Proton spin decomposition from lattice QCD

Yi-Bo Yang

August 2016

Proton Spin decomposition

The quantities in the light cone gauge

R. L. Jaffe and A. V. Manohar, Nucl. Phys. B 337, 509 (1990).

$$J = \boxed{\frac{1}{2}\vec{\Sigma}_q} + \boxed{\vec{L}_q} + \boxed{\Delta G} + \boxed{L_g}$$

quark spin

quark OAM

$$= \boxed{\int d^3x\, \tfrac{1}{2}\, \overline{\psi}\, \vec{\gamma}\, \gamma^5\, \psi} + \boxed{\int d^3x \psi^\dagger \{\vec{x} \times (i\vec{\nabla})\} \psi}$$

$$+ \boxed{\int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}]} + \boxed{\int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \vec{\nabla} A^i]}$$

glue helicity

glue OAM

From the non-symmetric canonical energy momentum tensor.
Directly related to the experiment, but can't be calculated on lattice,
except the quark spin

Proton Spin decomposition

Frame independent decomposition

X.D. Ji., Phys. Rev. Lett. 78, 610-613 (1997).

$$\vec{J} = \underbrace{\int d^3x\, \bar{\psi}\, \{\vec{x} \times \tfrac{i}{2}(\gamma^4 \vec{D} + \vec{\gamma} D^4)\}\, \psi}_{\text{quark AM}} + \int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

glue AM

$$= \underbrace{\int d^3x\, \tfrac{1}{2}\, \overline{\psi}\, \vec{\gamma}\, \gamma^5\, \psi}_{\text{quark spin}} + \underbrace{\int d^3x \psi^\dagger\, \{\vec{x} \times (i\vec{D})\}\, \psi}_{\text{quark OAM}} + \int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

From the symmetric energy momentum tensor, gauge invariant, frame independent, and well defined on the lattice

In this talk,

I will focus on:

- *The quark spin decomposition based on the anomalous Ward Identity (AWI)*

$$\boxed{\int d^3x\, \tfrac{1}{2}\,\overline{\psi}\,\vec{\gamma}\gamma^5\,\psi = \int d^3x\,\vec{x}\,m_q\overline{\psi}\,\gamma^5\,\psi + \int d^3x\,\vec{x}\,\tfrac{i\alpha_s}{4\pi}\,\mathrm{Tr}G_{\mu\nu}\tilde{G}_{\mu\nu}}$$

- *The quark/glue angular momentum in proton*

$$\boxed{\vec{J} = \int d^3x\,\bar{\psi}\,\{\vec{x}\times\tfrac{i}{2}(\gamma^4\vec{D}+\vec{\gamma}D^4)\}\,\psi + \int d^3x 2\{\vec{x}\times\mathrm{Tr}[\vec{E}\times\vec{B}]\}}$$

- *The glue spin and helicity*

$$\boxed{\begin{array}{c}\int d^3x 2\mathrm{Tr}[\vec{E}\times \vec{A^{phys}}] \\ \lim_{P^z\to\infty} S_g^z = \Delta G\end{array}}$$

Outline

- *The proton spin decomposition*

  Two types of decompositions.

- ***The quark spin decomposition based on anomalous Ward Identity***

  The contributions from 2mP and the quantum anomaly.

- *The quark/glue angular momentum in proton*

  The preliminary results and the perturbative matching to the MS-bar scheme.

- *The glue spin and helicity*

  The glue spin based on Chen's decomposition and the connection to the glue helicity.

The role of the anomaly

in the proton spin

$$\bar{\psi}\partial_\mu\gamma^\mu\gamma_5\psi = 2m_q\bar{\psi}\gamma_5\psi - 2\frac{\alpha_s}{4\pi}G\tilde{G}$$

The anomalous Ward Identity (AWI) connects the quark spin to the contribution from the pseudo-scalar current and that from the triangle anomaly,

$\bar{\psi}\gamma^\mu\gamma_5\psi$ $\bar{\psi}\gamma_5\psi$

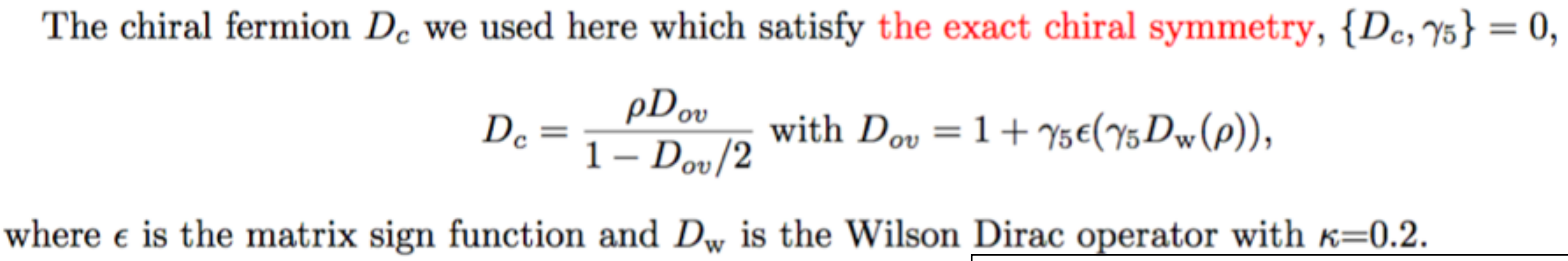
$$\begin{aligned}\langle ps\,|\mathcal{A}_\mu|\,ps\rangle s_\mu &= \lim_{\vec{q}\to 0}\frac{i|\vec{s}|}{\vec{q}\cdot\vec{s}}\langle p', s|2m_f\mathcal{P} - 2i\frac{\alpha_s}{4\pi}G\tilde{G}\,|p,s\rangle \\ &= 2m_f\langle p,s|\int d^3x\,\vec{x}\cdot\vec{s}\,\mathcal{P}(x)|p,s\rangle - 2i\langle p,s|\int d^3x\,\vec{x}\cdot\vec{s}\,\frac{\alpha_s}{4\pi}G(x)\tilde{G}(x)\,|p,s\rangle\end{aligned}$$

ignoring the 2-loop flavor mixing which will be addressed later.

Then we can check whether the anomaly plays an important role in the proton spin.

Overlap fermion

as the best action on the lattice


The chiral fermion D_c we used here which satisfy the exact chiral symmetry, $\{D_c, \gamma_5\} = 0$,

$$D_c = \frac{\rho D_{ov}}{1 - D_{ov}/2} \text{ with } D_{ov} = 1 + \gamma_5 \epsilon(\gamma_5 D_{\rm w}(\rho)),$$

where ϵ is the matrix sign function and $D_{\rm w}$ is the Wilson Dirac operator with κ=0.2.

Herbert Neuberger Phys.Lett. B417 (1998) 141
T.-W. Chiu and S. V. Zenkin, Phys. Rev. D59, 074501 (1999)

Advantage:

- Exact chiral symmetry as in continuum,
- No O(a) errors (no dimension 5 chirally symmetric action) and small O(a^2) errors,
- No additive quark mass renormalization,
- Well defined topology both globally and locally,
- No mixing of operators in different chiral sectors,
- Critical slowing down of m_π is gentle with the mass-independent deflation.

Disadvantage:

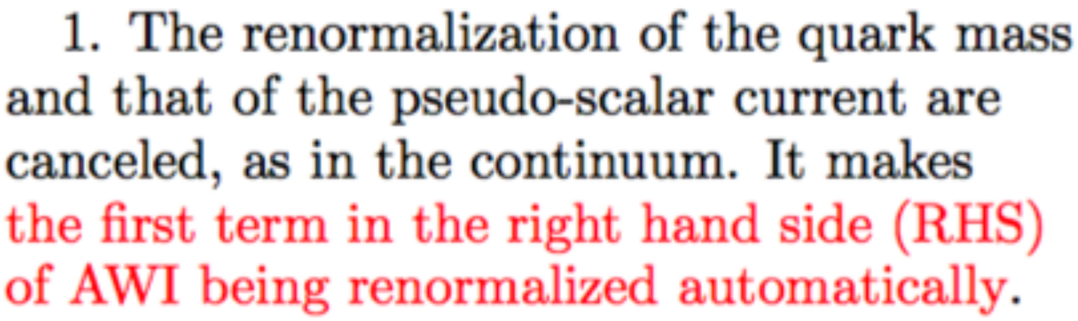
- Numerically intensive (~100 times of Wilson fermion to invert Zolotarev approximation of matrix sign function (< 10^{-9}).

Overlap fermion

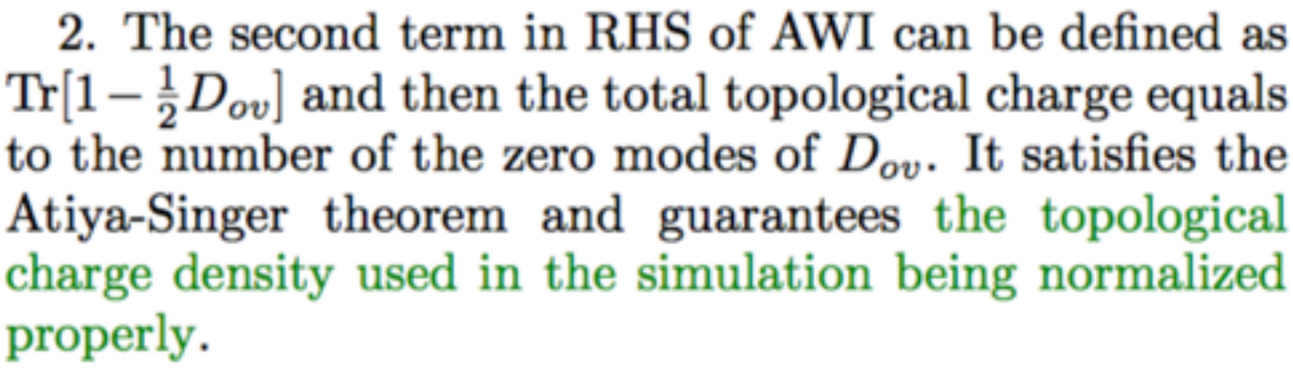
as a natural candidate for the simulation of the quark spin decomposition

The overlap fermion sacrifices *the ultra local property* $(D(x,y)_{|x-y|>r}=0)$

and *the cheapness of the simulation* to obtain:



1. The renormalization of the quark mass and that of the pseudo-scalar current are canceled, as in the continuum. It makes the first term in the right hand side (RHS) of AWI being renormalized automatically.



2. The second term in RHS of AWI can be defined as $\mathrm{Tr}[1-\frac{1}{2}D_{ov}]$ and then the total topological charge equals to the number of the zero modes of D_{ov}. It satisfies the Atiya-Singer theorem and guarantees the topological charge density used in the simulation being normalized properly.

$$\bar{\psi}\partial_\mu\gamma^\mu\gamma_5\psi = 2m_q\bar{\psi}\gamma_5\psi - 2\frac{\alpha_s}{4\pi}G\tilde{G}$$

Y. Kikukawa and A. Yamada, Phys. Lett. B448, 265 (1999)

Anomalous Ward Identity

at finite Q^2

$$2m_f\langle p,s|\int d^3x\,\underline{\vec{x}\cdot\vec{s}}\,\mathcal{P}(x)|p,s\rangle - 2i\langle p,s|\int d^3x\,\underline{\vec{x}\cdot\vec{s}}\,\frac{\alpha_s}{4\pi}G(x)\tilde{G}(x)\,|p,s\rangle$$

Since the explicit space coordinate poses a problem for direct calculation of matrix elements on the lattice, we have to return to the original AWI,

$$\bar\psi\partial_\mu\gamma^\mu\gamma_5\psi = 2m_q\bar\psi\gamma_5\psi - 2\frac{\alpha_s}{4\pi}G\tilde{G}$$

With the definition of the form factors in the proton state,

$$\begin{aligned}
\langle p's\left|\bar\psi\gamma^\mu\gamma_5\psi\right|ps\rangle &= \bar u(p',s)[\gamma_\mu\gamma_5 g_A(q^2) - q_\mu\gamma_5 h_A(q^2)]u(p,s)\\
\langle p's\left|\bar\psi\gamma_5\psi\right|ps\rangle &= \bar u(p',s)i\gamma_5[g_P(q^2)]u(p,s)\\
\langle p's\left|\frac{\alpha_s}{4\pi}G\tilde G\right|ps\rangle &= \bar u(p',s)i\gamma_5[Mg_{G\tilde G}(q^2)]u(p,s)
\end{aligned}$$

AWI can be expressed in terms of the form factors,

$$\frac{q_j}{2E_q}(2mg_A^R(Q^2) - Q^2h_A^R(Q^2)) = \frac{q_j}{2E_q}(2m_q g_P(Q^2) - 2g_{G\tilde G}(Q^2))$$

Quark Spin decomposition

The lattice ensembles

L ~ 4.3 fm
m_π ~ 170 MeV
32^3 x 64, a =0.143 fm

Smaller lattice spacing

L ~ 2.8 fm
m_π ~ 330 MeV
24^3 x 64, a =0.111 fm

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Larger Volume

Lighter sea quark

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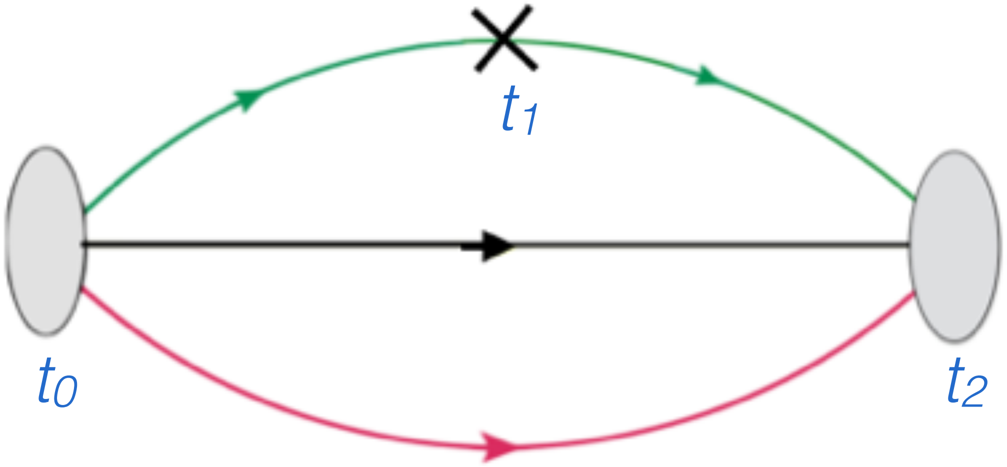
2+1 flavor DWF configurations (RBC-UKQCD)

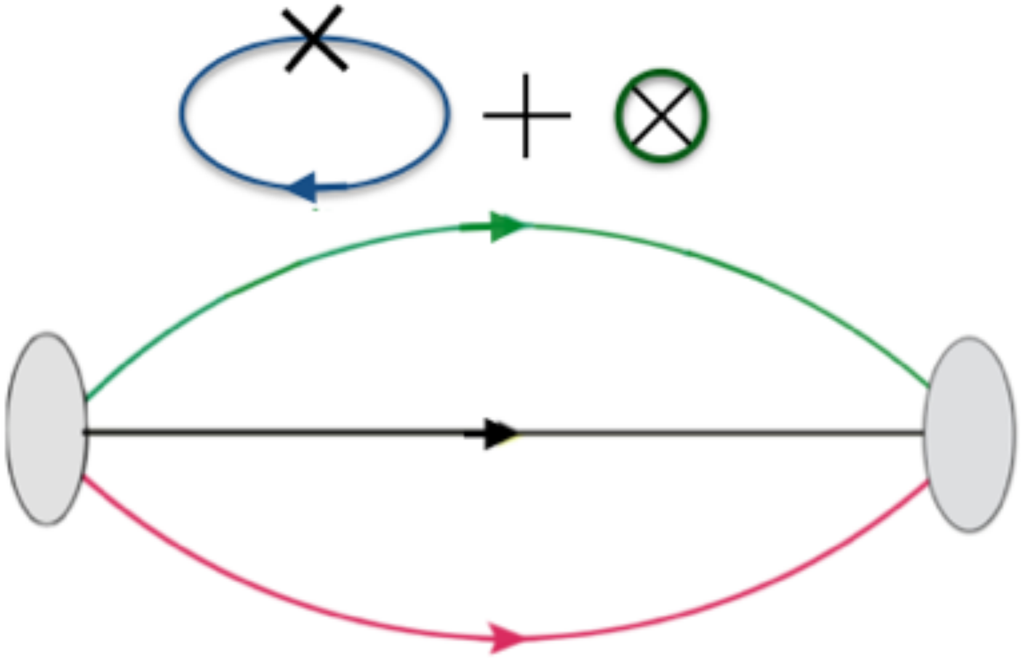
T. Blum et al. (RBC, UKQCD), Phys. Rev. D93, 074505 (2016)

