Higgs Modes
in Cold Atoms and Superconductor

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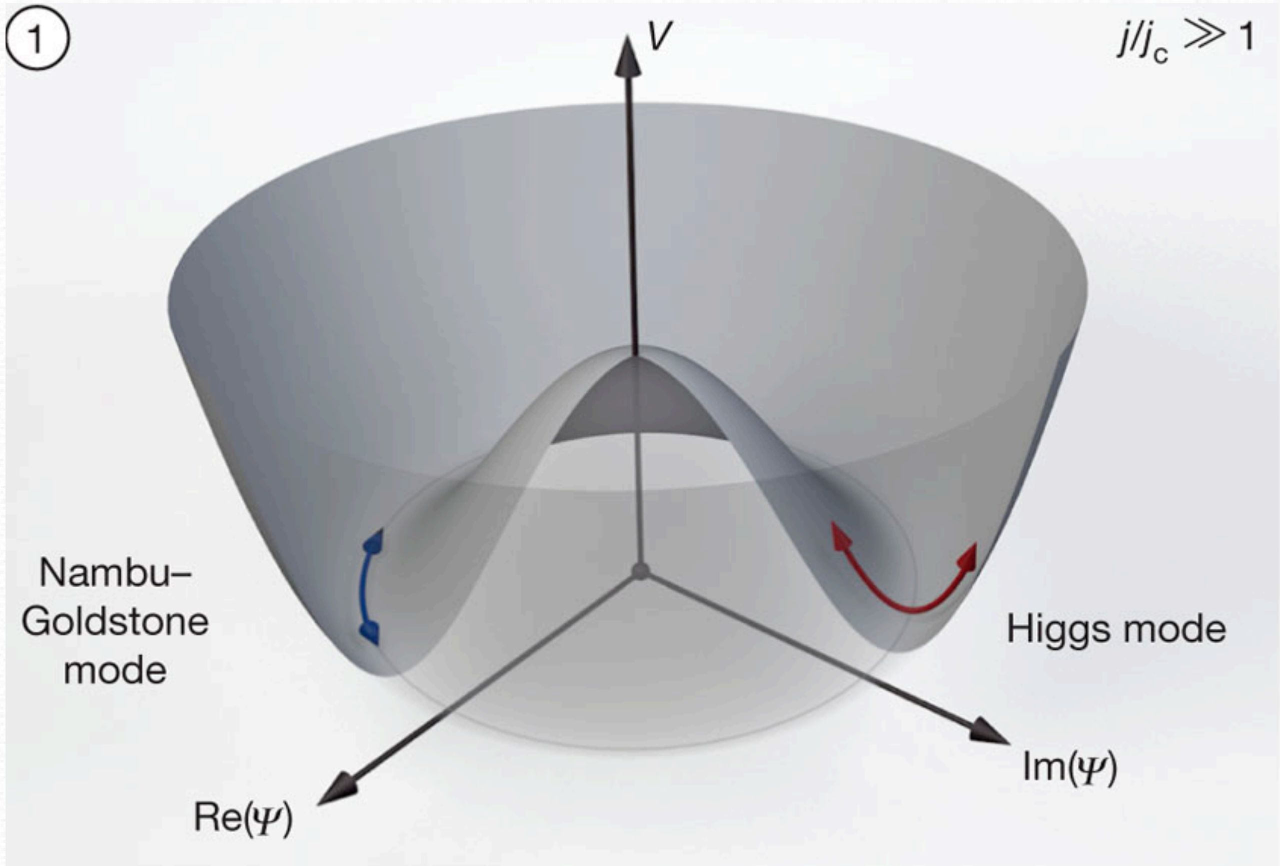
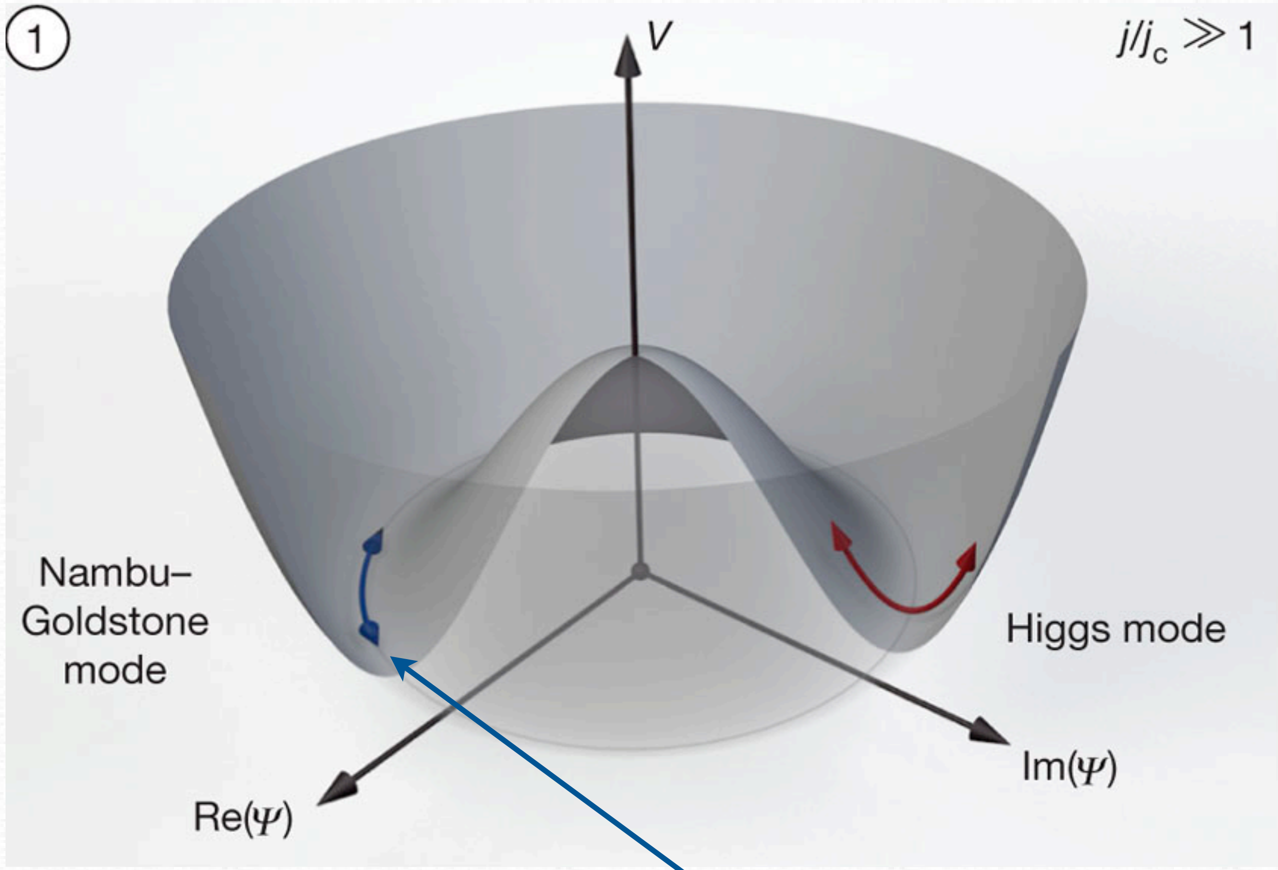
December, 2014

