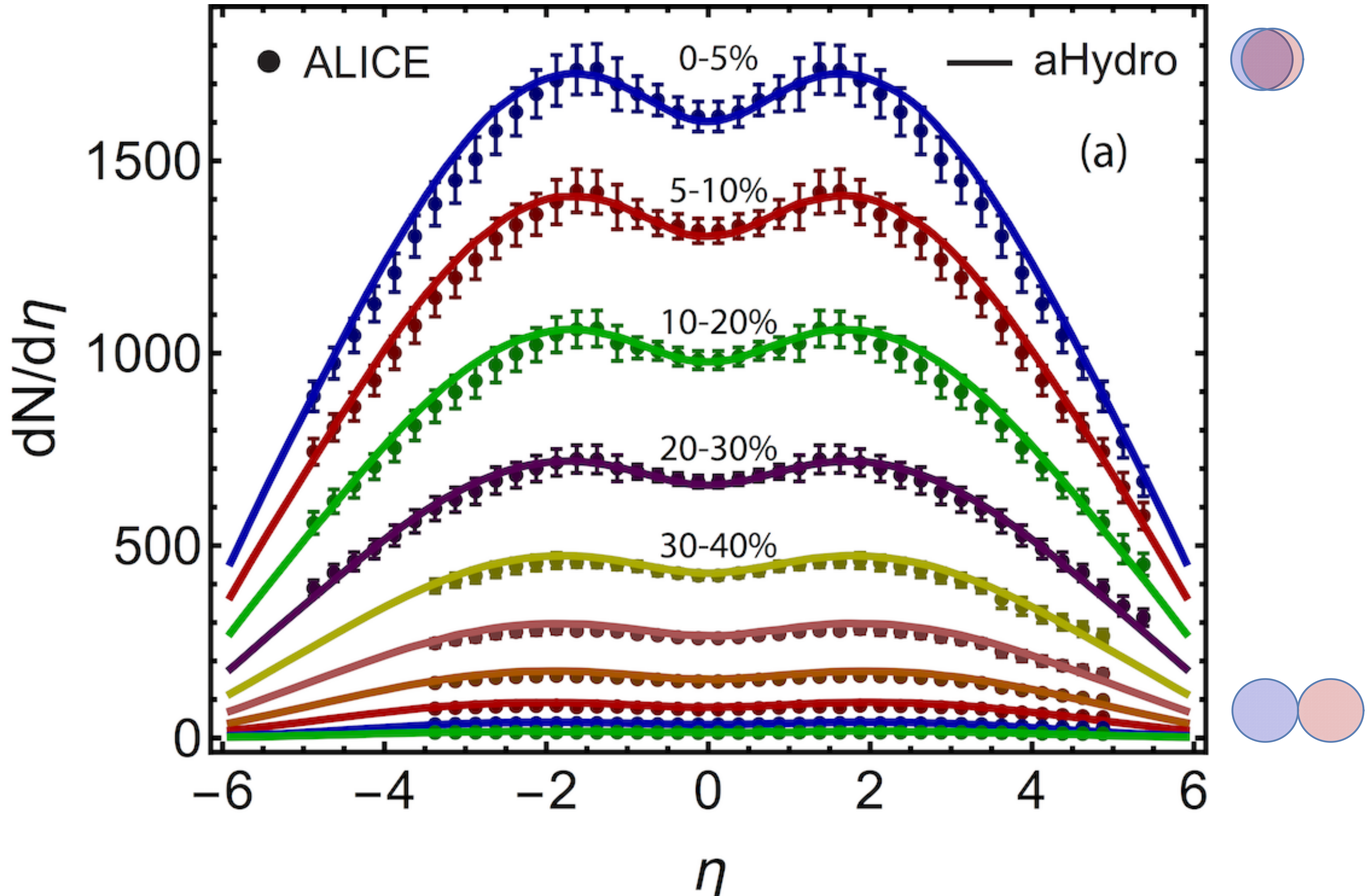
Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191