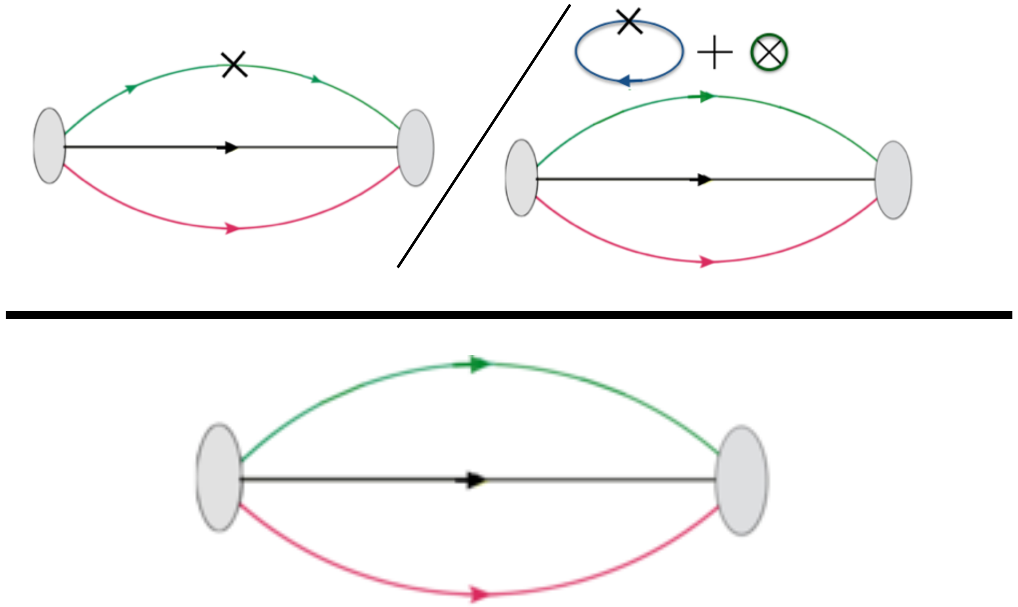
Two kinds of insertions

$2m_q\bar{\psi}\gamma_5\psi$



$2m_q\bar{\psi}\gamma_5\psi - 2\frac{\alpha_s}{4\pi}G\tilde{G}$

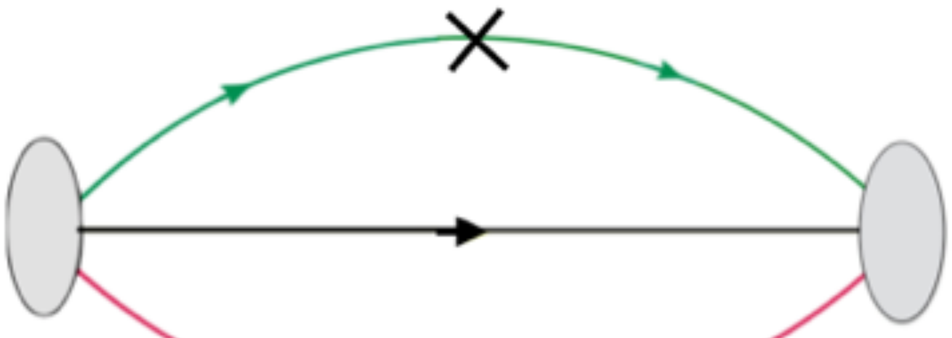




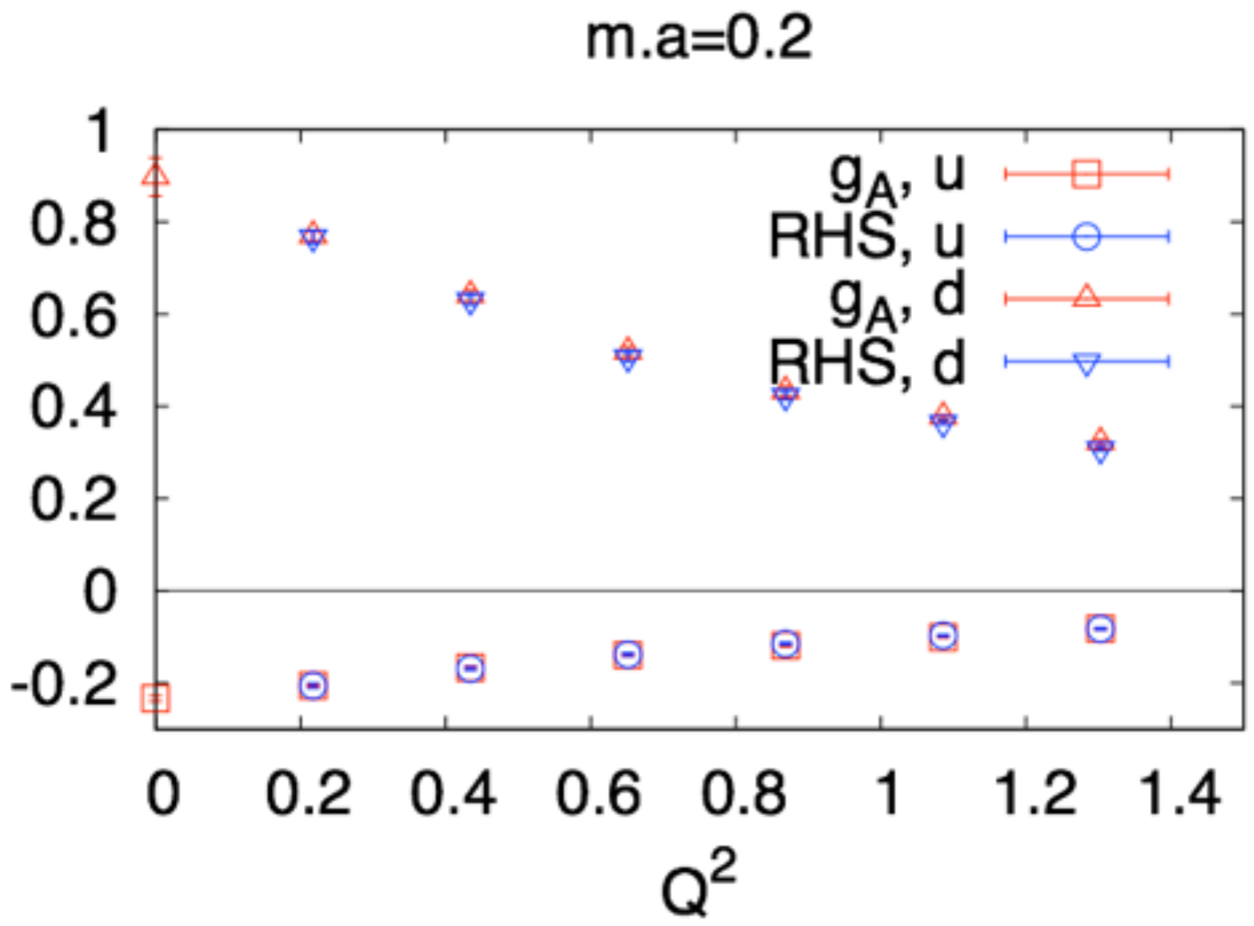
$$= g_{A,P,G\tilde{G}\ldots} + C_1 e^{\Delta_{m_1}(t_1-t_0)} + C_2 e^{\Delta_{m_2}(t_2-t_1)}$$
$$+ C_3 e^{\Delta_{m_2}(t_2-t_0)} \ldots.$$
$$\xrightarrow[t_1-t_0\to\infty,t_2-t_1\to\infty]{} \quad g_{A,P,G\tilde{G}\ldots}$$

Contributions from

the connected insertion



Connected insertion

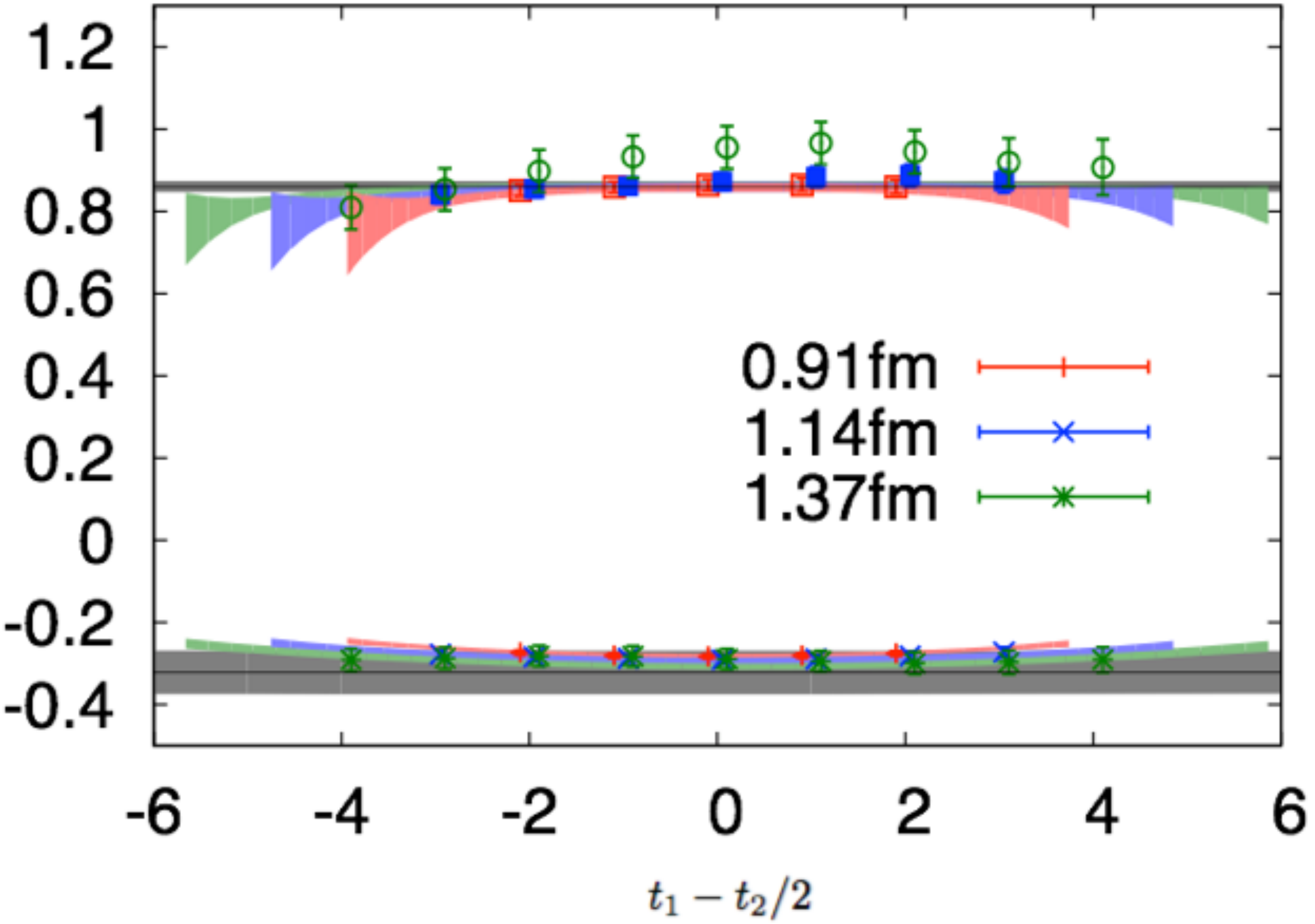


- The anomaly doesn’t contribute: $g^R_{A,CI}(Q^2) = \frac{Q^2}{2M} h^R_{A,CI}(Q^2) + \frac{m_q}{M} g_{P,CI}(Q^2)$
- AWI is confirmed with the simulation with relatively heavier quark mass.
- Calculate the axial vector current directly in the following discussion.

Contributions from

the connected insertion

$$R(t_2,t_1,t_0) = g_A + C_1 e^{\Delta_{m_1}(t_1-t_0)} + C_2 e^{\Delta_{m_2}(t_2-t_1)} + C_3 e^{\Delta_{m_2}(t_2-t_0)} \dots \xrightarrow[t_1-t_0\to\infty, t_2-t_1\to\infty]{} g_A$$



At the unitary point:

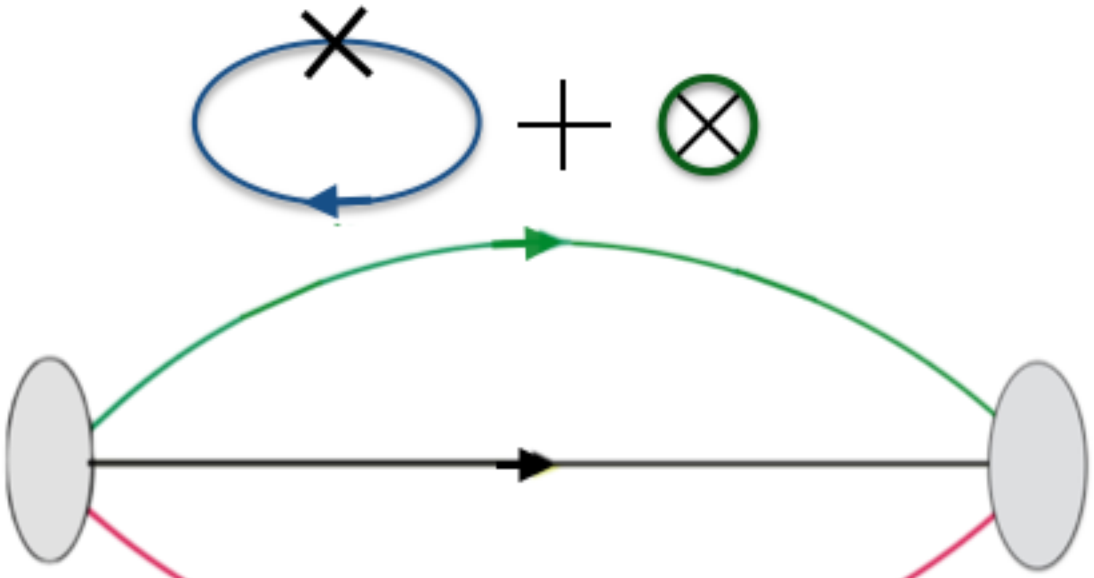
Δu_{CI} = 0.860(14)

Δd_{CI} = -0.322(53)