Higgs Mode



$$\phi = \sqrt{\rho + \delta\rho}\, e^{i\theta}$$

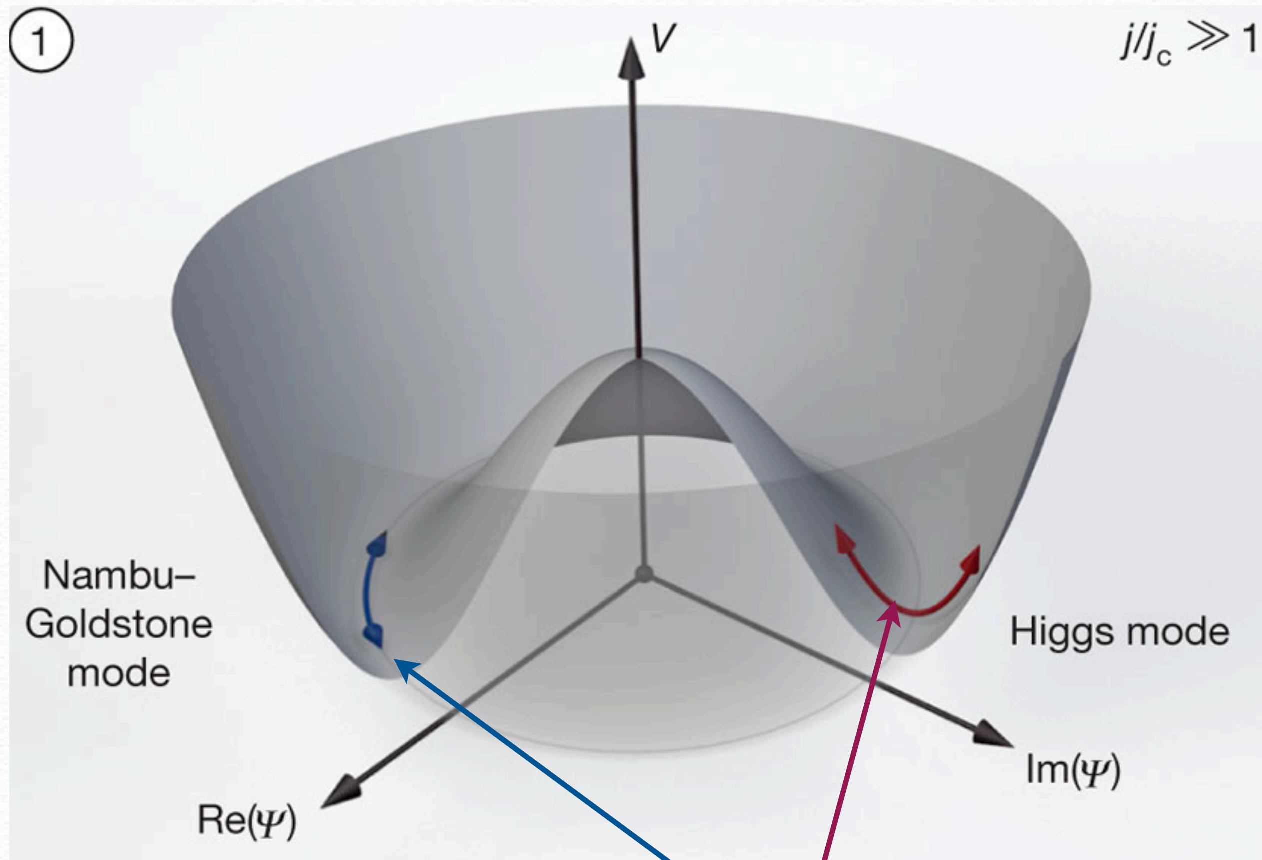
Higgs Mode



$$\phi = \sqrt{\rho + \delta\rho}e^{i\theta}$$

Phase fluctuation
Nambu-Goldstone mode

Higgs Mode



$$\phi = \sqrt{\rho + \delta\rho}\, e^{i\theta}$$

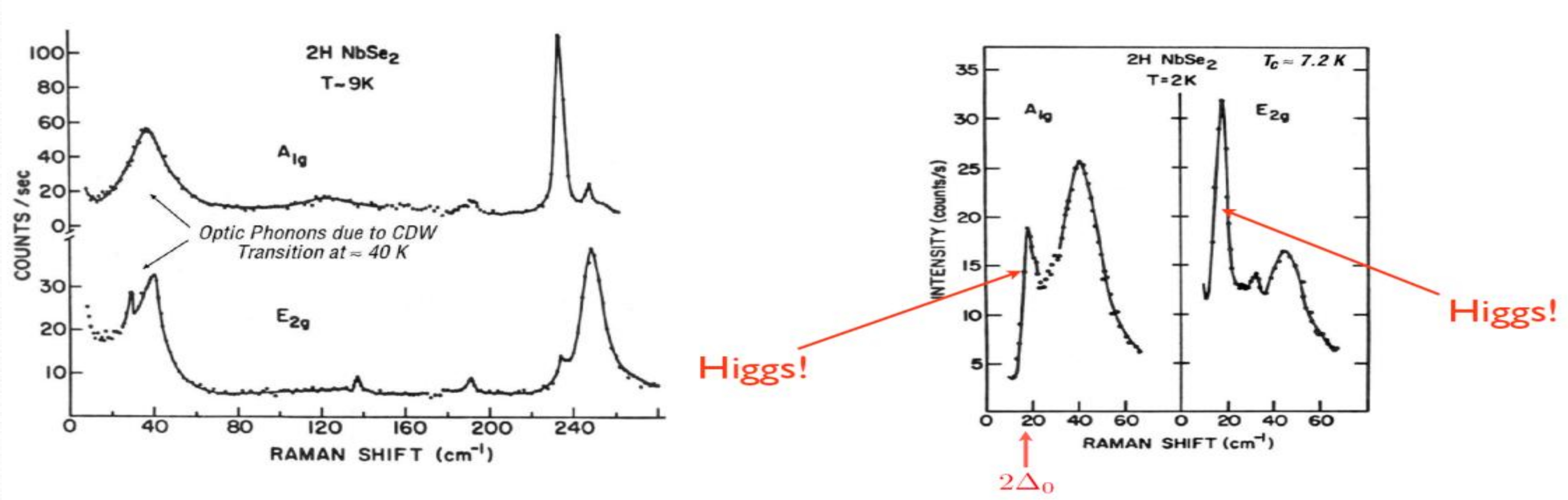
Phase fluctuation
Nambu-Goldstone mode

Amplitude fluctuation
Higgs mode

Experiments about Higgs modes

Superconducting charge density wave compound

$NbSe_2$



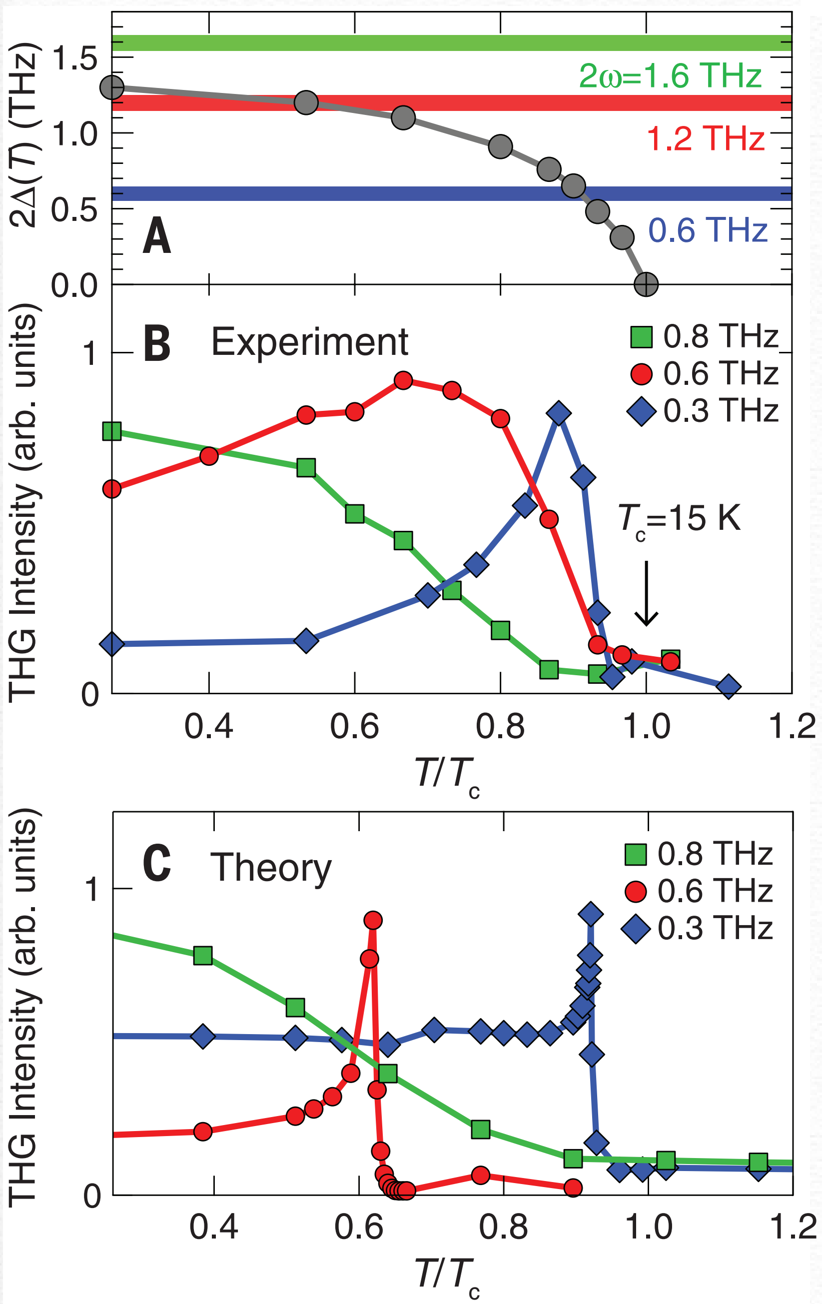
Raman scattering experiment

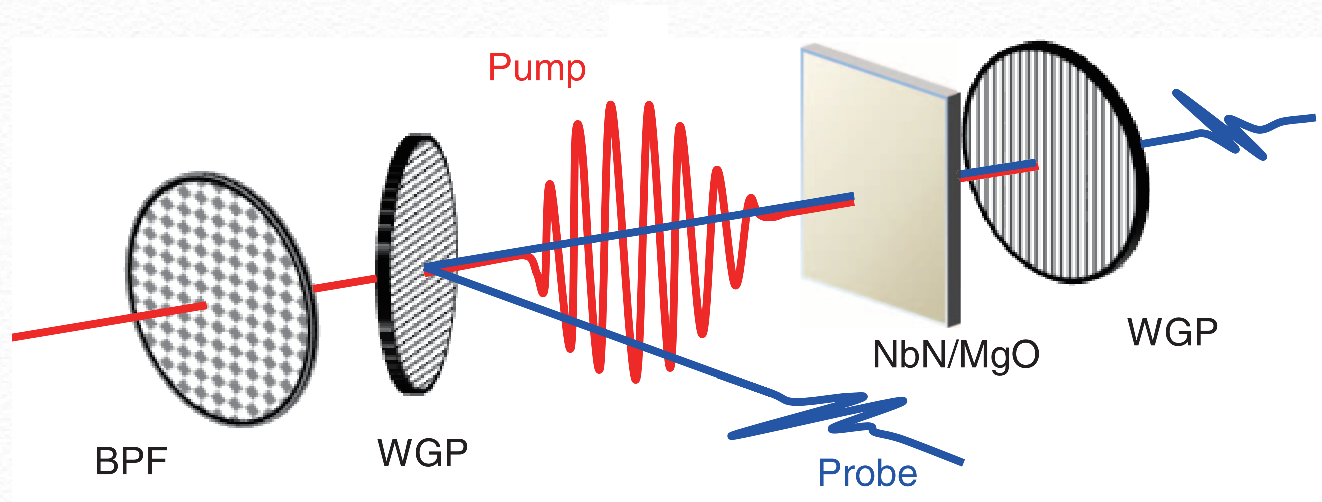
Sooryakumar and Klein, PRL, 45, 660 (1980);
Littlewood and Varma, PRB, 26, 4883 (1992); PRL, 47, 811 (1981)