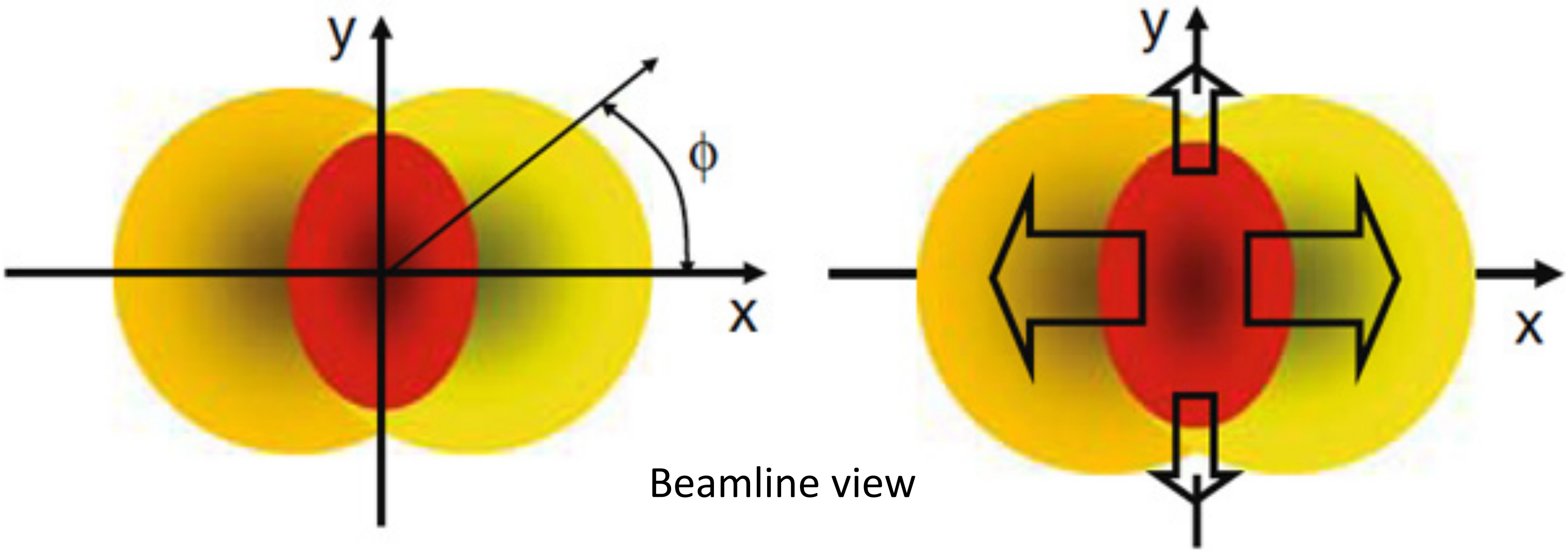
Data are from the ALICE collaboration data for **Pb-Pb collisions @ 2.76 TeV/nucleon**