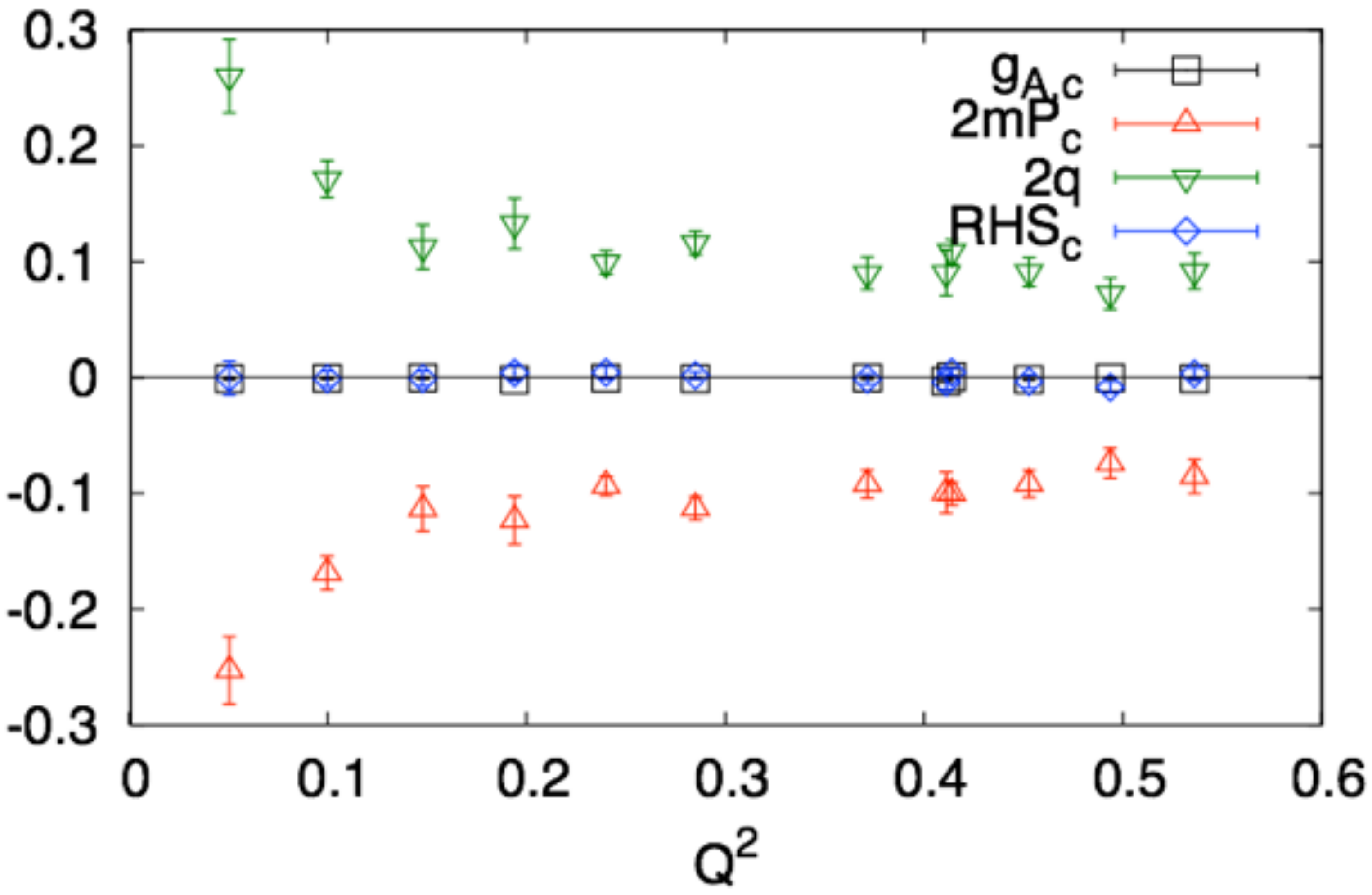
The separation dependence of Δd_{CI} is much larger than that of Δu_{CI}, thus the final uncertainty is also larger.

Contributions from

the disconnected insertions



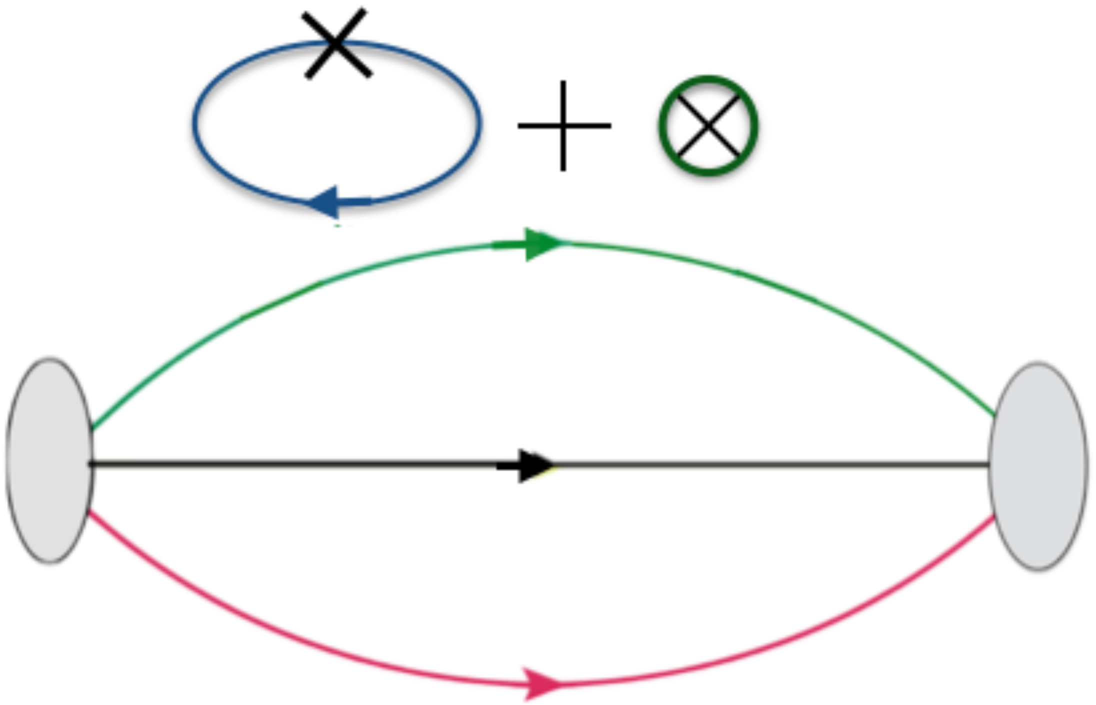
Disconnected insertion



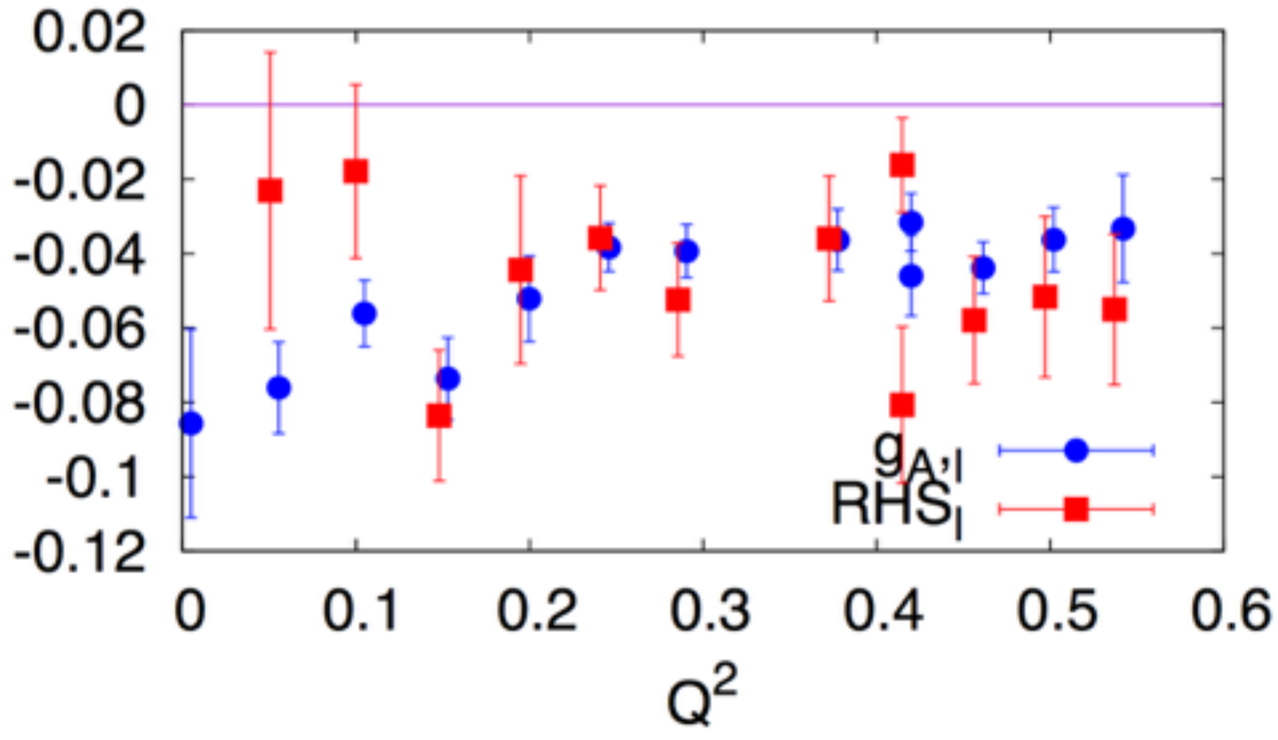
- The anomaly contributes: $g^R_{A,CI}(Q^2) = \frac{Q^2}{2M} Q^2 h^R_{A,CI}(Q^2) + \frac{m_q}{M} g_{P,CI}(Q^2) - \frac{1}{M} g_{G\tilde{G}}(Q^2)$

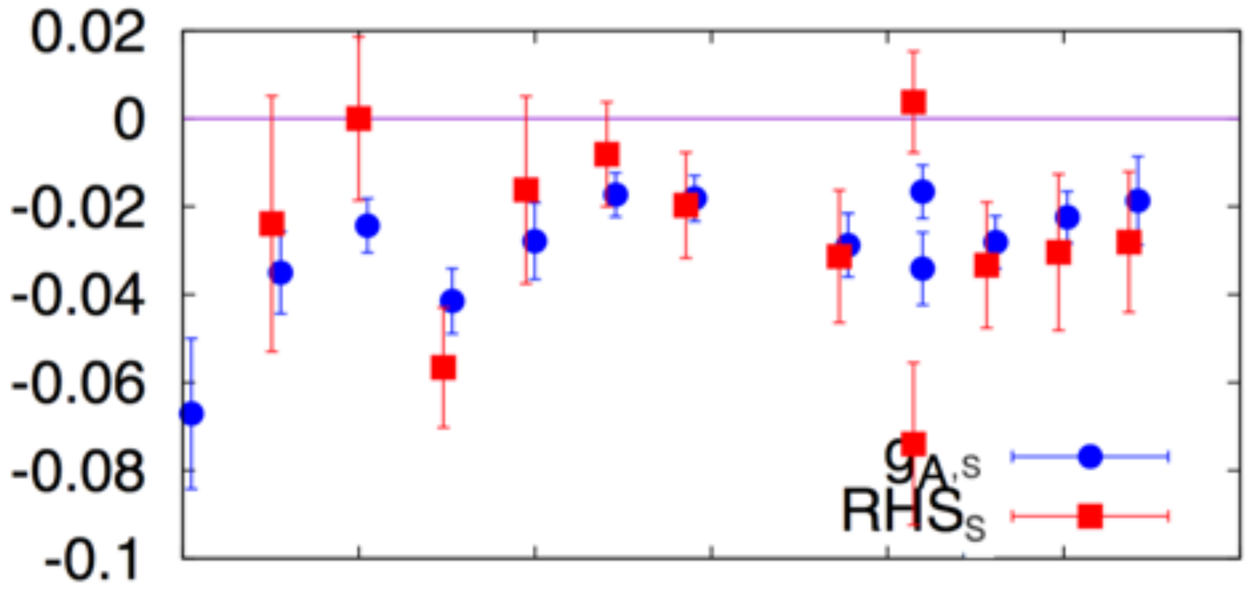
- The contribution from the 2mP term of the charm quark is canceled with that of the anomaly term.

Contributions from the disconnected insertions



Disconnected insertion





- The anomaly contributes:

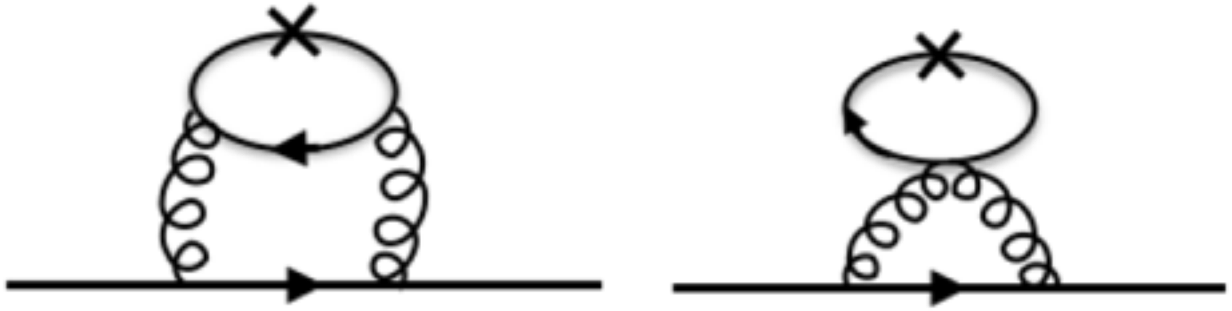
$$g^R_{A,CI}(Q^2) = \frac{Q^2}{2M}Q^2 h^R_{A,CI}(Q^2) + \frac{m_q}{M} g_{P,CI}(Q^2) - \frac{1}{M} g_{G\tilde{G}}(Q^2)$$

- The residual contributions from the DI

part of the light flavors are: $g^{DI}_{A,l}$= - 0.082(22), $g^{DI}_{A,s}$= - 0.064(18).

Systematic uncertainty:

the two-loop mixing



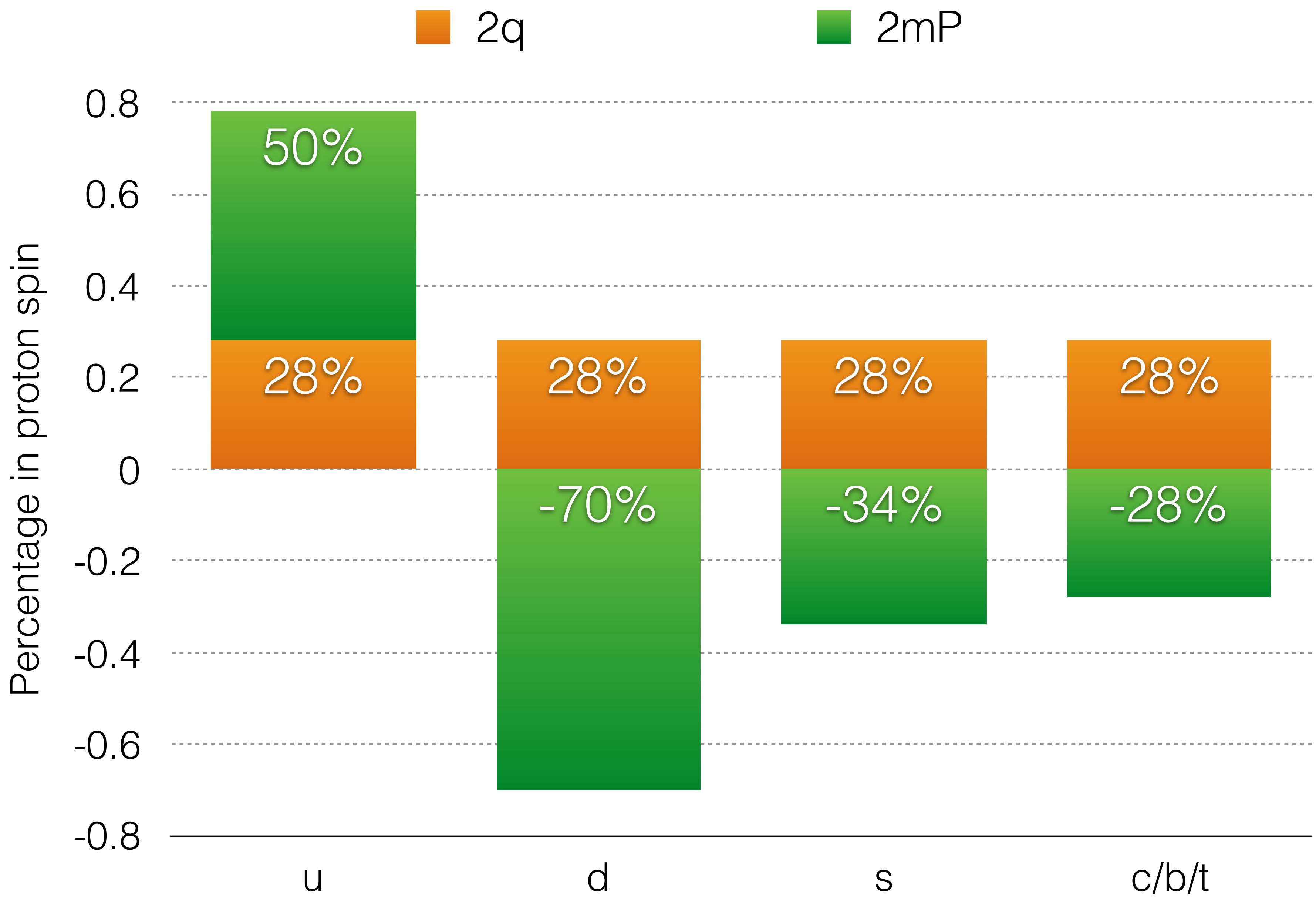
$$\mathcal{A}_f^{r,\overline{MS}} = \mathcal{A}_f^{L,WI} + (\frac{\alpha_s}{4\pi})^2 4C_F$$
$$(\frac{3}{2}\mathrm{Log}(\mu^2 a^2) + 2 + B_A)\sum_f \mathcal{A}_f^{L,WI}$$
$$+O(\alpha_s^3),$$

The lattice regularization induces additional flavor mixing at the 2-loop level. Supposing B_A to be O(10), then it can contribute 0.005 per flavor which is smaller than the statistical error.