Experiments about Higgs modes

Superconducting compound

NbN



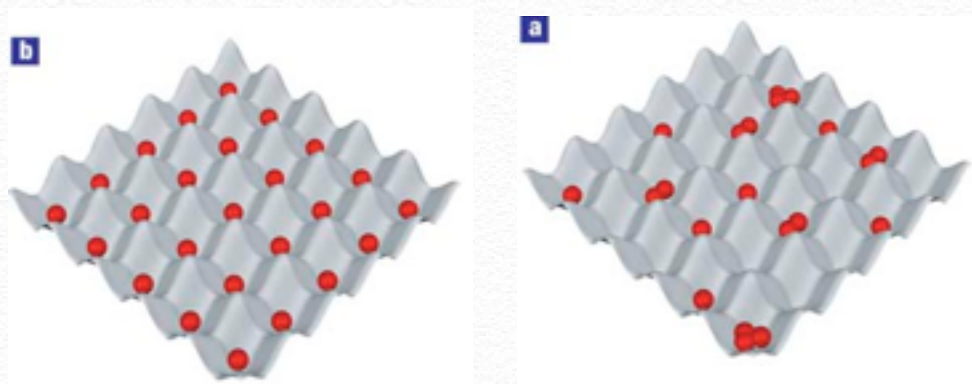


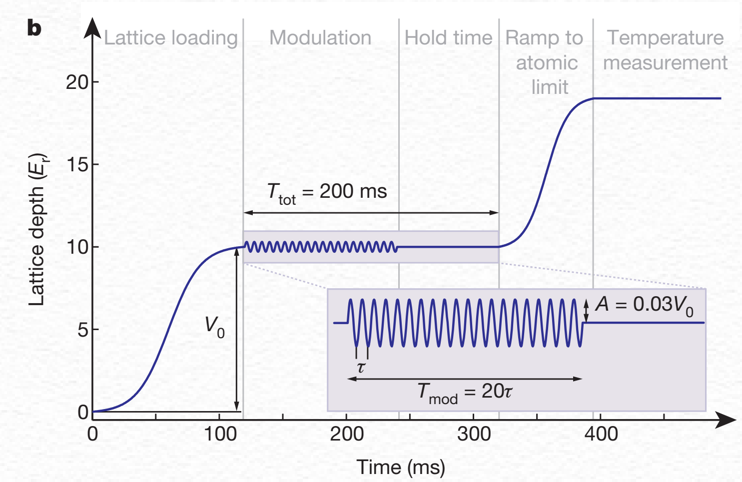
Pump-Probe experiment

Tokyo group, Science, 345, 1145 (2014)

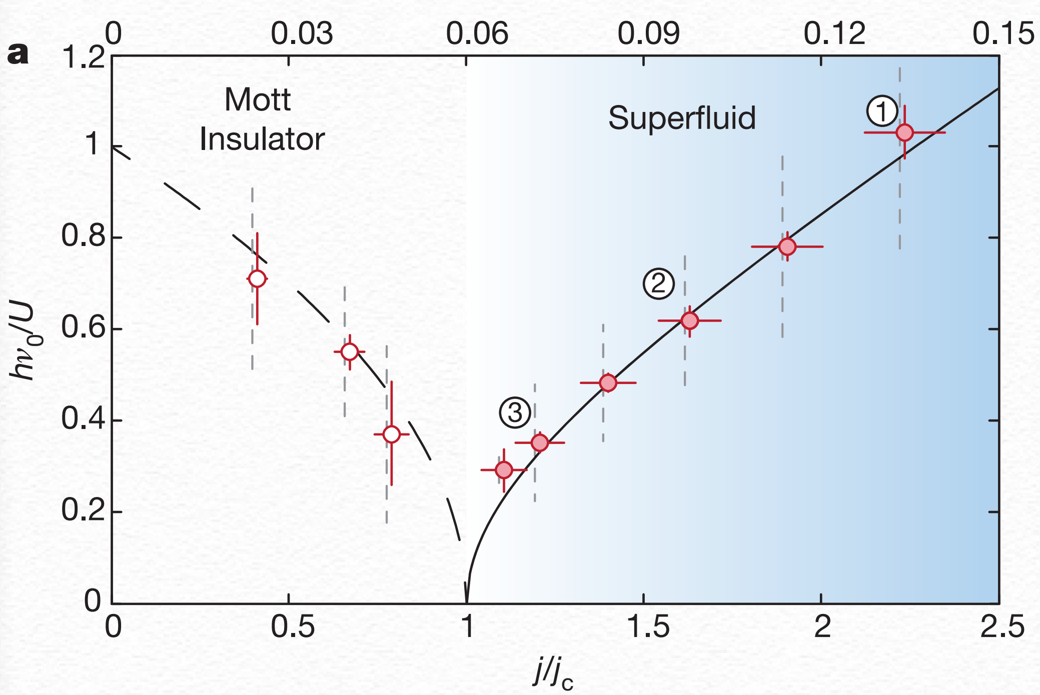
Experiments about Higgs modes

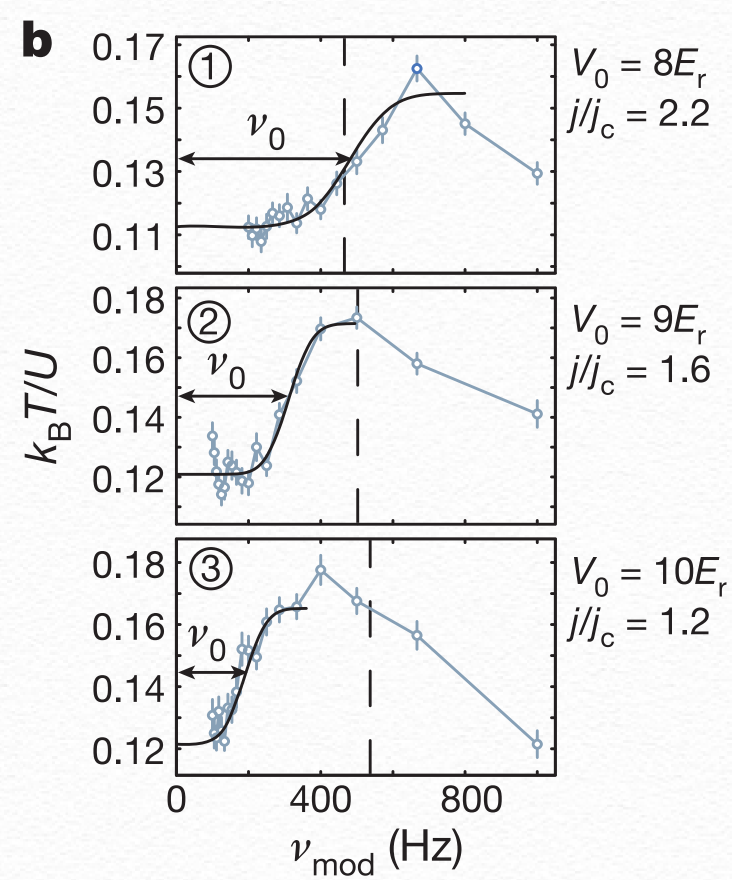
Cold atom system: bosons in optical lattices





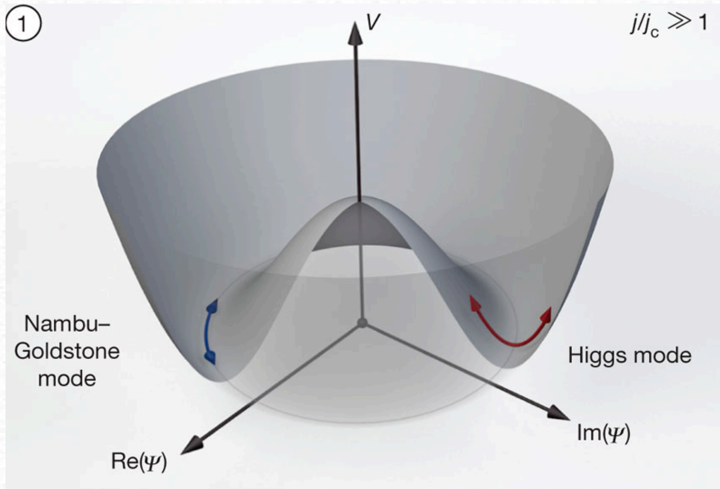
Lattice modulation spectroscopy





Munich group, Nature, 487, 454 (2012)