Charged particle multiplicity

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191



Elliptic flow

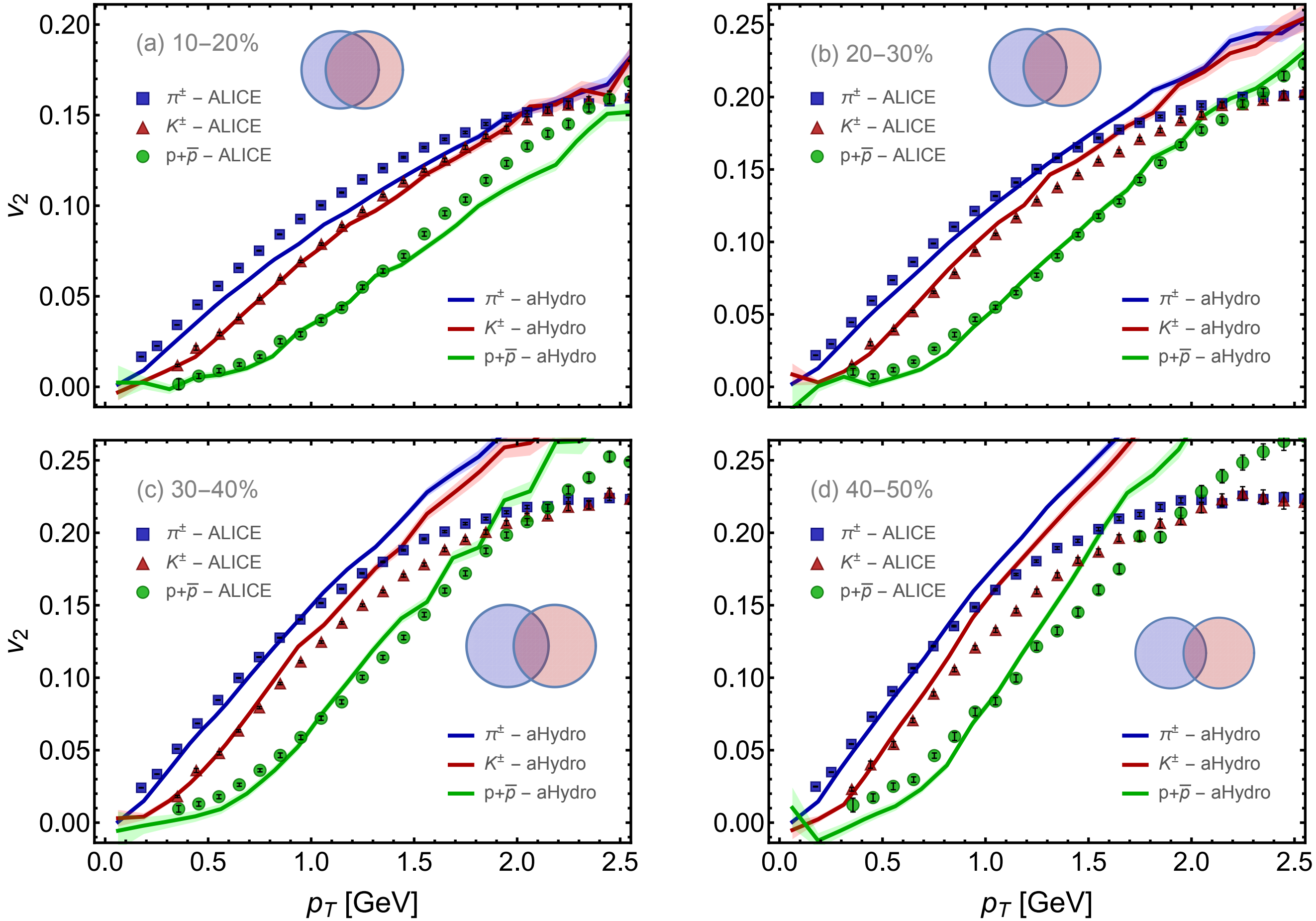


Beamline view

Geometry of overlap region creates anisotropic pressure gradients which result in “anisotropic flow” of plasma constituents.