For the other quantities, the lattice regularization effect are at the 1-loop level and can be large.
Will be discussed in the angular momentum case.

Results

Anomaly as the spin saver



Preliminary

The simulation with the conserved current is in progress to confirm the results.

The role of the anomaly

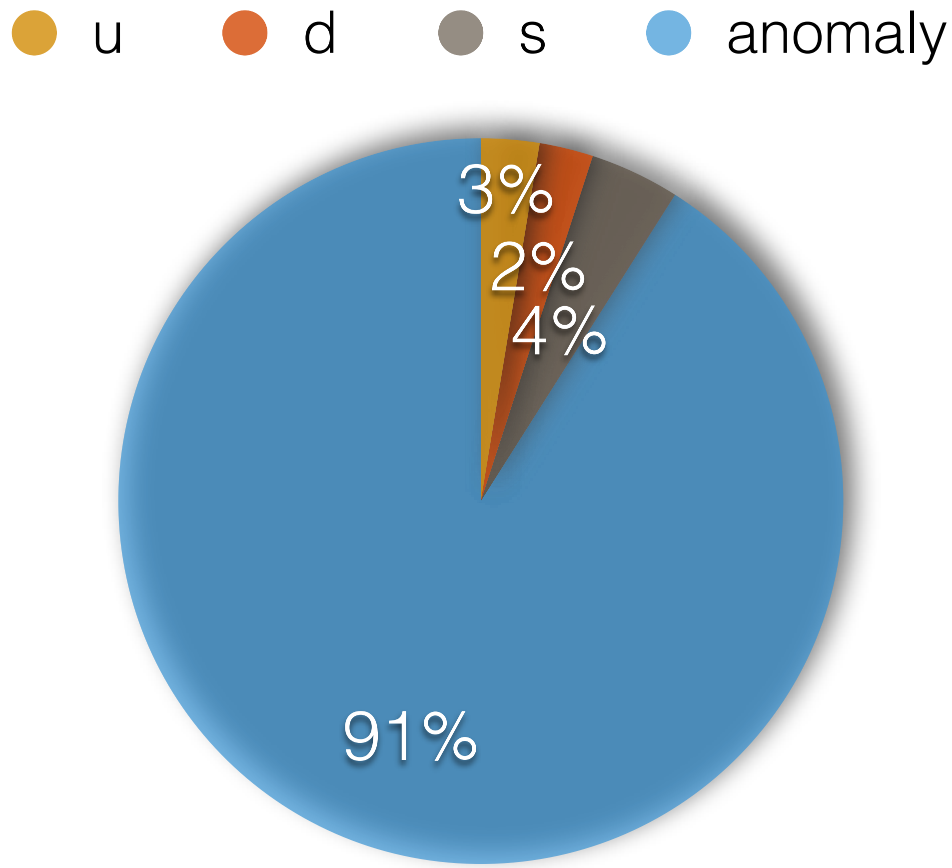
in the proton mass

The trace anomaly

$$m_N = \sum_{q=u,d,s} m_q(1+\gamma_m)\langle N|\bar{q}q|N\rangle + \sum_{Q=c,b,t} m_Q(1+\gamma_m)\langle N|\bar{Q}Q|N\rangle \boxed{-\frac{\beta(g)}{2g}\langle N|G_{\mu\nu}G_{\mu\nu}|N\rangle}$$

$$\parallel$$

$$\frac{\alpha_s}{4\pi}\left(\frac{11}{2}-\frac{n_f}{3}\right)+O(\alpha_s^2)$$

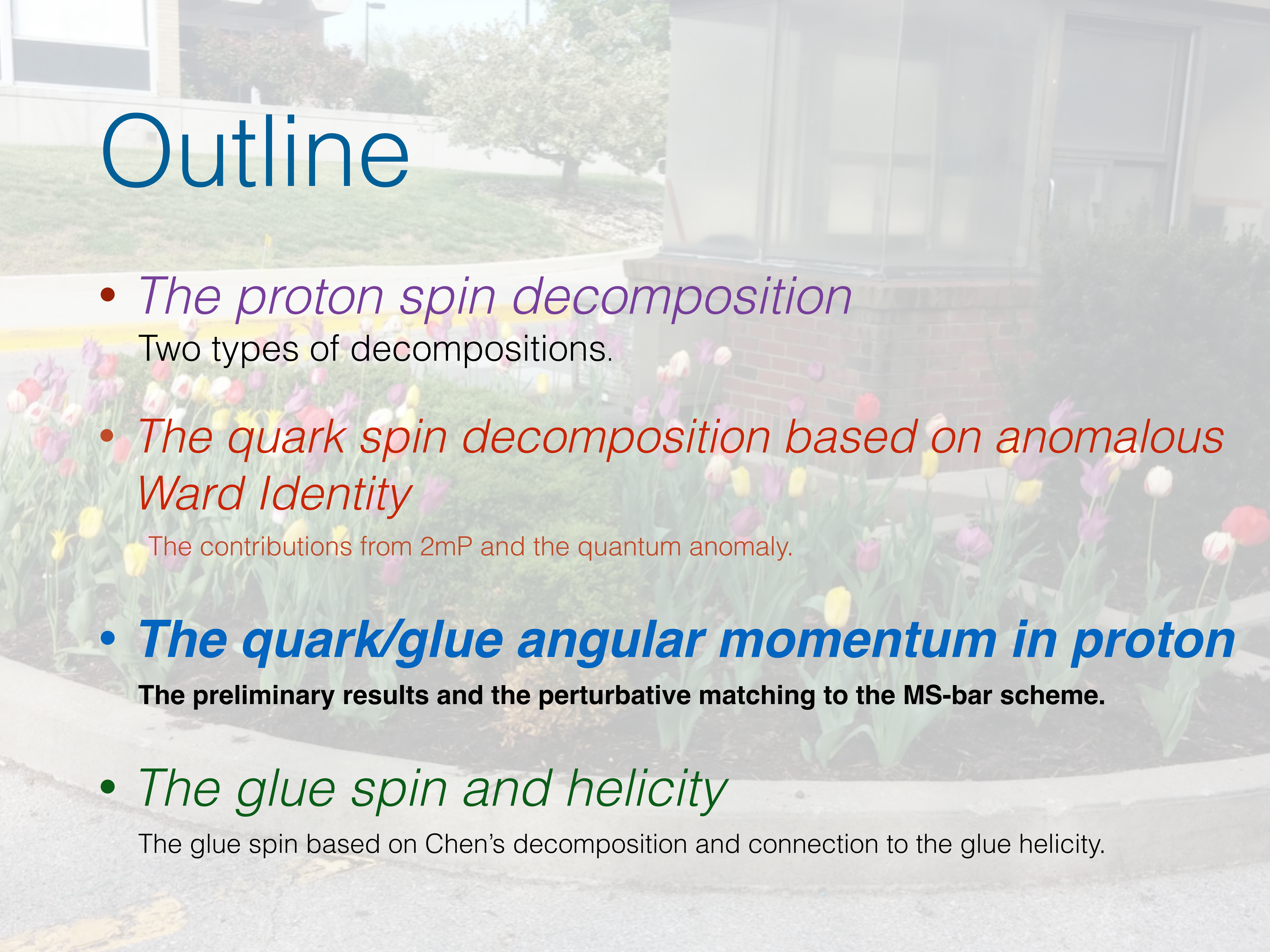


- The light two flavors contribute ~46(8) MeV and the strange quark mass contributes 40(12) MeV, in total ~9(2)% of the proton mass.

Y.B. Yang et al., χQCD collaboration arXiv:1511.09089

- The heavy quark contribution ~70MeV, is canceled by the anomaly term in the 1-loop level,

$$m_Q\langle N|\bar{Q}Q|N\rangle \rightarrow \frac{\alpha_s}{12\pi}\langle N|G_{\mu\nu}G_{\mu\nu}|N\rangle + O(\alpha_s^2).$$



Outline

- *The proton spin decomposition*

  Two types of decompositions.

- *The quark spin decomposition based on anomalous Ward Identity*

  The contributions from 2mP and the quantum anomaly.

- ***The quark/glue angular momentum in proton***

  The preliminary results and the perturbative matching to the MS-bar scheme.

- *The glue spin and helicity*

  The glue spin based on Chen’s decomposition and connection to the glue helicity.

Proton Spin decomposition

Calculation through the EMT form factors

X.D. Ji., Phys. Rev. Lett. 78, 610–613 (1997).

Ji’s angular momentum (AM) can be written in terms of the symmetrized energy momentum tensor (EMT) as,

$$J^{q,g} = \langle p, s| \int d^3x \, x \times \mathcal{T}^{\{0i\}q,g} |p, s\rangle, \qquad \mathcal{T}^{\{0i\}q} = \frac{1}{4}\bar{\psi}\gamma^{(0}\overleftrightarrow{D}^{i)}, \; \mathcal{T}^{\{0i\}g} = \vec{E} \times \vec{B}.$$

, with the form factors of the off-diagonal part of EMT defined by,

$$\begin{aligned}(p', s'|\mathcal{T}^{\{0i\}q,g}|p, s) = \left(\frac{1}{2}\right) \bar{u}(p', s') \Big[& T_1(q^2)(\gamma^0\bar{p}^i + \gamma^i\bar{p}^0) + \frac{1}{2m}T_2(q^2)\left(\bar{p}^0(i\sigma^{i\alpha}) + \bar{p}^i(i\sigma^{0\alpha})\right) q_\alpha \\ & + \frac{1}{m}T_3(q^2)q^0q^i\Big]^{q,g} u(p, s),\end{aligned}$$

Ji’s quark and glue AM correspond to the forward limit of the form factor combination,

$$\boxed{J^{q,g} = \frac{1}{2}\left[T_1(0) + T_2(0)\right]^{q,g}}$$

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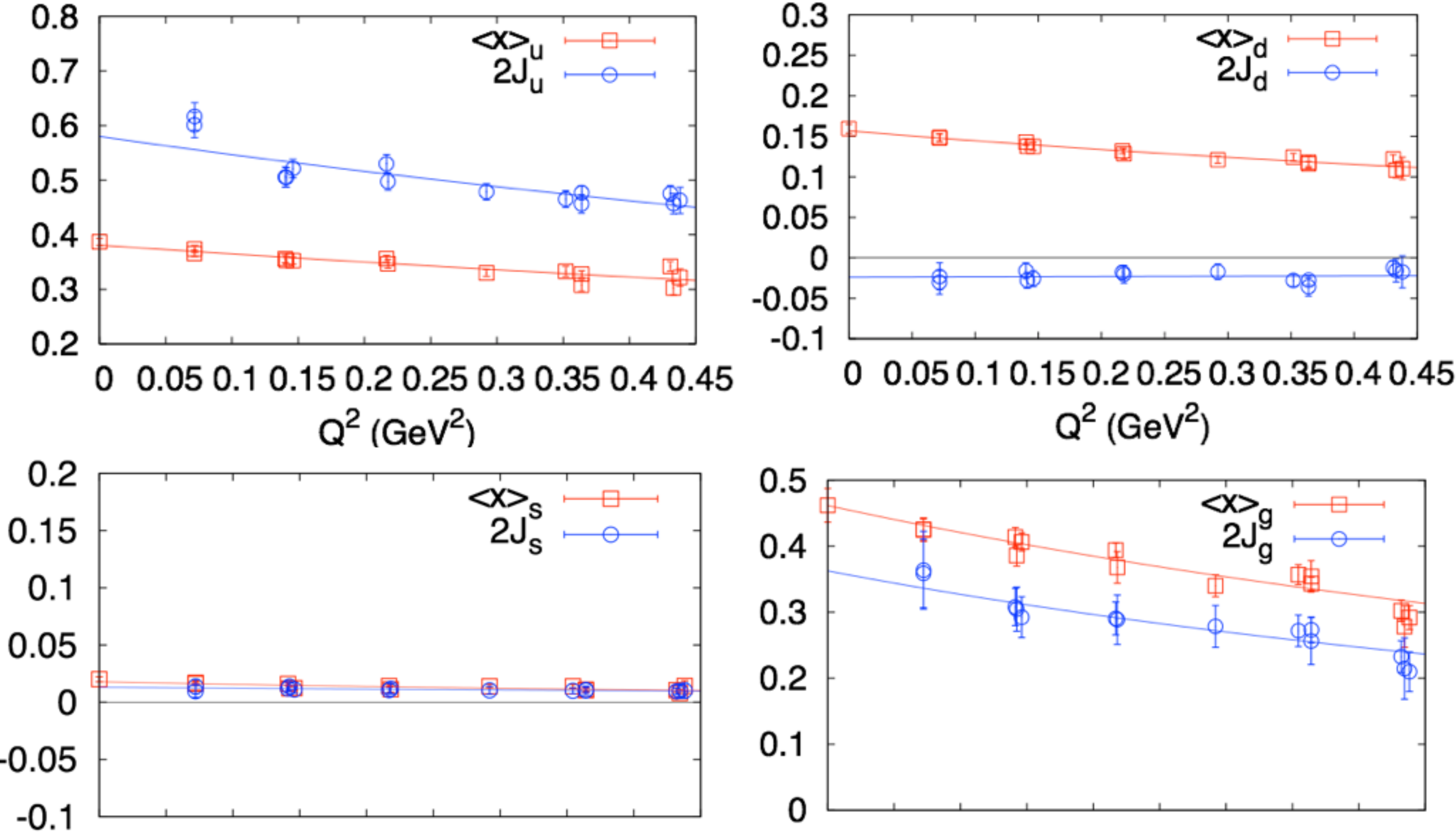
2+1 flavor DWF configurations (RBC-UKQCD)

T. Blum et al. (RBC, UKQCD), Phys. Rev. D93, 074505 (2016)

Quark and glue angular momentums

m_π=400 MeV

The bare results on lattice



Next steps?

- *Repeat the calculation on the other ensembles to access the systematic uncertainty (lattice spacing, volume, sea quark mass, etc.)*

  Costly but the framework has been set up.