Higgs Mode



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Higgs Mode

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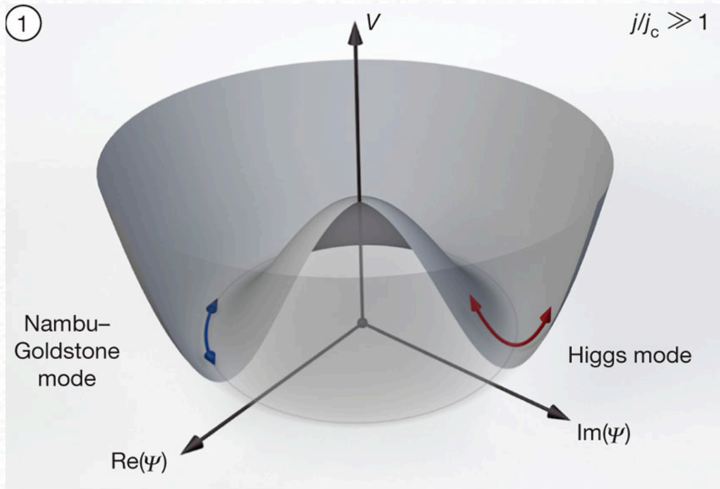
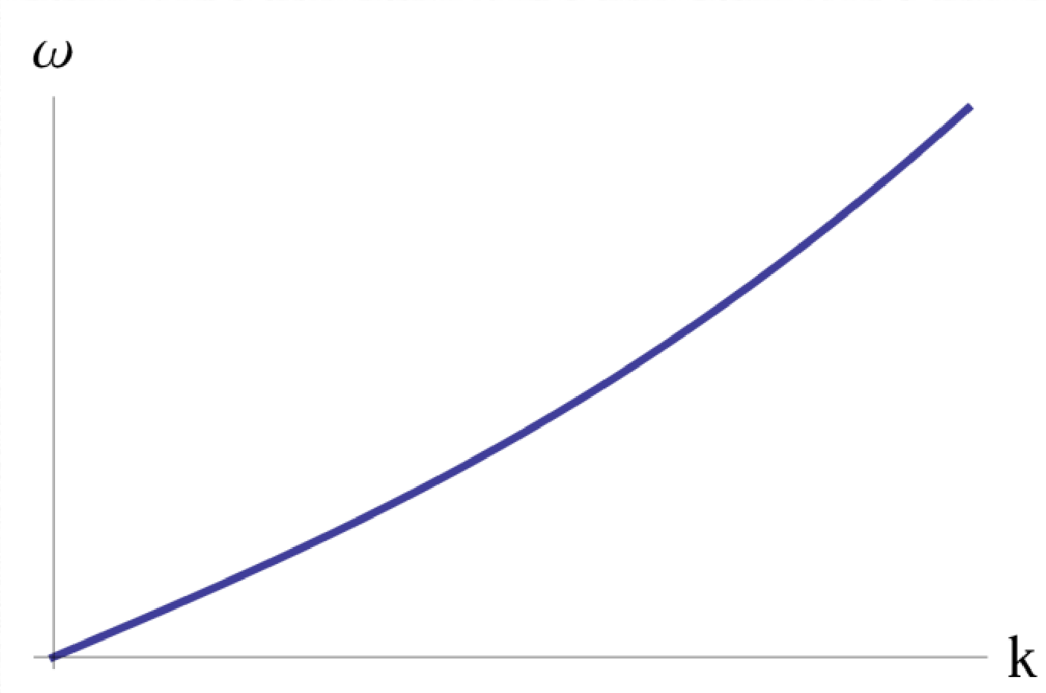


Non-relativistic theory:

$$S = \int dt d^3\mathbf{x} \left\{ \phi^* (-i\partial_t - \frac{\nabla^2}{2m} - r)\phi + \frac{b}{2}|\phi|^4 \right\}$$

Bogoliubov spectrum:

$$\omega = \sqrt{k^2/2m(k^2/2m + 2r)}$$



Higgs Mode

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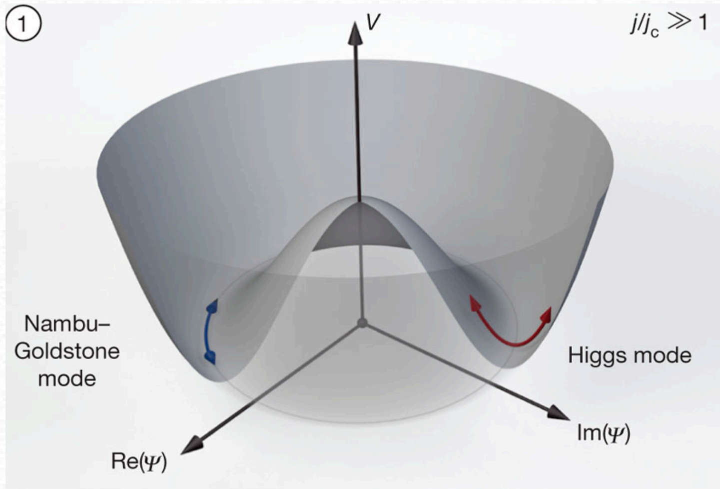


Relativistic theory:

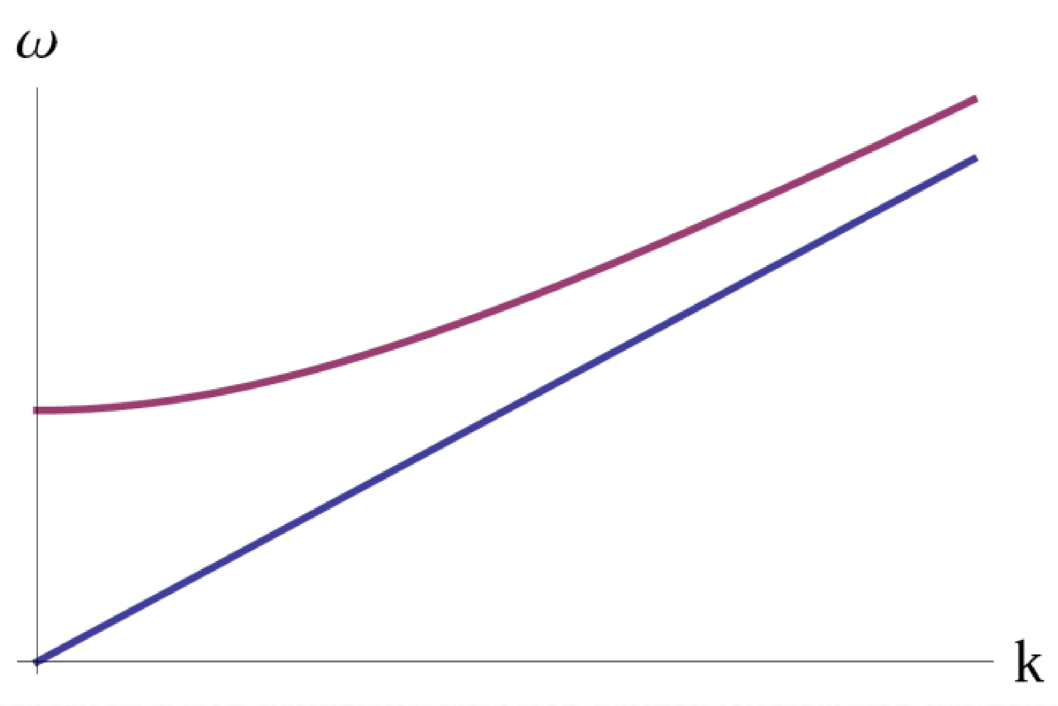
$$S = \int dt d^3\mathbf{x} \left\{ \phi^* (\partial_t^2 - \frac{\nabla^2}{2m} - r)\phi + \frac{b}{2}|\phi|^4 \right\}$$

Gapless Numbu-Goldstone mode:

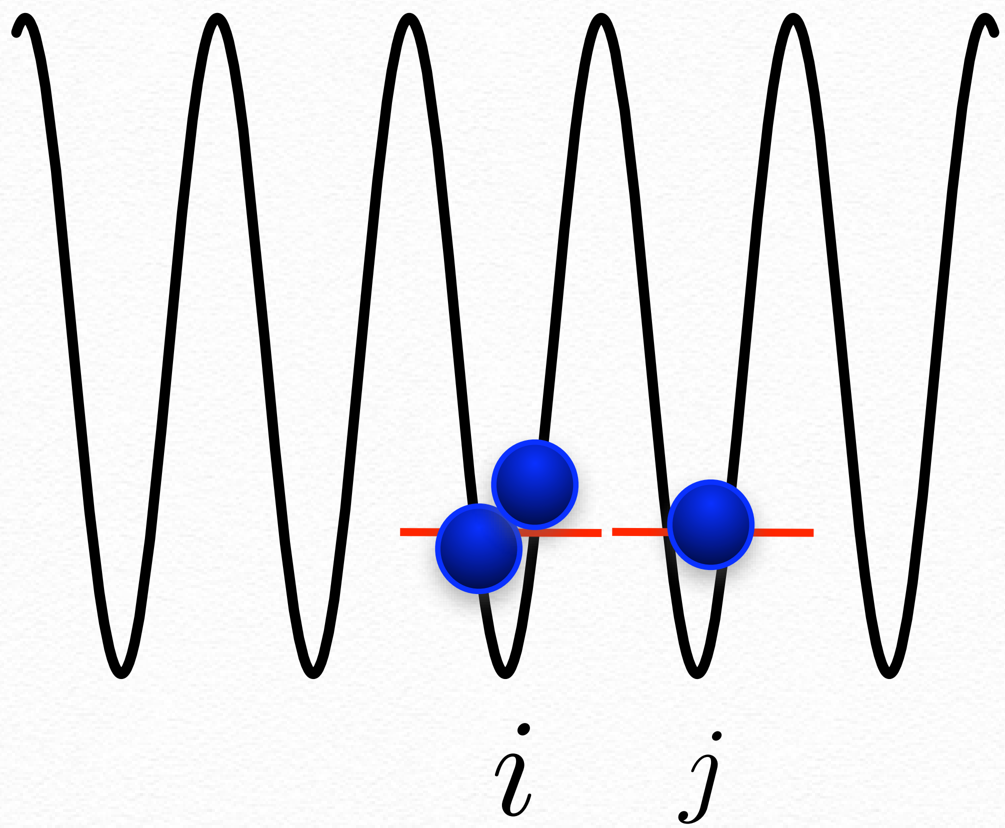
$$\omega = \frac{k}{\sqrt{2m}}$$

Gapped Higgs mode:

$$\omega = \sqrt{\frac{k2}{2m} + 2r}$$



Bose-Hubbard Model



$$\hat{H}_{\mathrm{BH}} = -t \sum_{\langle ij \rangle} \hat{b}_i^\dagger \hat{b}_j + U \sum_i \hat{n}_i(\hat{n}_i - 1) - \mu \sum_i \hat{n}_i$$

Nearest neighboring hopping

On-site interaction

Chemical potential

Quantum Phases in Bose-Hubbard Model

$$\hat{H}_{\rm BH} = -t\sum_{\langle ij\rangle}\hat{b}_i^\dagger\hat{b}_j + U\sum_i \hat{n}_i(\hat{n}_i-1) - \mu\sum_i \hat{n}_i$$

$$U = 0 \qquad\qquad |\Psi\rangle = \left(\frac{1}{\sqrt{M}}\sum_{i=1}^{m}\hat{b}_i^\dagger\right)^N |0\rangle$$

- On-site particle number fluctuation

$$P[n_i] = e^{-\bar{n}}\frac{\bar{n}_i^n}{n_i!} \qquad \bar{n} = \langle n_i\rangle \qquad \langle \delta n_i^2\rangle = \bar{n}$$

- Long-range correlations, U(1) symmetry breaking

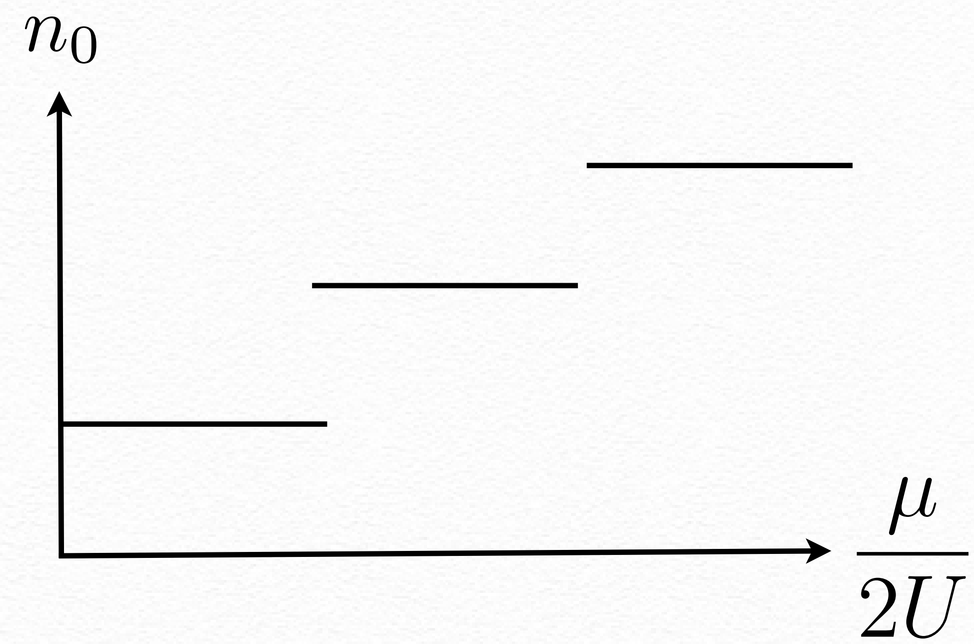
$$\langle \hat{b}_i^\dagger \hat{b}_j\rangle \to \mathcal{C} \qquad |i-j| \to \infty$$

- Gapless Goldstone mode

Quantum Phases in Bose-Hubbard Model

$$\hat{H}_{\mathrm{BH}} = -t\sum_{\langle ij\rangle}\hat{b}_i^\dagger\hat{b}_j + U\sum_i \hat{n}_i(\hat{n}_i - 1) - \mu\sum_i \hat{n}_i$$

$$t = 0 \qquad\qquad |\Psi\rangle = \prod_i (\hat{b}_i^\dagger)^{n_0}|0\rangle$$



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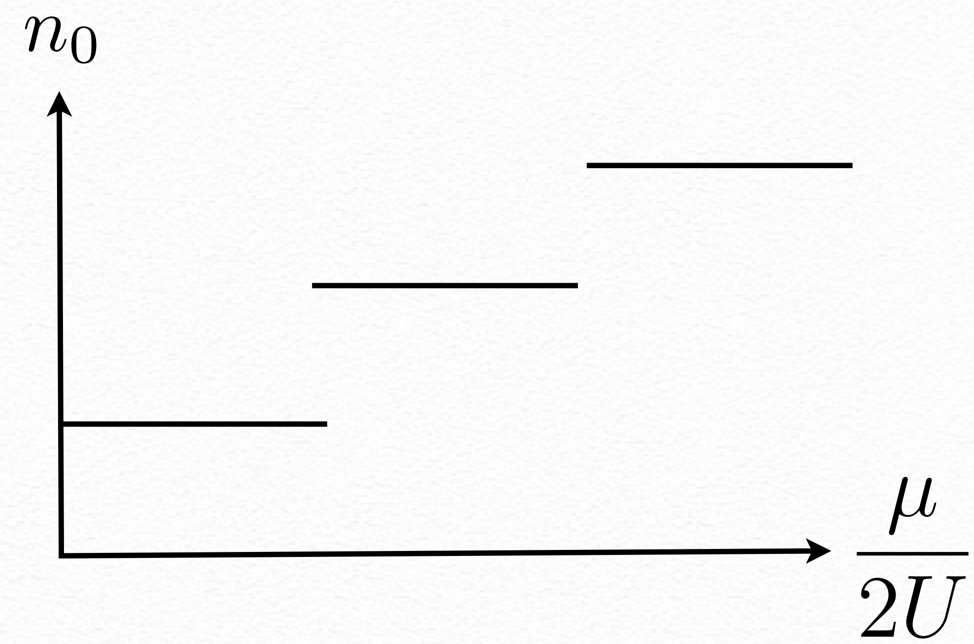

- No particle number fluctuation

- No Long-range correlations, and no U(1) symmetry breaking

$$\langle \hat{b}_i^\dagger \hat{b}_j \rangle = 0$$

Quantum Phases in Bose-Hubbard Model

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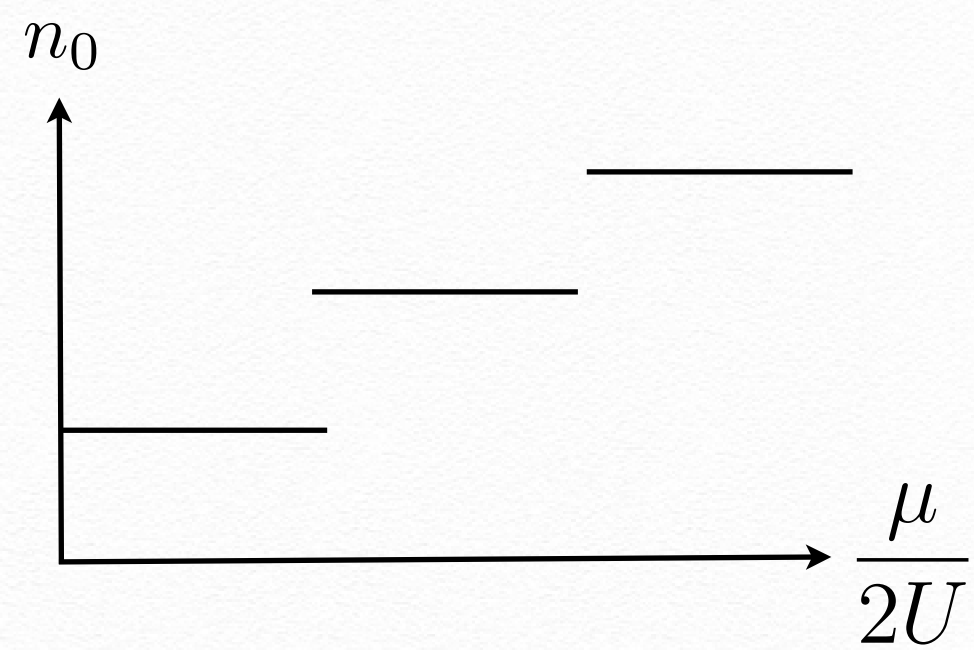