Elliptic flow

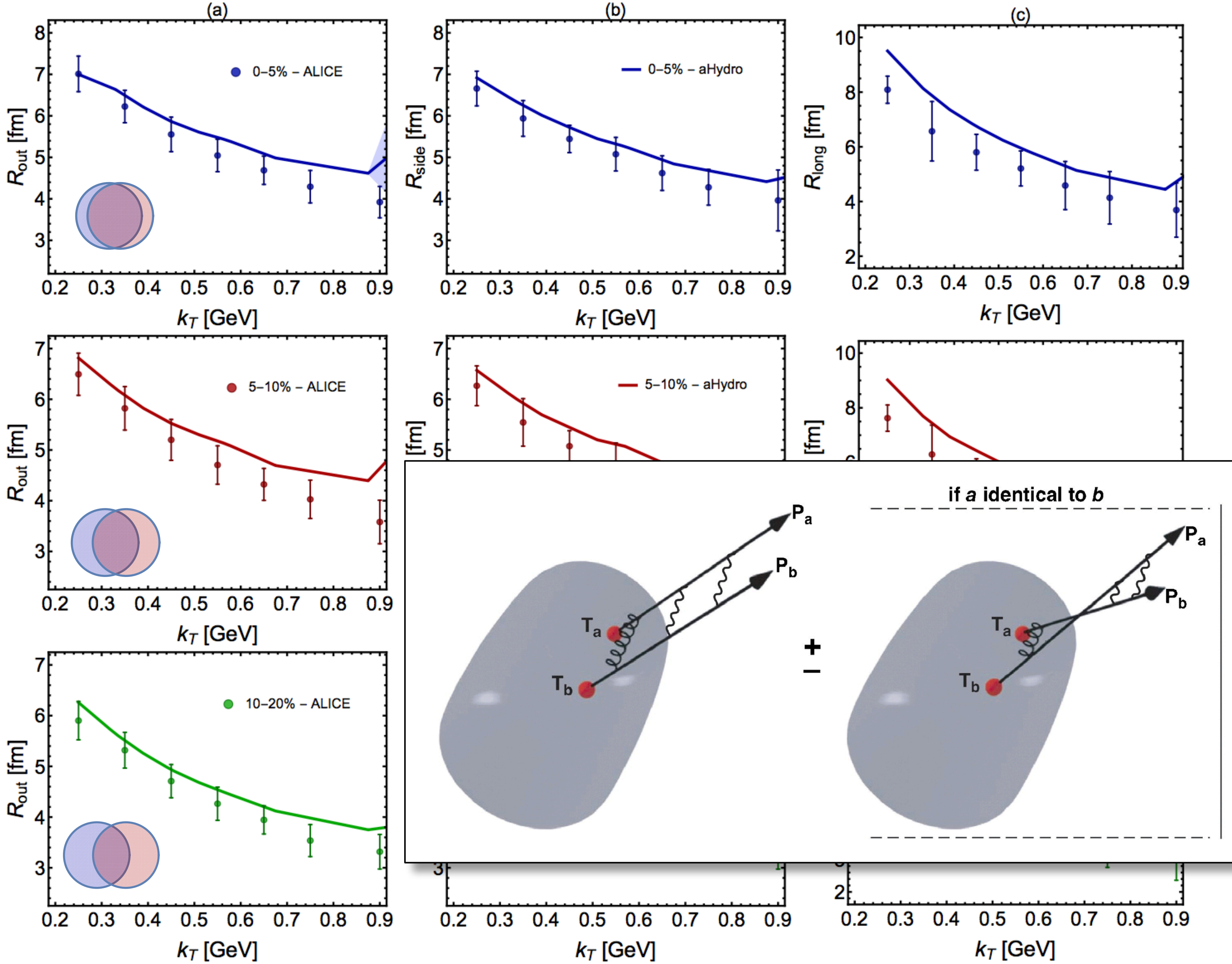
- Quite good description of identified particle elliptic flow as well
- Central collisions → need to include fluctuating init. Conditions!



Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191

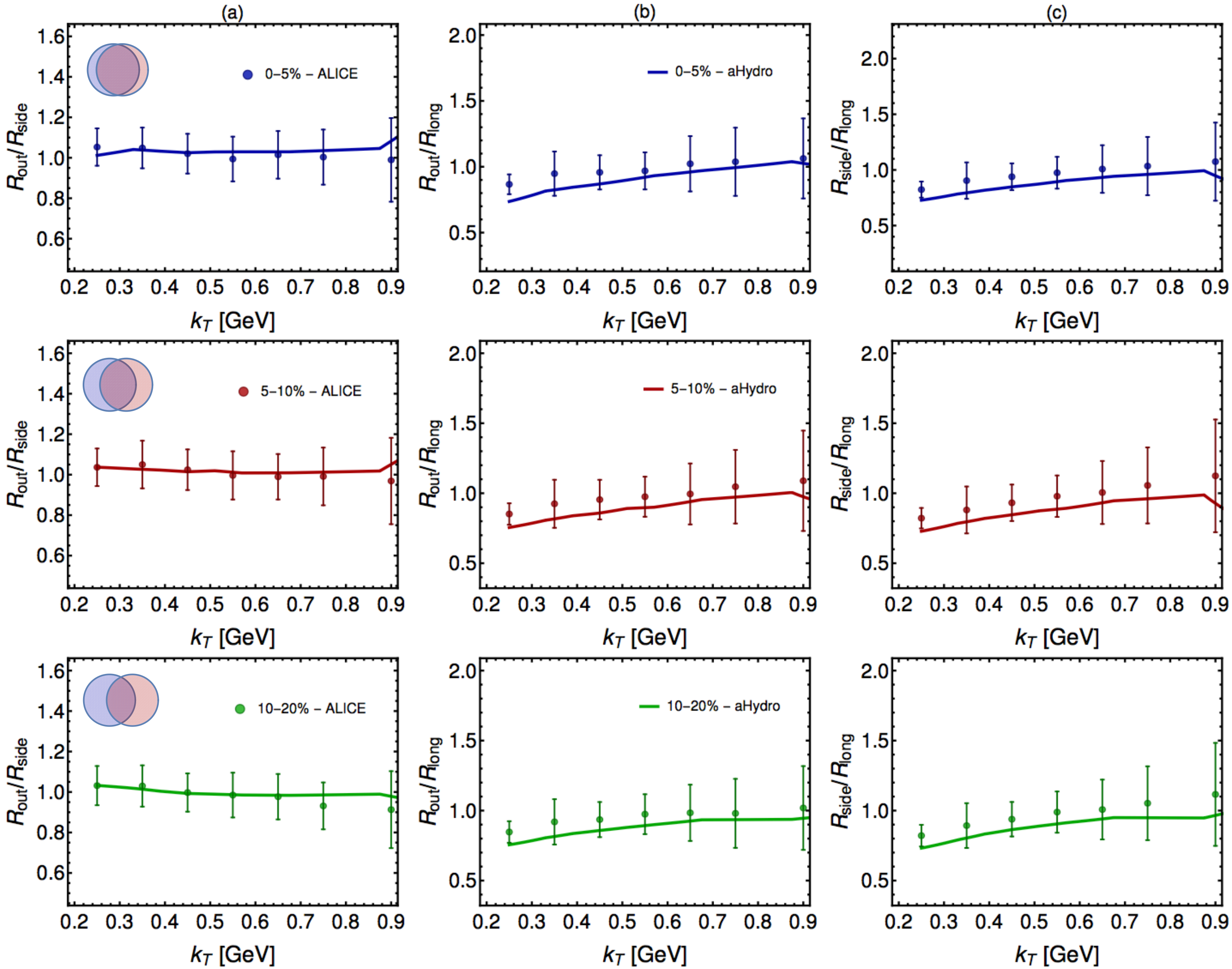
HBT Radii

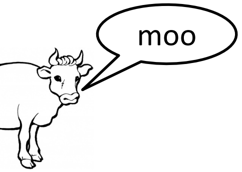
Alqahtani, Nopoush, Ryblewski, MS, 1705.10191



HBT Radii Ratios

Alqahtani, Nopoush, Ryblewski, MS, 1705.10191





Conclusions and Outlook

- Anisotropic hydrodynamics builds upon prior advances in relativistic hydrodynamics in an attempt to **create an even more quantitatively reliable model of QGP evolution**.
- It incorporates some "facts of life" specific to the conditions generated in relativistic heavy ion collisions and, in doing so, **optimizes the dissipative hydrodynamics approach for HIC**.
- We now have a running 3+1d "ellipsoidal" aHydro code with realistic EoS, anisotropic freeze-out, and fluctuating initial conditions.
- **Our preliminary fits to experimental data using smooth Glauber initial conditions look quite nice.**
- **Future:** off-diagonal anisotropies, turn on the fluctuating initial conditions, lower-energies/finite μ_B, small systems…

Backup slides

Why spheroidal form at LO?

- What is special about this form at leading order?

$$f_{\rm aniso}^{LRF} = f_{\rm iso}\left(\frac{\sqrt{\mathbf{p}^2 + \xi(\mathbf{x},\tau)p_z^2}}{\Lambda(\mathbf{x},\tau)}\right)$$

- Gives the **ideal hydro limit** when $\xi=0$ ($\Lambda \to T$)
- For longitudinal (0+1d) **free streaming**, the LRF distribution function is of spheroidal form; **limit emerges automatically** in conformal 0+1d aHydro

$$\xi_{\rm FS}(\tau) = (1+\xi_0)\left(\frac{\tau}{\tau_0}\right)^2 - 1$$

- Since $f_{iso} \geq 0$, the one-particle distribution function and pressures are ≥ 0 (not guaranteed in standard 2nd-order viscous hydro)
- **Reduces to 2nd-order viscous hydrodynamics in limit of small anisotropies**

M. Martinez and MS, 1007.0889

$$\frac{\Pi}{\mathcal{E}_{\rm eq}} = \frac{8}{45}\xi + \mathcal{O}(\xi^2)$$

For general (3+1d) proof of equivalence to second-order viscous hydrodynamics using generalized RS form in the near-equilibrium limit see Tinti 1411.7268.