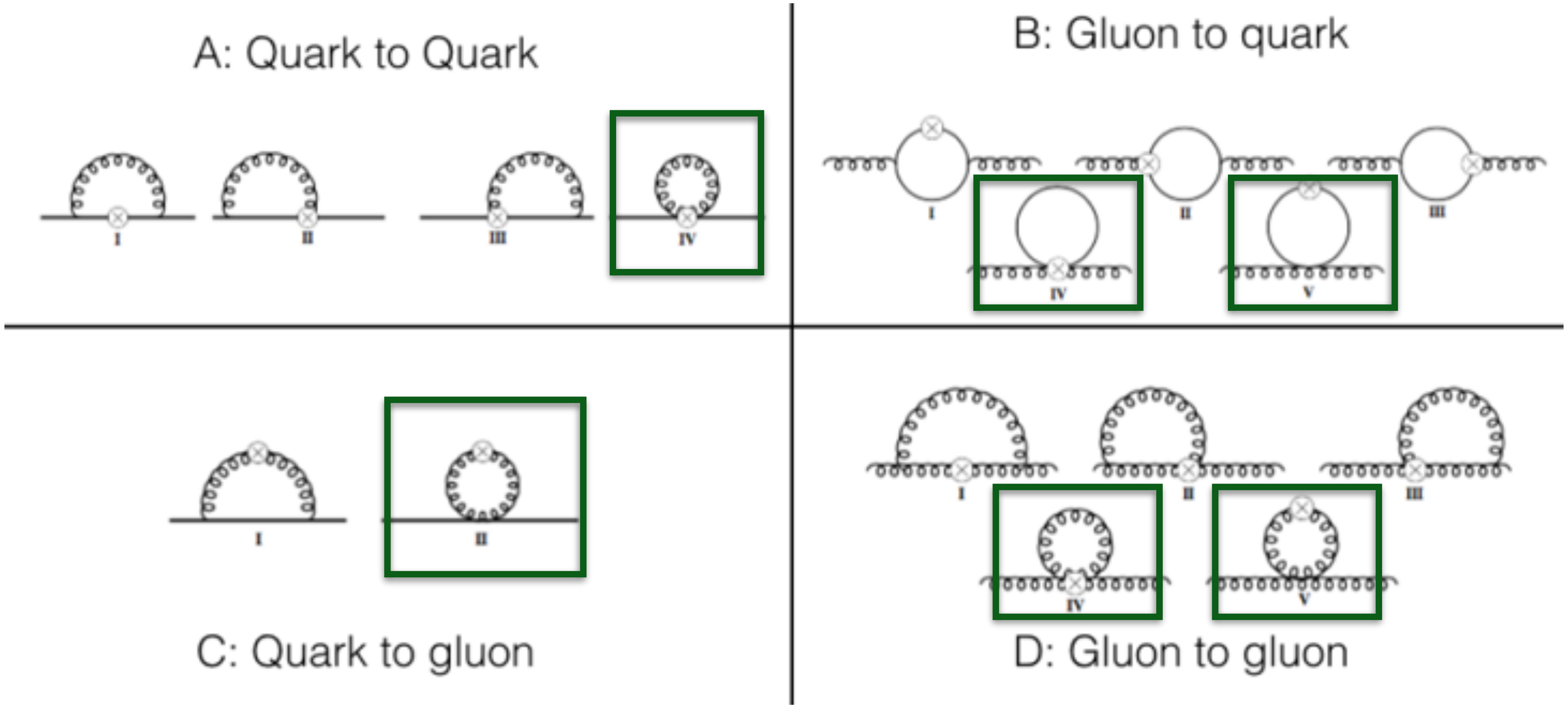
- *Matching the lattice bare results to that under MS-bar scheme at 2GeV.*

  A non-trivial lattice perturbative calculation (will be addressed in the following a few pages).

The renormalization of AM for quark and glue

See
S. Capitani and G. Rossi, Nuclear Physics B 433 (1995) 351-389, as example

☐ *Diagrams don’t exist or contribute in the continuum*

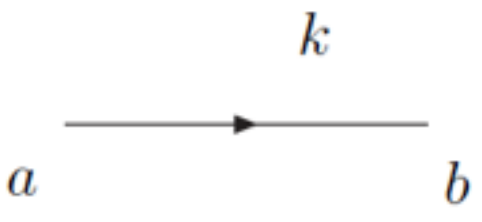


The Feynman rules of LatPT with the extra vertices

Taking the simplest Wilson fermion as example,

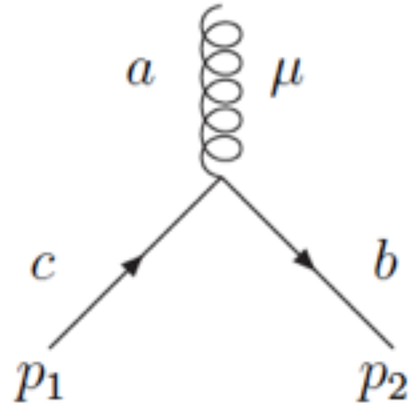
See
S. Capitani, Phys.Rept. 382 (2003) 113–302,
as example

$$S_W = a^4\sum_x\Big[-\frac{1}{2a}\sum_\mu\big[\overline{\psi}(x)(r-\gamma_\mu)U_\mu(x)\psi(x+a\hat{\mu}) + \overline{\psi}(x+a\hat{\mu})(r+\gamma_\mu)U_\mu^\dagger(x)\psi(x)\big] + \overline{\psi}(x)\left(m_0+\frac{4r}{a}\right)\psi(x)\Big]$$



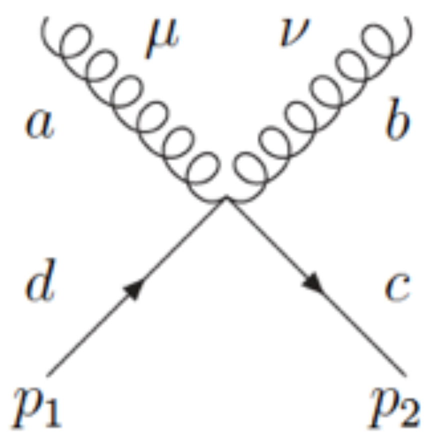
$$\delta^{ab}\cdot a\,\frac{-\mathrm{i}\sum_\mu\gamma_\mu\sin ak_\mu + am_0 + 2r\sum_\mu\sin^2\frac{ak_\mu}{2}}{\sum_\mu\sin^2 ak_\mu + \left(2r\sum_\mu\sin^2\frac{ak_\mu}{2} + am_0\right)^2}$$

Ordinary vertices



$$-g_0(T^a)^{bc}\left(\mathrm{i}\gamma_\mu\cos\frac{a(p_1+p_2)_\mu}{2} + r\sin\frac{a(p_1+p_2)_\mu}{2}\right)$$

Extra vertex



$$-\frac{1}{2}ag_0^2\,\delta_{\mu_1\mu_2}\left(\frac{1}{N_c}\delta^{ab} + d^{abe}T^e\right)^{cd}\left(-\mathrm{i}\gamma_\mu\sin\frac{a(p_1+p_2)_\mu}{2} + r\cos\frac{a(p_1+p_2)_\mu}{2}\right)$$

The renormalization under MS-bar scheme

for the lattice bare quantities

The renormalization of the quark EMT $\mathcal{T}^{\{0i\}q} = \frac{1}{4}\bar{\psi}\gamma^{(0}\overleftrightarrow{D}^{i)}$ with the lattice regularization and under RI-MOM scheme is,

$$Z_L^{MOM} = 1 - \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\log(a^2p^2) + B_{QQ} + \xi] + O(g^4),$$

where B_{QQ} with $B_{QQ}|_{a\to 0} \neq 0$ is the gauge independent finite piece which is sensitive to the lattice quark and gluon actions.

The continuum field renormalization with the dimensional regularization and under RI-MOM and $\overline{MS}$ scheme is,

$$\begin{aligned} Z_{DR}^{\overline{MS}} &= 1 + \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\frac{1}{\epsilon}] + O(g^4), \\ Z_{DR}^{MOM} &= 1 + \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\frac{1}{\epsilon} + \frac{8}{3}\log(\mu^2/p^2) + \frac{40}{9} - \xi] + O(g^4). \end{aligned}$$

So the final renormalization under $\overline{MS}$ scheme for the lattice quantity is,

$$\begin{aligned} Z_L^{\overline{MS}}(a,\mu) &= \frac{Z_{DI}^{\overline{MS}}}{Z_{DI}^{MOM}} Z_L^{MOM}(a,\mu) \\ &= 1 - \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\log(a^2\mu^2) + \frac{40}{9} + B_{QQ}] + O(g^4). \end{aligned}$$

The renormalization of AM

the formulas

From the lattice bare quantities to that under the MOM scheme,

$$\begin{pmatrix} O^{MOM}_{Q,(1)} \\ O^{MOM}_{G,(1)} \end{pmatrix} = \begin{pmatrix} 1-\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(a^2p^2)+B_{QQ}+\xi] & -\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(a^2p^2)+B_{GQ}] \\ +\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(a^2p^2)+B_{QG}] & 1+\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(a^2p^2)+B^f_{GG}]+\frac{g^2N_c}{16\pi^2}[B_{GG}+2\xi-\frac{\xi^2}{4}] \\ +O(g^2)O_{E.O.M.}+O(g^2)O_{G.V.}+O(g^4) & \end{pmatrix} \begin{pmatrix} O^{L}_{Q,(1)} \\ O^{L}_{G,(1)} \end{pmatrix}$$

From the MOM scheme to the MS-bar scheme,

$$\begin{pmatrix} O^{\overline{MS}}_{Q,(1)} \\ O^{\overline{MS}}_{G,(1)} \end{pmatrix} = \begin{pmatrix} 1-\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(\mu^2/p^2)+\frac{40-9\xi}{9}] & -\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(\mu^2/p^2)+\frac{4}{9}] \\ +\frac{g^2C_F}{16\pi^2}[\frac{8}{3}\log(\mu^2/p^2)+\frac{22}{9}] & 1+\frac{g^2N_f}{16\pi^2}[\frac{2}{3}\log(\mu^2/p^2)+\frac{10}{9}]+\frac{g^2N_c}{16\pi^2}(\frac{4}{3}-2\xi+\frac{\xi^2}{4}) \\ +O(g^2)O_{E.O.M.}+O(g^2)O_{G.V.}+O(g^4) & \end{pmatrix} \begin{pmatrix} O^{MOM}_{Q,(1)} \\ O^{MOM}_{G,(1)} \end{pmatrix}$$

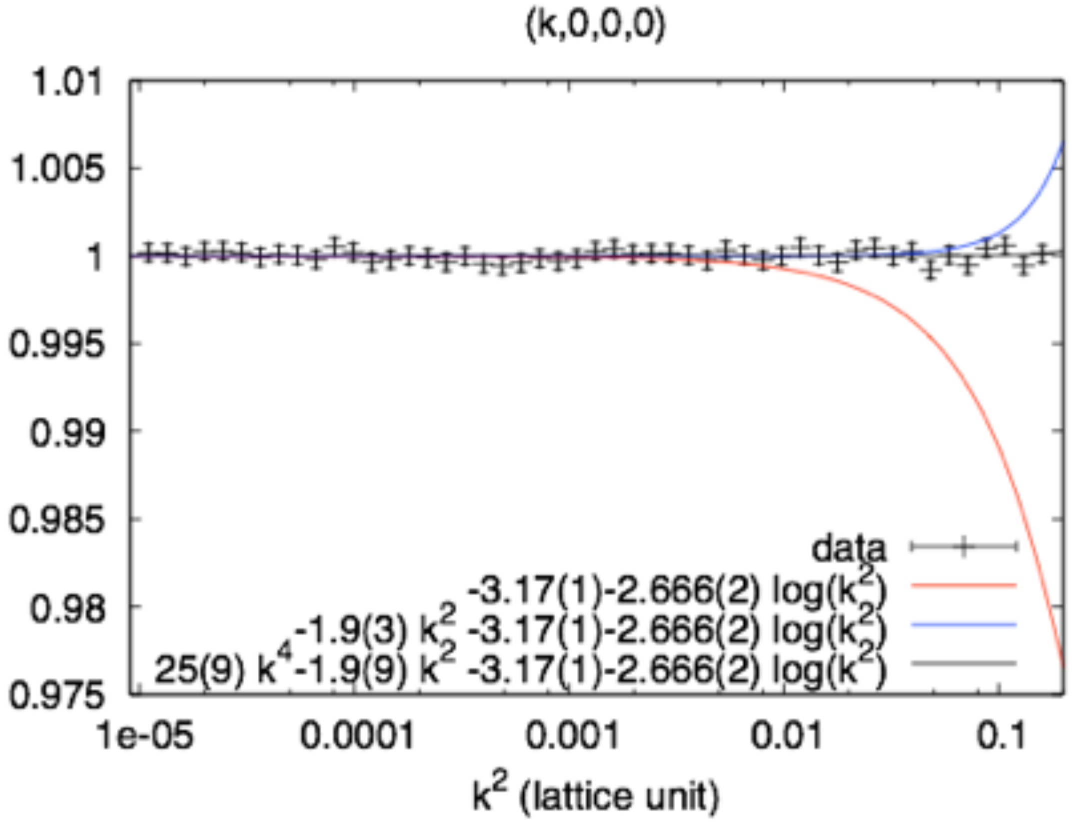
Based on Package-X described in
H. H. Patel, Comput.Phys.Commun.
197 (2015) 276-290

B_{XY} are sensitive to the fermion and gauge action, but ξ independent. One can focus on the case under the Feynman gauge to simplify the calculation.

Do the loop integration numerically…

$$Z_L^{MOM} = 1 - \boxed{\frac{g^2C_F}{16\pi^2}}[\frac{8}{3}\log(a^2p^2) + B_{QQ}] + O(g^4)$$

~0.02



The ratio of the fit v.s. the numerical integration on different $k^2=a^2p^2$

- With the higher order of a^2p^2, the larger a^2p^2 region can be well described with the constant part unchanged.
- $B_{QQ}|_{a\rightarrow 0}$=3.17(1) is precise enough given our statistical error in the simulation.
- We will focus on the constant part of B_{XY} in the following discussions.

The finite pieces

with kinds of actions

$$Z_L^{\overline{MS}}(a,\mu) = \frac{Z_{DI}^{\overline{MS}}}{Z_{DI}^{MOM}} Z_L^{MOM}(a,\mu) = (1 - \frac{g^2 C_F}{16\pi^2}[\frac{8}{3}\log(a^2\mu^2) + \frac{40}{9} + B_{QQ}] + O(g^4)$$

The gluon actions

B_{QQ}	Wilson	Iwasaki	Iwasaki^{HYP}
wilson	-3.17	-2.59	-1.53
overlap	-34.90	-18.83	-4.89
D_c	-42.10	-24.25	-8.63

The quark actions

- The values are sensitive to both the quark and gluon actions.
- The values with the unimproved Wilson glue action can be very large.
- The HYP smearing can make the values smaller and become less sensitive to the quark action.