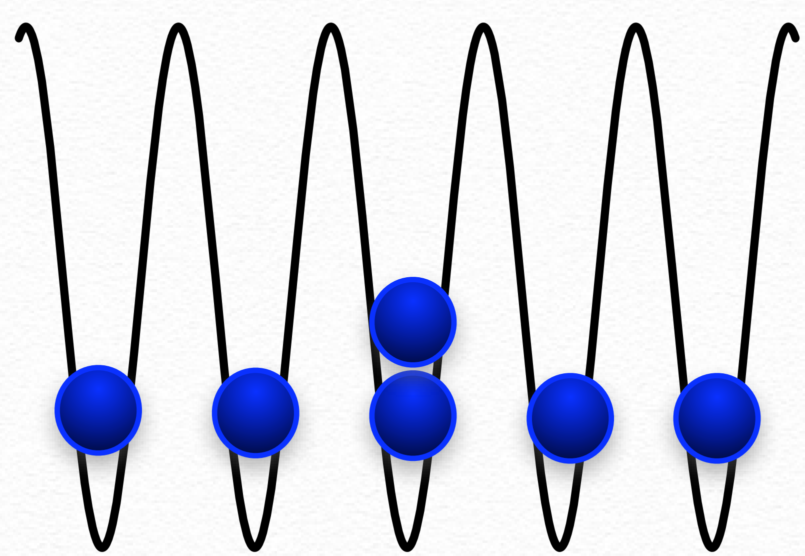

Particle excitation:

$$\Delta E = E(n_0 + 1) - E(n_0) = 2Un_0 - \mu > 0$$

Quantum Phases in Bose-Hubbard Model

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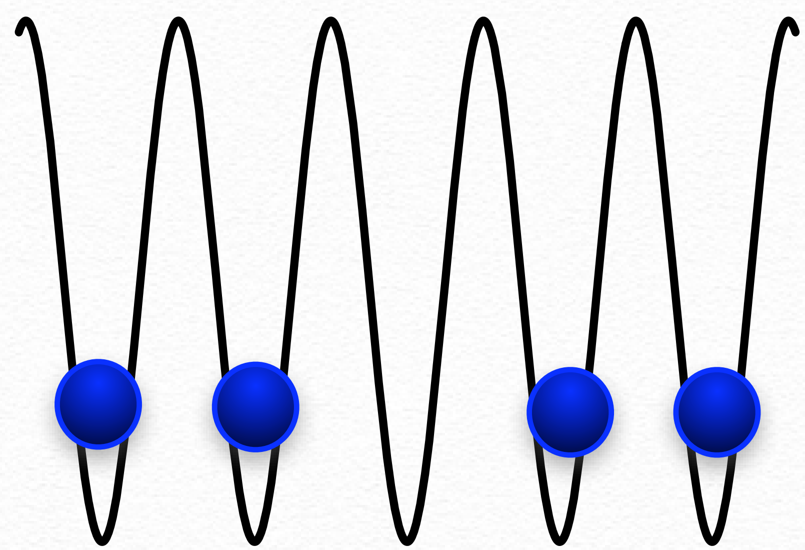

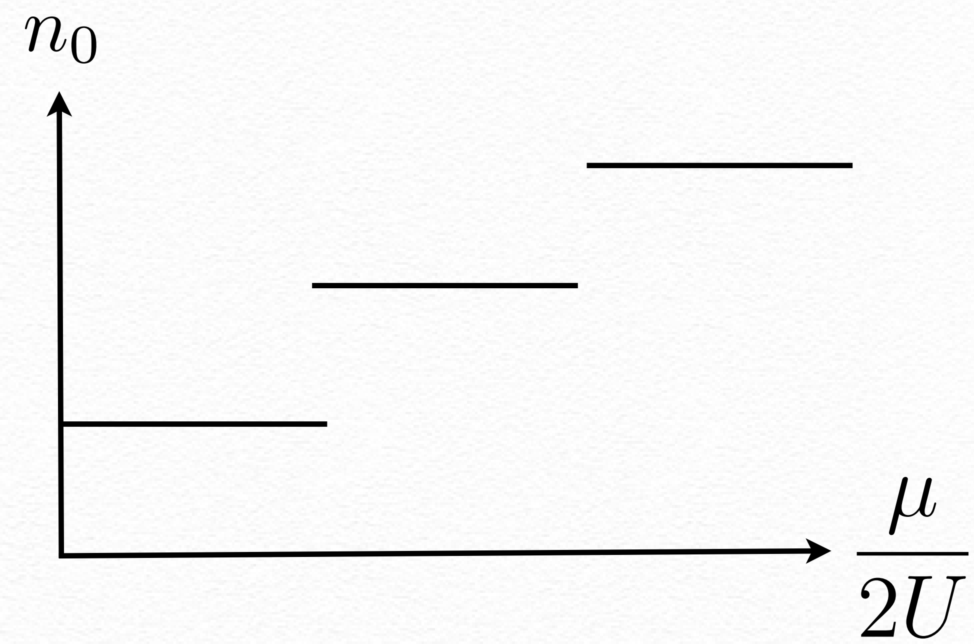

hole excitation:

$$\Delta E = E(n_0 - 1) - E(n_0) = \mu + 2U - 2Un_0 > 0$$

Thursday, December 25, 14

Quantum Phases in Bose-Hubbard Model

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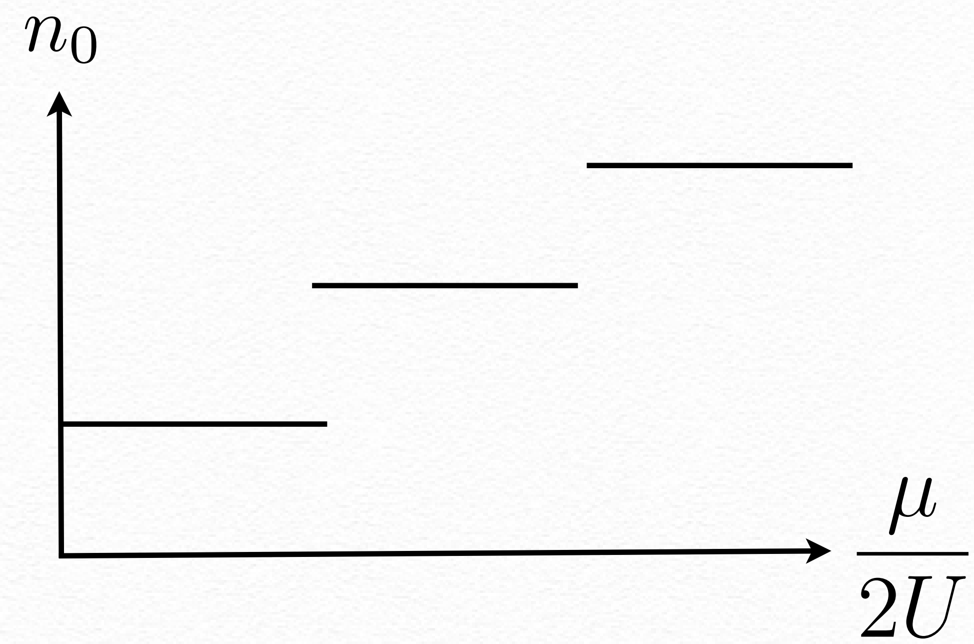