The renormalization of AM

the results

From the lattice bare quantities with the chiral fermion and HYP smeared Iwasaki gluon to that under the MS-bar scheme,
at a scale μ=1/a,

$$\begin{pmatrix} O^{\overline{MS}}_{Q,(1)} \\ O^{\overline{MS}}_{G,(1)} \end{pmatrix} = \begin{pmatrix} 1-\frac{g^2C_F}{16\pi^2}[\frac{40}{9}-8.63(1)] & -\frac{g^2N_f}{16\pi^2}[\frac{4}{9}+0.20(1)] \\ +\frac{g^2C_F}{16\pi^2}[\frac{22}{9}+3.56(1)] & 1+\frac{g^2N_f}{16\pi^2}[\frac{10}{9}-3.52(1)] \\ & +\frac{g^2N_c}{16\pi^2}[\frac{4}{3}+1.54(1)+V.T.] \end{pmatrix} \begin{pmatrix} O^{L}_{Q,(1)} \\ O^{L}_{G,(1)} \end{pmatrix} + O(g^4)$$

$$\xrightarrow[g^2\sim 3]{} \begin{pmatrix} 1.1060(2) & -0.0122(2)N_f \\ 0.1521(2) & 1.1637(6)-0.0458(2)N_f+0.0570V.T.] \end{pmatrix} \begin{pmatrix} O^{L}_{Q,(1)} \\ O^{L}_{G,(1)} \end{pmatrix} + O(g^4)$$

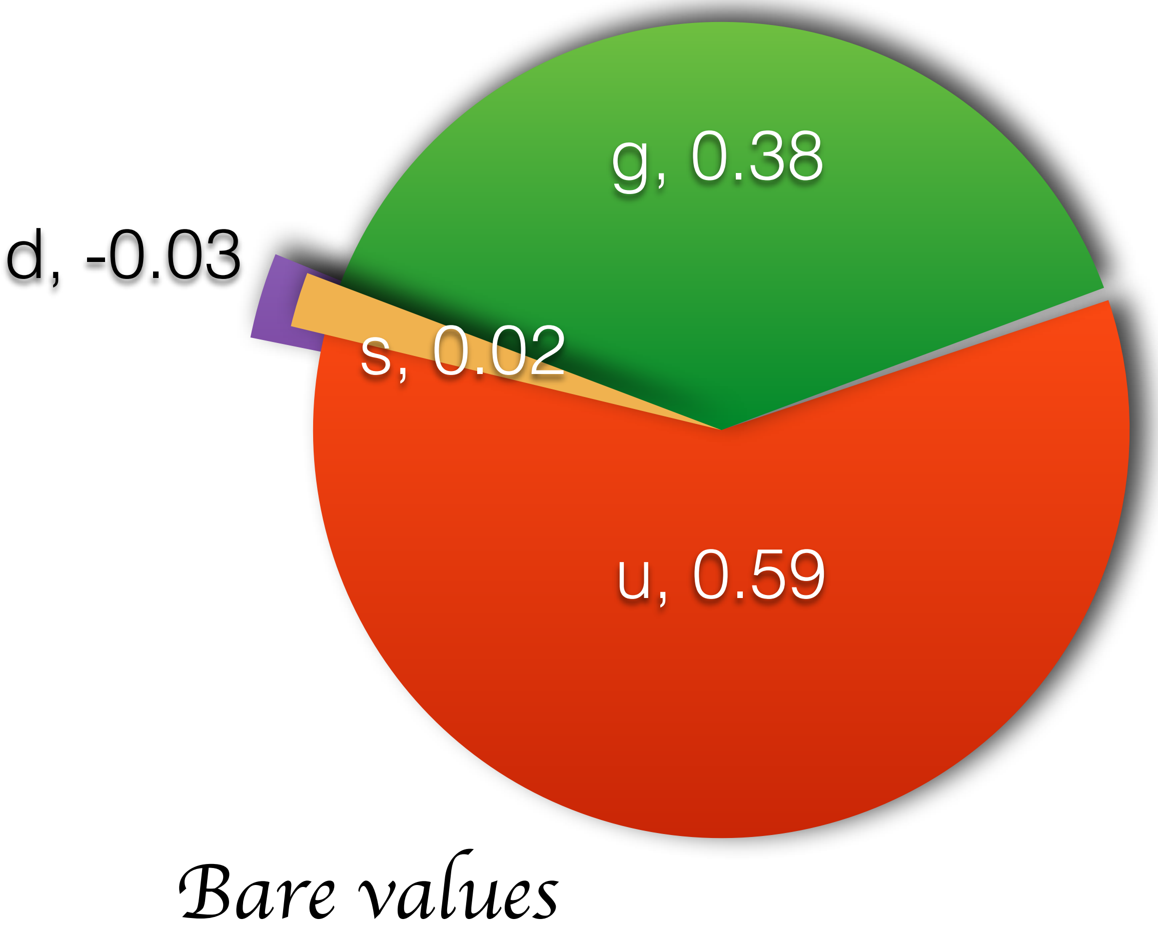
V.T.: The 4-gluon vertex tadpole contribution, in progress.

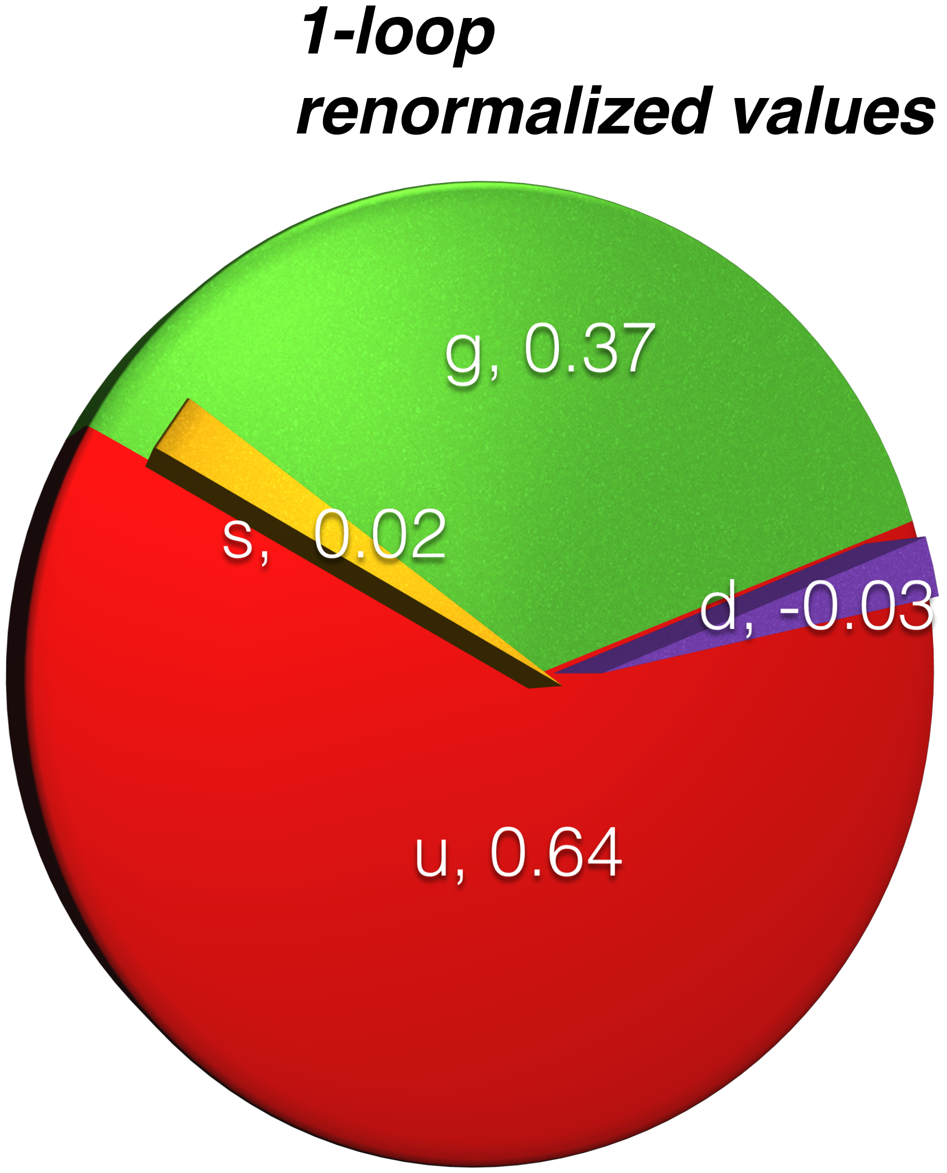
We can force the sum rule of the momentum fractions to avoid the calculation of V.T., and the final normalization factor for the gluon operator is ***~0.8.***

The pie charts

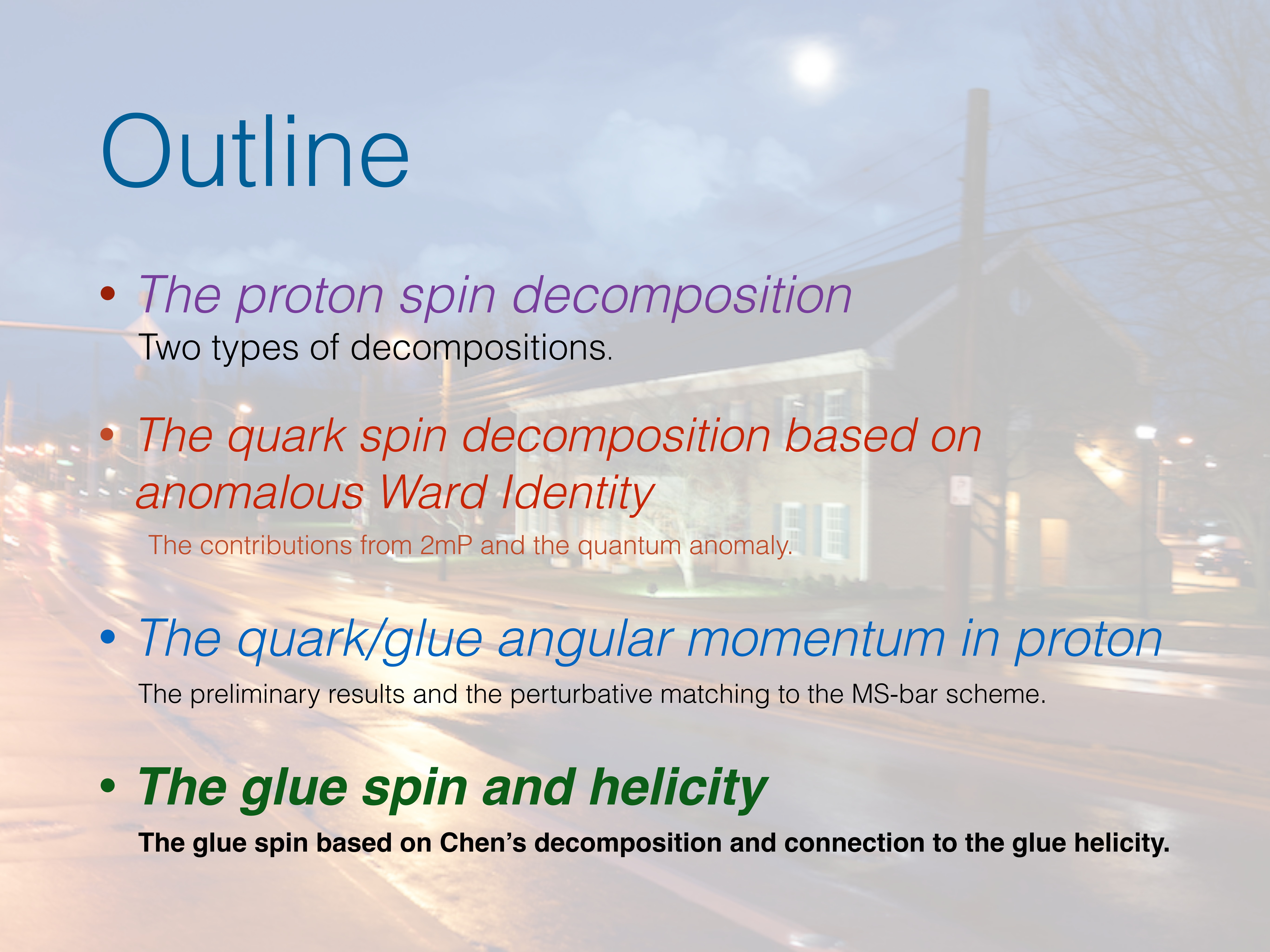
of the quark and gluon AM in proton

m_π=400 MeV, preliminary





The percentage of the angular momentum in proton



Outline

- *The proton spin decomposition*

  Two types of decompositions.

- *The quark spin decomposition based on anomalous Ward Identity*

  The contributions from 2mP and the quantum anomaly.

- *The quark/glue angular momentum in proton*

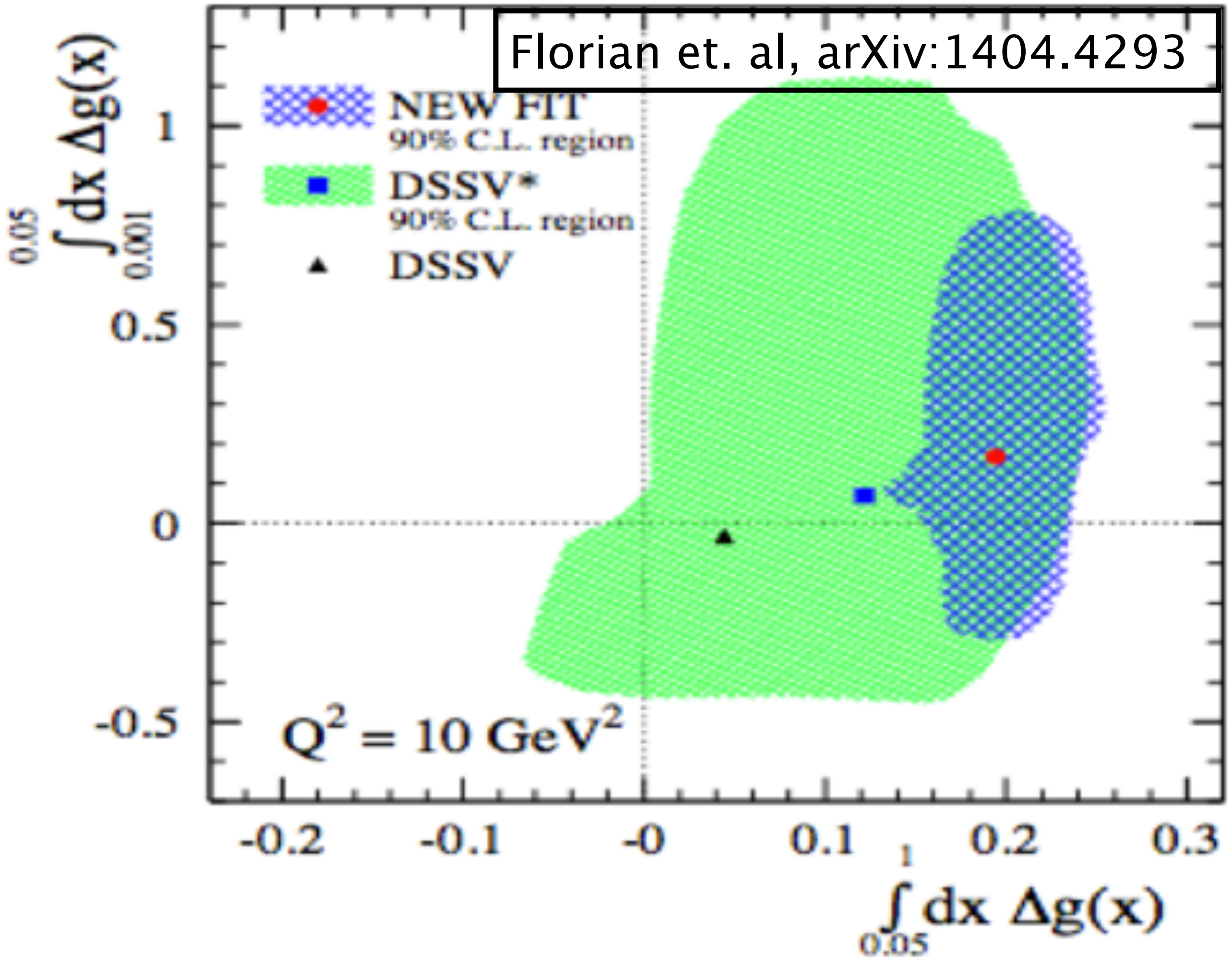
  The preliminary results and the perturbative matching to the MS-bar scheme.

- ***The glue spin and helicity***

  The glue spin based on Chen's decomposition and connection to the glue helicity.

Glue spin

The glue helicity



Florian et. al, arXiv:1404.4293

The global fit based on recent experimental data (2009 RHIC) shows evidence of nonzero polarization of gluon in the proton. The glue helicity is defined as,

$$\Delta G = \int_0^1 \Delta g(x) dx$$

$$= \int dx \frac{i}{2xP+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle$$

A. V. Manohar, Phys. Lett. B255, 579 (1991)

After integrating the longitudinal momentum x,

$$O_{\Delta G} = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla +}(\vec{\nabla} A^{+,b})\mathcal{L}^{ba}(\xi^-,0))\right]$$

Y. Hatta, Phys. Rev. D84, 041701 (2011),
X. Ji, J.H. Zhang, and Y. Zhao, Phys. Rev. Lett. 111 112002 (2013)

Proton Spin decomposition

Decomposition targets to the IMF quantities

X. –S. Chen et al., Phys. Rev. Lett. 100, 232002 (2008).
X. –S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009).

quark spin *quark OAM*

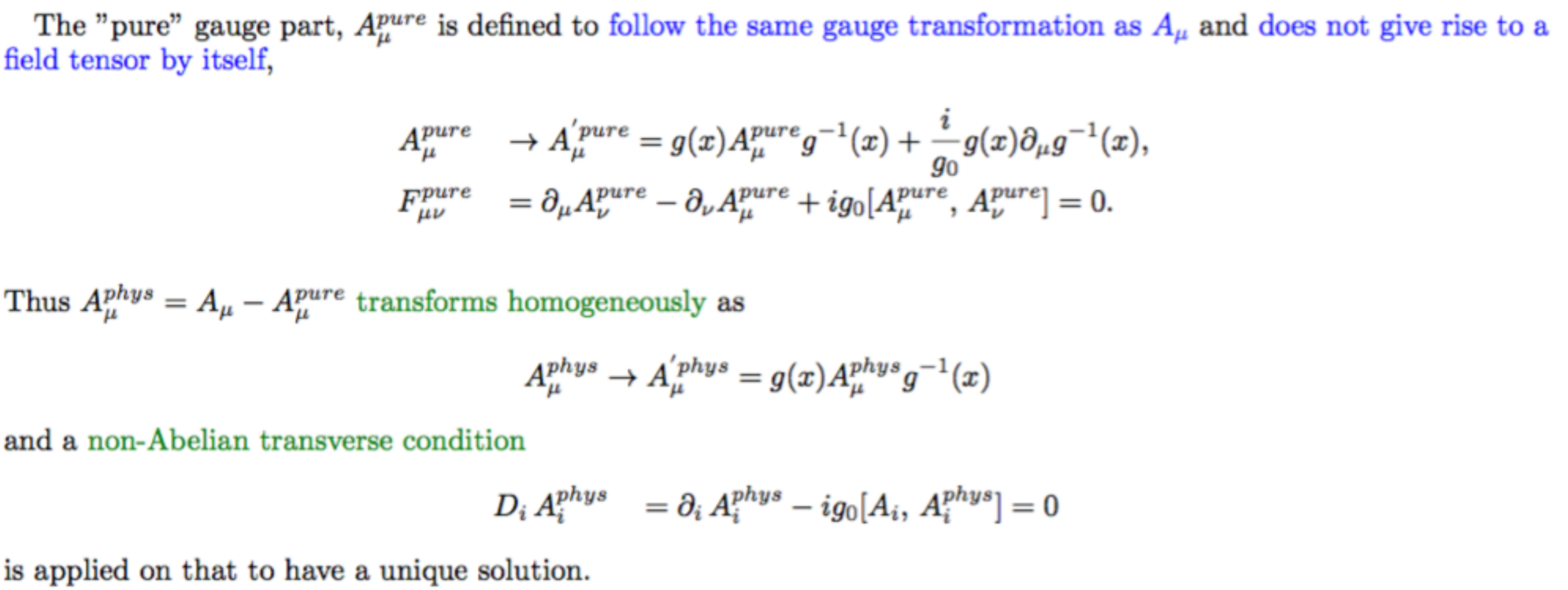
$$\vec{J} = \boxed{\int d^3x\, \frac{1}{2}\, \overline{\psi}\, \vec{\gamma}\, \gamma^5\, \psi} + \boxed{\int d^3x \psi^\dagger \left\{ \vec{x} \times (i\vec{D}^{pure}) \right\} \psi}$$

$$+ \boxed{\int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}^{phys}} + \boxed{\int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \overrightarrow{D^{pure}} A^{i,phys}]}$$

glue spin *glue OAM*

Gauge invariant but frame dependent

What is A^{phys}?



The "pure" gauge part, A_μ^{pure} is defined to follow the same gauge transformation as A_μ and does not give rise to a field tensor by itself,

$$\begin{aligned} A_\mu^{pure} &\to A_\mu^{\prime pure} = g(x) A_\mu^{pure} g^{-1}(x) + \frac{i}{g_0} g(x) \partial_\mu g^{-1}(x), \\ F_{\mu\nu}^{pure} &= \partial_\mu A_\nu^{pure} - \partial_\nu A_\mu^{pure} + i g_0 [A_\mu^{pure}, A_\nu^{pure}] = 0. \end{aligned}$$

Thus $A_\mu^{phys} = A_\mu - A_\mu^{pure}$ transforms homogeneously as

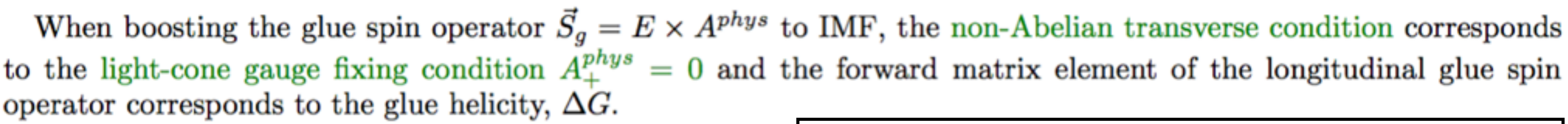
$$A_\mu^{phys} \to A_\mu^{\prime phys} = g(x) A_\mu^{phys} g^{-1}(x)$$

and a non-Abelian transverse condition

$$D_i \, A_i^{phys} \quad = \partial_i \, A_i^{phys} - i g_0 [A_i, \, A_i^{phys}] = 0$$

is applied on that to have a unique solution.

X. -S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009).



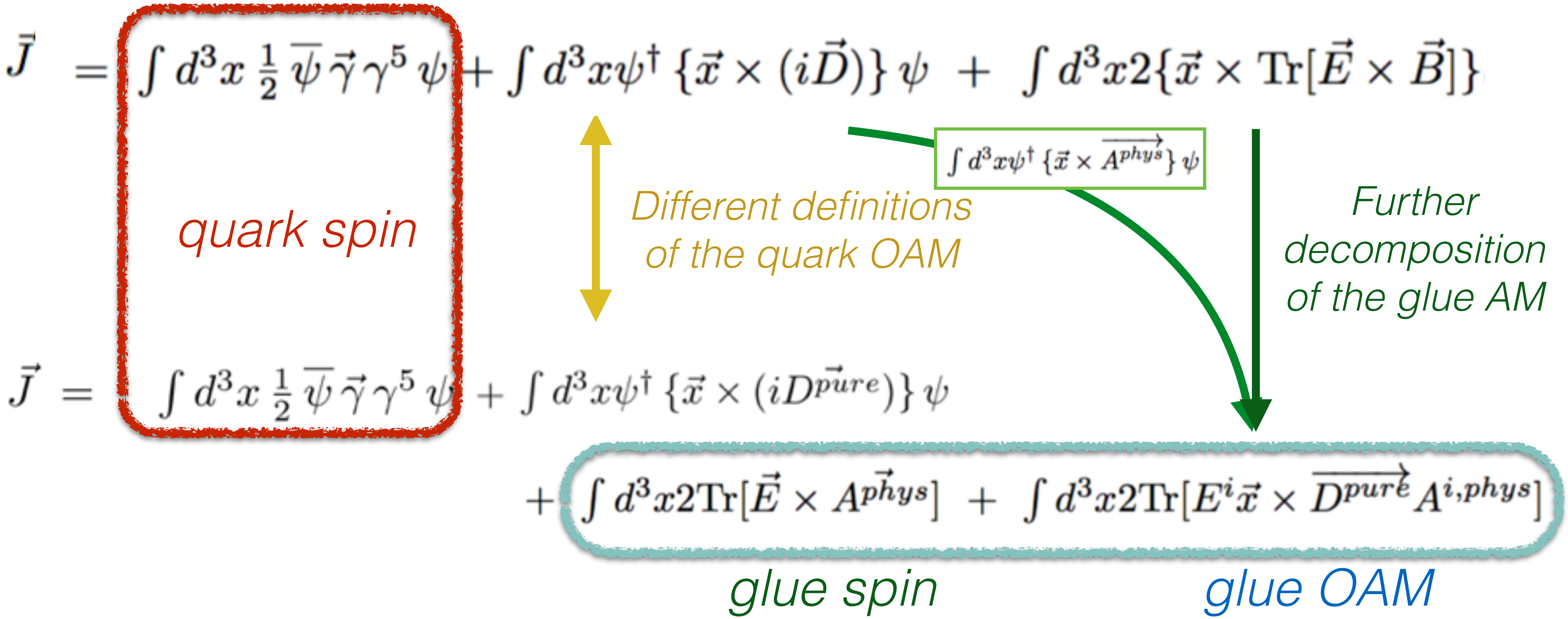
When boosting the glue spin operator $\vec{S}_g = E \times A^{phys}$ to IMF, the non-Abelian transverse condition corresponds to the light-cone gauge fixing condition $A_+^{phys} = 0$ and the forward matrix element of the longitudinal glue spin operator corresponds to the glue helicity, ΔG.

X. Ji, J.H. Zhang, and Y. Zhao, Phys. Rev. Lett. 111 112002 (2013)

Proton Spin decomposition

Two decompositions

X.D. Ji., Phys. Rev. Lett. 78, 610-613 (1997).



$$\vec{J} = \int d^3x\, \frac{1}{2}\,\overline{\psi}\,\vec{\gamma}\,\gamma^5\,\psi + \int d^3x\psi^\dagger\,\{\vec{x}\times(i\vec{D})\}\,\psi + \int d^3x2\{\vec{x}\times\mathrm{Tr}[\vec{E}\times\vec{B}]\}$$

$$\int d^3x\psi^\dagger\,\{\vec{x}\times\overrightarrow{A^{phys}}\}\,\psi$$

quark spin

Different definitions of the quark OAM

Further decomposition of the glue AM

$$\vec{J} = \int d^3x\, \frac{1}{2}\,\overline{\psi}\,\vec{\gamma}\,\gamma^5\,\psi + \int d^3x\psi^\dagger\,\{\vec{x}\times(iD^{\vec{pure}})\}\,\psi$$

$$+ \int d^3x2\mathrm{Tr}[\vec{E}\times A^{\vec{phys}}] + \int d^3x2\mathrm{Tr}[E^i\vec{x}\times\overrightarrow{D^{pure}}A^{i,phys}]$$

glue spin

glue OAM

X. -S. Chen et al., Phys. Rev. Lett. 100, 232002 (2008).
X. -S. Chen et al., Phys. Rev. Lett. 103, 062001 (2009).

How to obtain S_g on the lattice?

If one can find a gauge transformation g_c to make the gauge potential after the rotation

$$A_{c,\mu} = g_c^{-1} A_\mu g_c + \frac{i}{g_0} g_c \partial_\mu g_c^{-1}, \text{ or equivalently } A_\mu = g_c A_{c,\mu} g_c^{-1} + \frac{i}{g_0} g_c \partial_\mu g_c^{-1}$$

to satisfy the condition $\partial \cdot A = 0$. Then it is easy to confirm that the decomposition defined by,

$$A_\mu^{pure} = \frac{i}{g_0} g_c \partial_\mu g_c^{-1}, \; A_\mu^{phys} = g_c A_{c,\mu} g_c^{-1}.$$

can satisfy all the requirement of the decomposition defined by Chen et.al. In the other word, Chen et.al's decomposition is equivalent to the gauge invariant extension of the Coulomb gauge.

C. Lorce, et al. Phys.Rev. D85 (2012) 114006
Yong Zhao, Keh-Fei Liu, Yibo Yang, arXiv:1506.08832

On the lattice, such a gauge transformation g_c can be obtained numerically with $O(a)$ corrections. So the glue spin operator on the lattice can be simply defined on the Coulomb gauge fixed configuration,

$$\boxed{\vec{S}_g = \int d^3x \; 2\mathrm{Tr}(\vec{E} \times g_c \vec{A}_c g_c^{-1}) = \int d^3x \; 2\mathrm{Tr}(\vec{E}_c \times \vec{A}_c)}$$

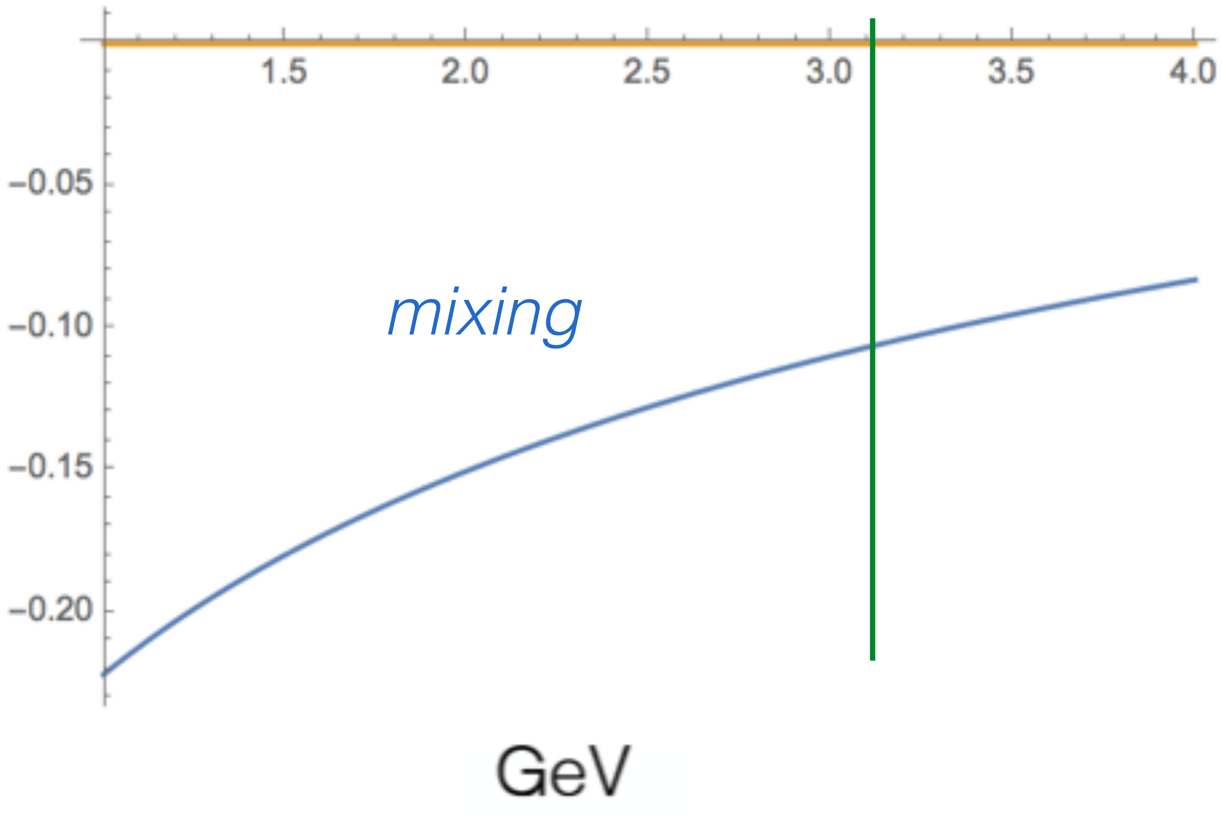
with E_c and A_c are the lattice version of the electric field and gauge potential.