- No particle number fluctuation
- No Long-range correlations, and no U(1) symmetry breaking

$$\langle \hat{b}_i^\dagger \hat{b}_j\rangle = 0$$

- Excitations are gapped

Quantum Phases in Bose-Hubbard Model

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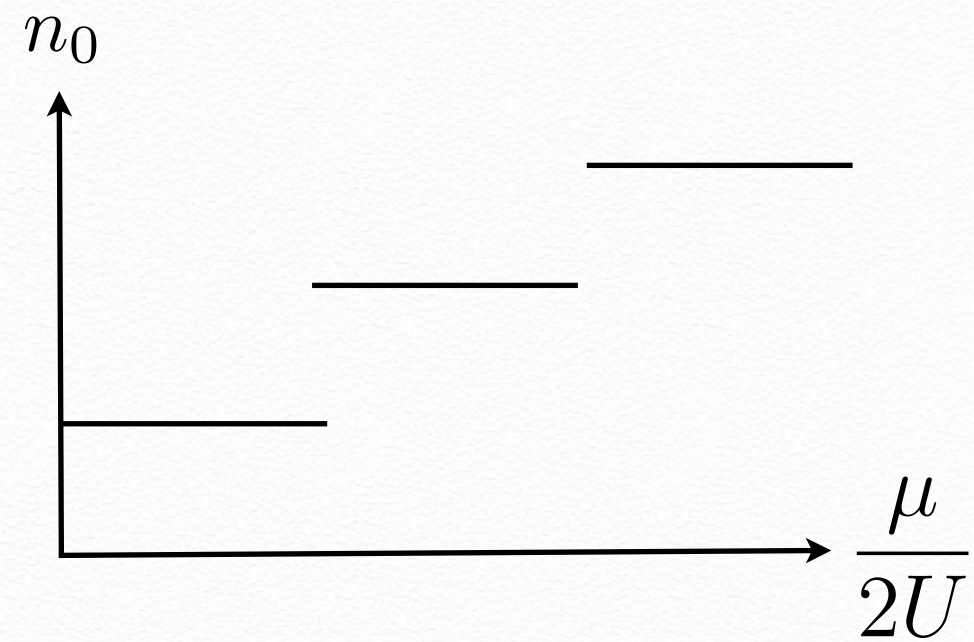
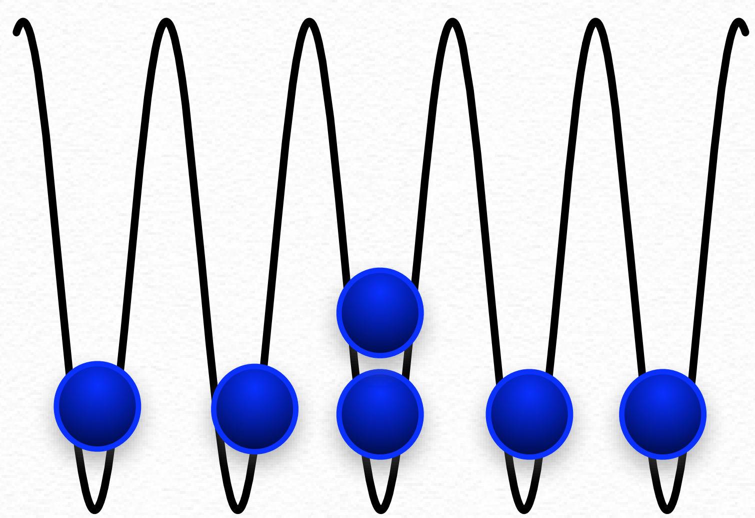
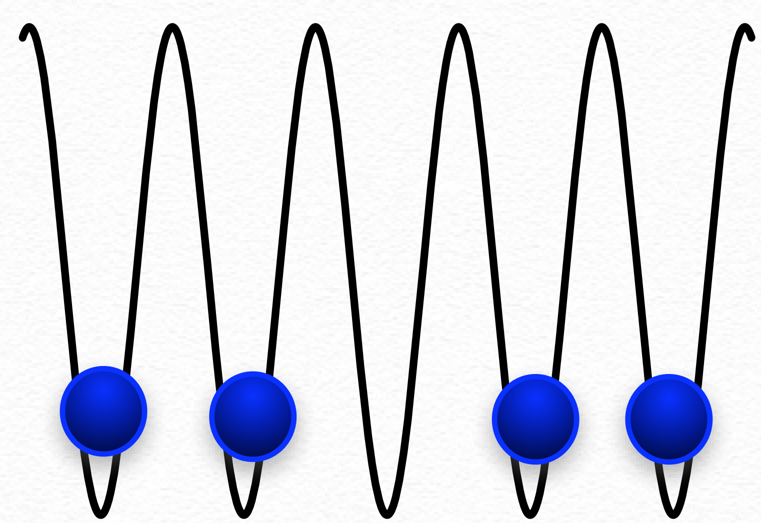




Particle excitation:

$$\Delta E = 2Un_0 - \mu > 0$$



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Quantum Phases in Bose-Hubbard Model

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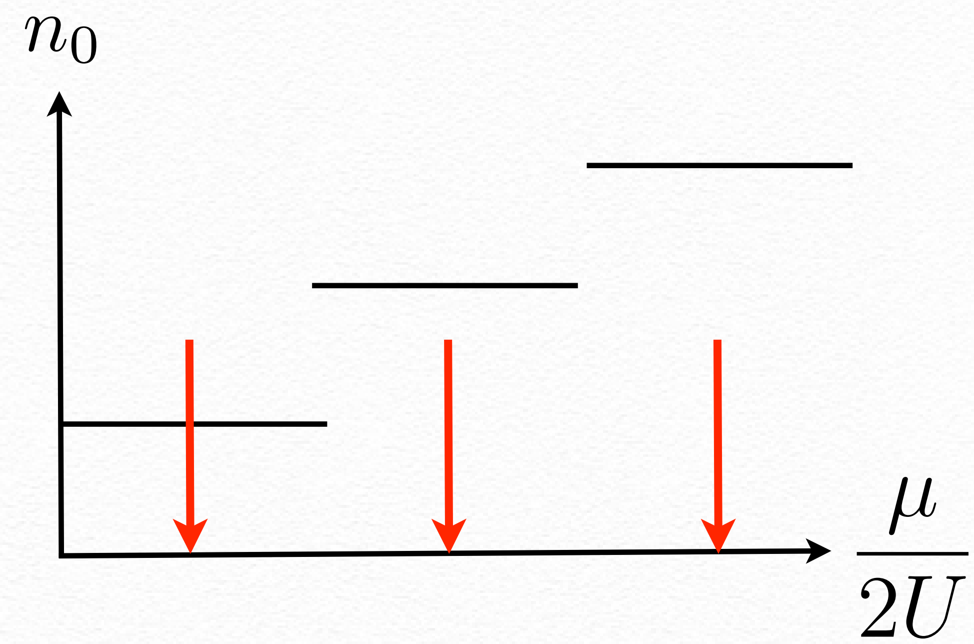

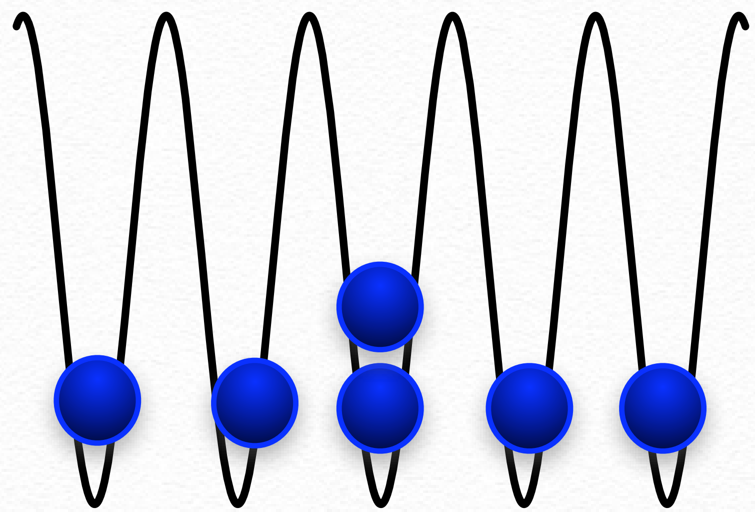

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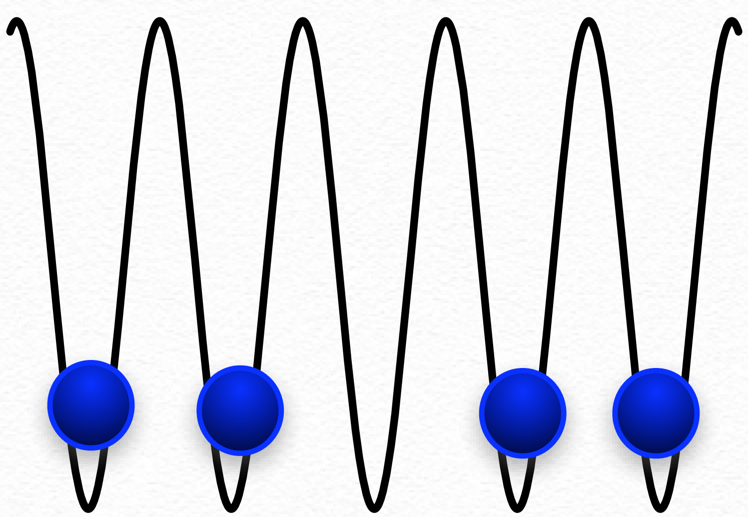