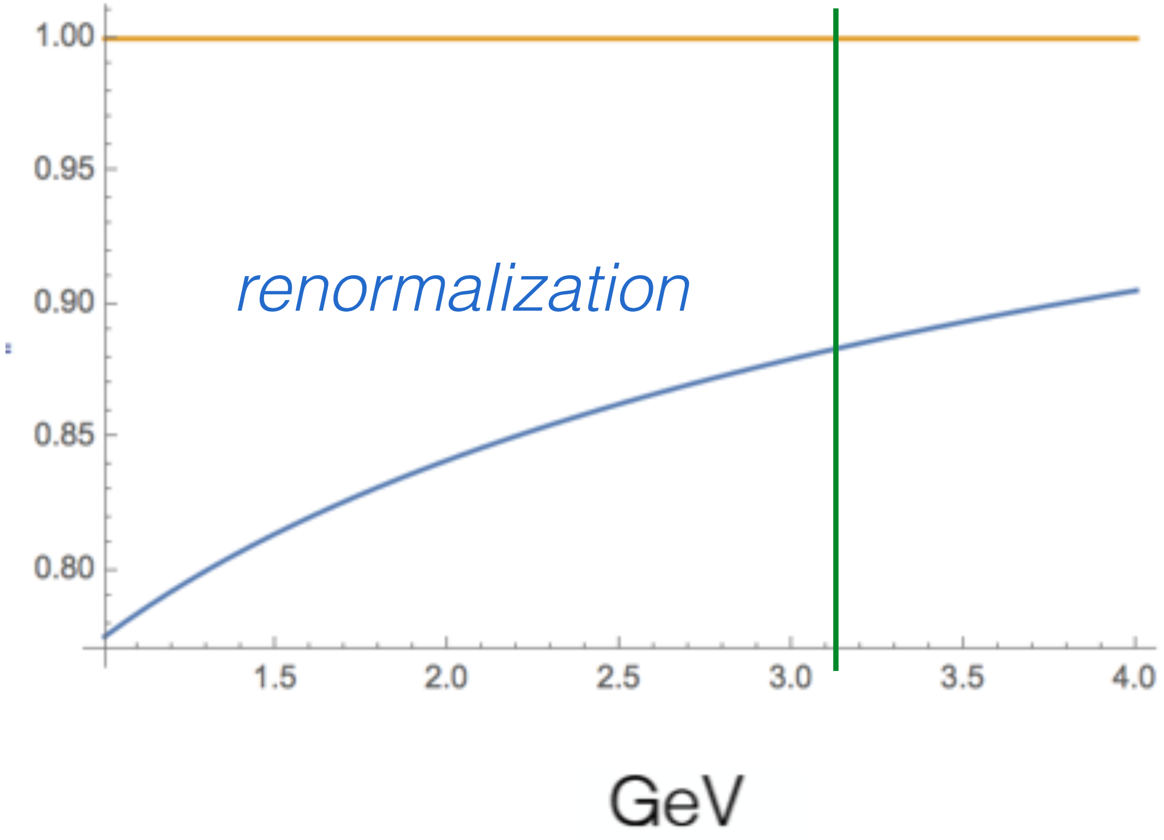
Glue spin

The renormalization and mixing

$$S^{\overline{MS}}_{G,(1)} = \left(1 + \frac{g^2 N_f}{16\pi^2}[\frac{2}{3}\log(\mu^2 a^2) + \frac{10}{9} - 3.52(1)] + \frac{g^2 C_A}{16\pi^2}[-\frac{4}{3}\log(\mu^2 a^2) + 1.6949 + C_{GG}]\right) S^L_{G,(1)}$$
$$- \frac{g^2 C_F}{16\pi^2}[\frac{5}{3}\log(\mu^2 a^2) + 3.1921 + 1.72(1)] \sum_{q=u,d,s...} \Delta_q^{L,(1)} + O(g^4),$$

The scale used by the experiment for the glue helicity is ***μ^2=10 GeV***

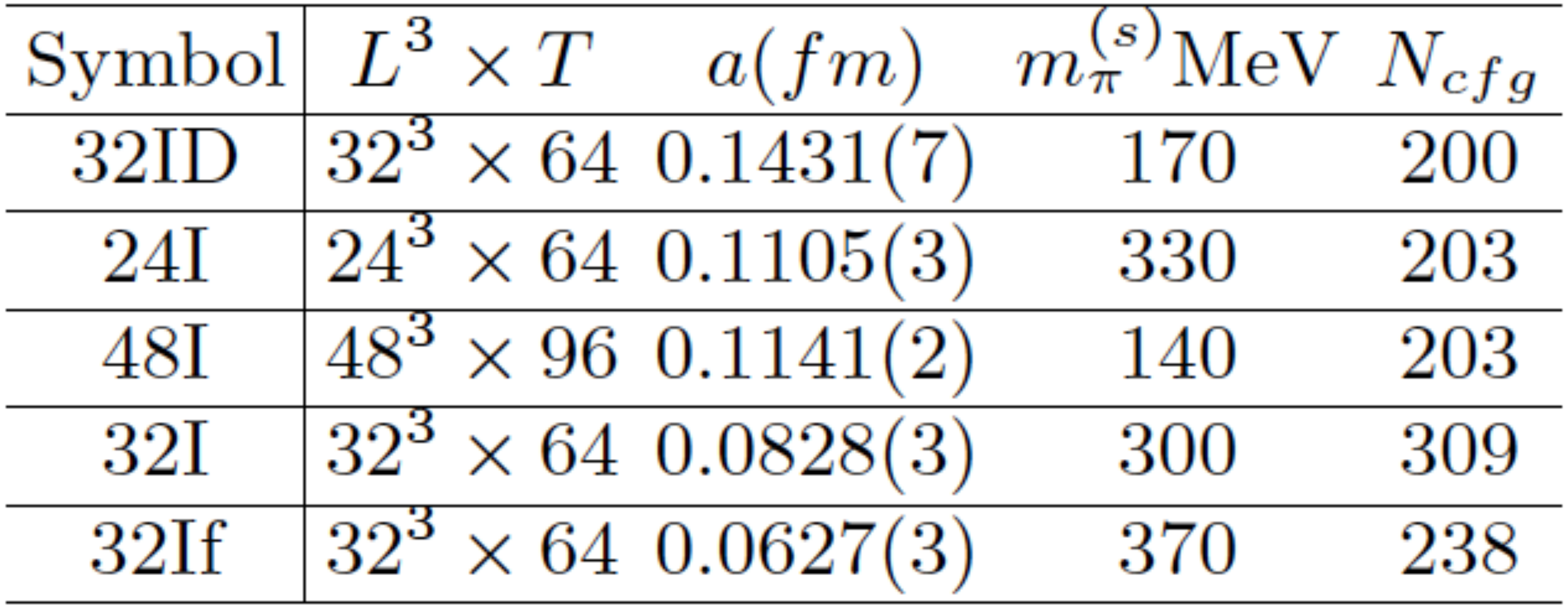




Glue Spin

Lattice setup

- Overlap valence quark on 2+1 Domain wall fermion configuration.



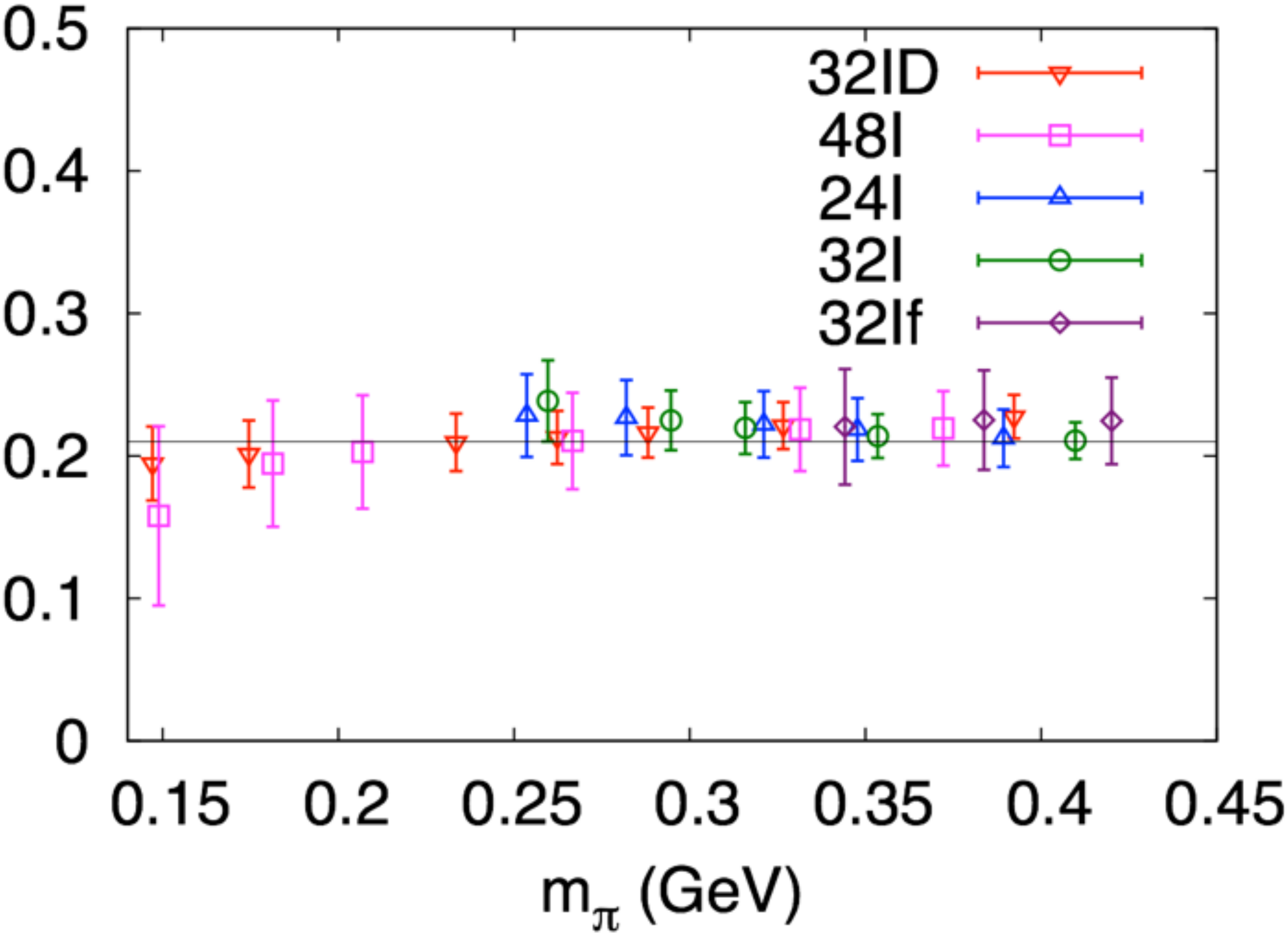
Symbol	$L^3 \times T$	$a(fm)$	$m_\pi^{(s)}$MeV	N_{cfg}
32ID	$32^3 \times 64$	0.1431(7)	170	200
24I	$24^3 \times 64$	0.1105(3)	330	203
48I	$48^3 \times 96$	0.1141(2)	140	203
32I	$32^3 \times 64$	0.0828(3)	300	309
32If	$32^3 \times 64$	0.0627(3)	370	238

T. Blum et al. (RBC, UKQCD), Phys. Rev. D93, 074505 (2016)

- 2pt: grid smear source, loop over t, with the low mode substitution.
- glue: clover operator based on 5 HYP smeared configuration.

The dependence

of m_π, a, and V



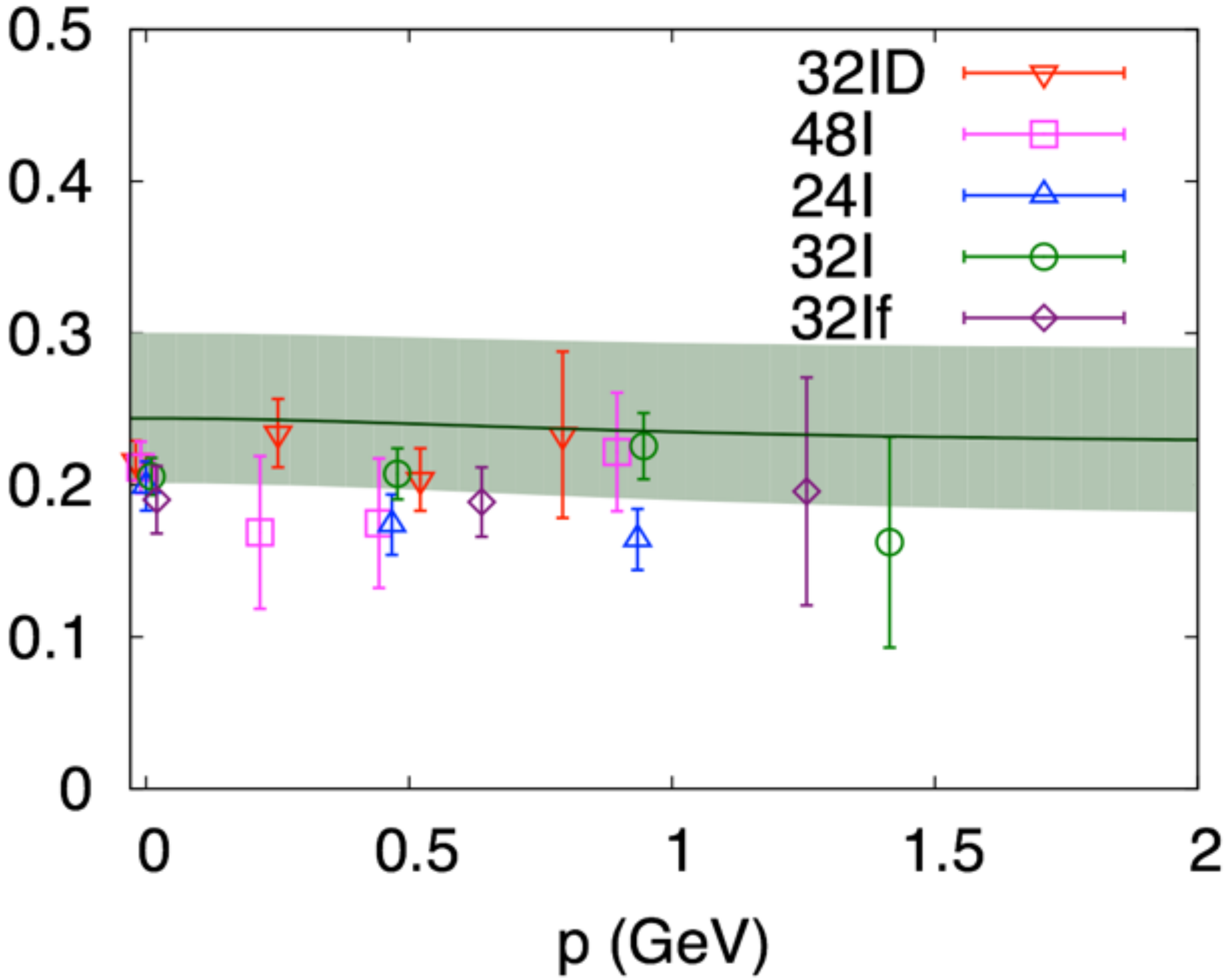
The pion mass (both valence and sea), lattice spacing and volume dependences are mild.

Glue spin

The frame dependence

$$S_G(p^2) = C_0 + \frac{C_1}{M^2 + p^2} + C_2 m_{\pi,vv}^2 + C_3 m_{\pi,ss}^2 + C_4 a,$$

$$M = 0.939 \text{ GeV}$$



The glue spin at the large momentum limit for the bare value at μ^2=10GeV^2:

S_G=0.226(53)

Summary

- **The anomalous Ward identity provides a new way to understand the structure of the quark spin in proton.**
- ***1. The cancelation of the 2mP and the topological charge term has been confirmed at the physical point on the 5.6 fm lattice.***
- ***2. The results need to be confirmed with the conserved current.***

- **The proton spin decomposition based on Ji's scheme is in progress.**
- ***1. The perturbative matching is almost done and the simulation is ongoing.***

- **The glue spin in the finite momenta <2 GeV^2 have been obtained at several lattice spacing and volumes, with the chiral extrapolation.**
- ***1. The glue spin and helicity at 10 GeV^2 are ~0.23(5) .***
- ***2. The perturbative matching is ongoing.***

- **Our goal is to obtain the final results with all the systematic uncertainty (lattice spacing, volume, physical pion mass etc.) under control.**