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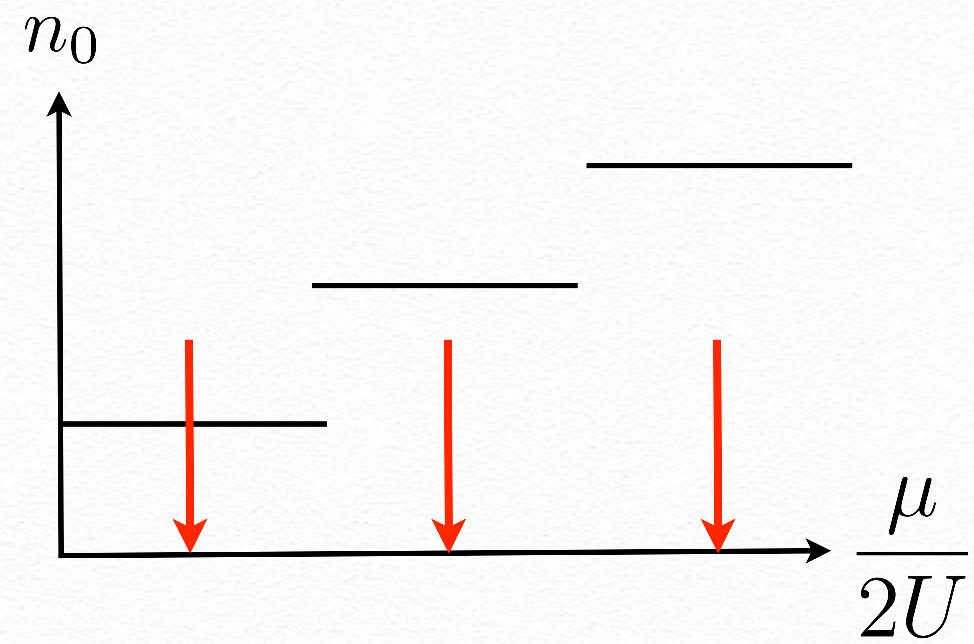
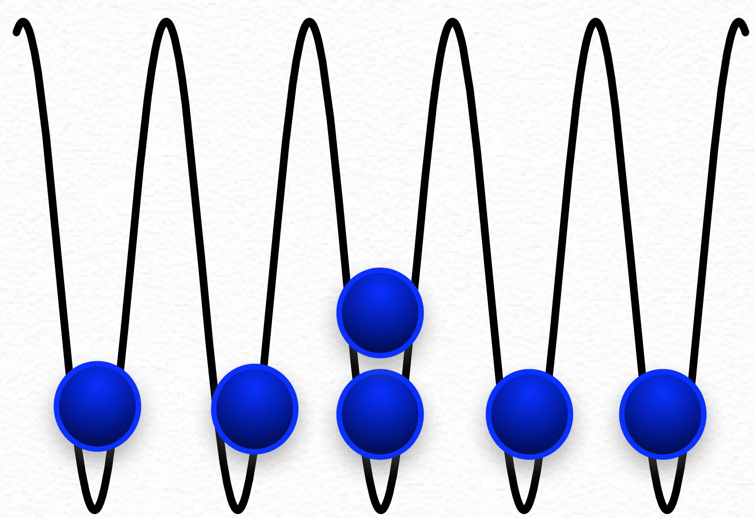


Particle-hole symmetric points



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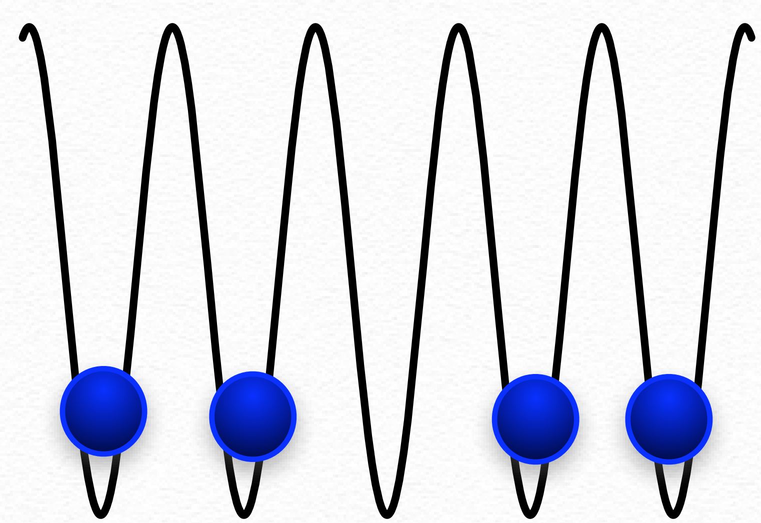

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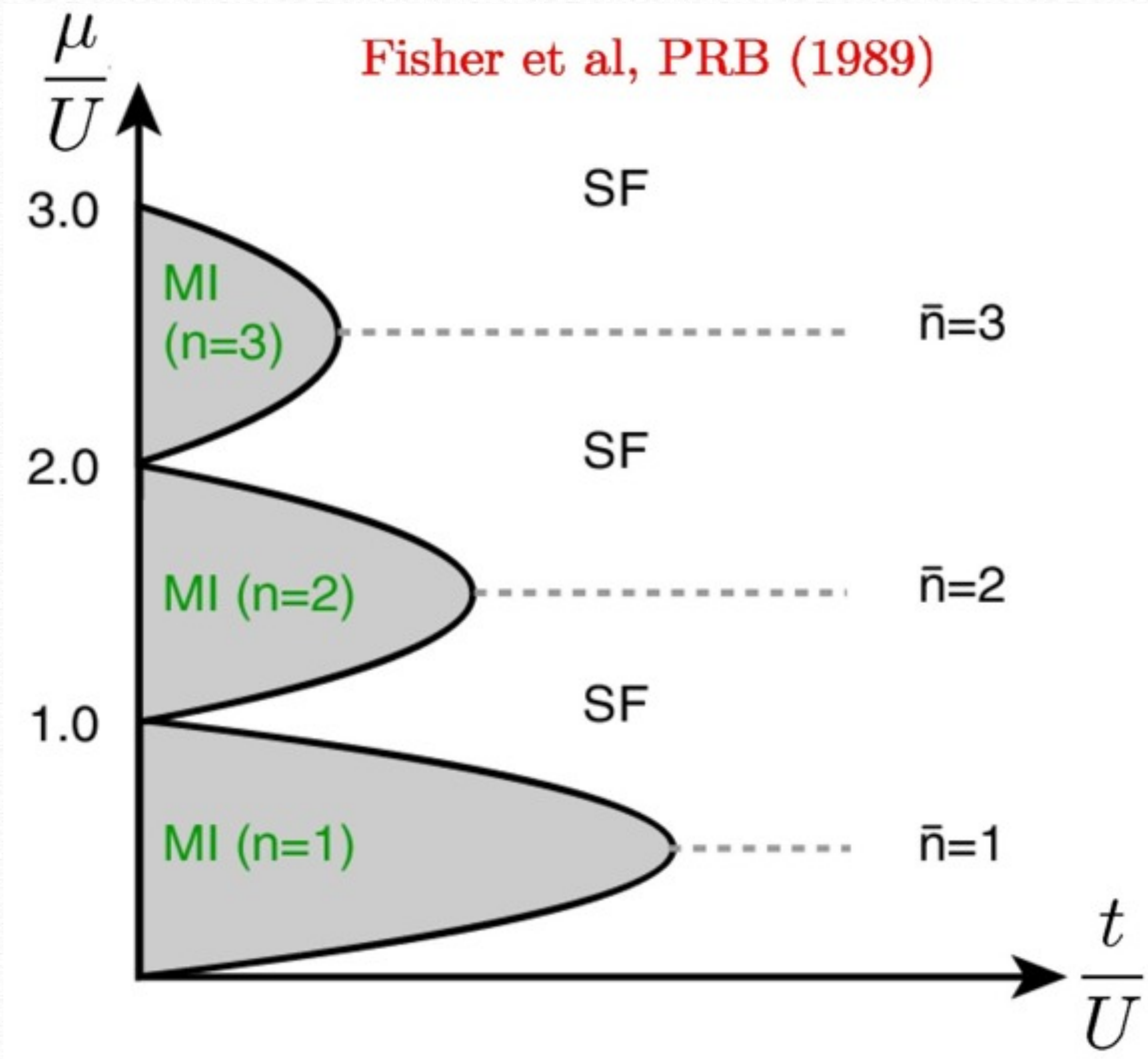
Phase Diagram



Field Theory Description

Path integral representation

$$\mathcal{Z} = \int \prod_i \mathcal{D}b_i^*(\tau)\mathcal{D}b_i(\tau) \exp\left\{\int_0^\beta \left[\sum_i b_i^*(\tau)\partial_\tau b_i(\tau) - H_{\mathrm{BH}}(b^*(\tau), b(\tau))\right]\right\}$$

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Introducing an auxiliary field to coupling hopping

$$\mathcal{Z} = \int \prod_i \mathcal{D}\varphi_i^* \mathcal{D}\varphi_i(\tau) e^{-S[\varphi^*,\varphi]}$$

$$S[\varphi^*,\varphi] = \int_0^\beta d\tau \sum_{ij} \varphi_i^* \frac{1}{t} \varphi_j - \sum_i \ln \int \mathcal{D}b^*(\tau)\mathcal{D}b(\tau) e^{-\int_0^\beta d\tau \mathcal{L}[b,\varphi_i]}$$

$$\mathcal{L} = -b^*(\tau)\partial_\tau b(\tau) - \mu|b(\tau)|^2 + \frac{U}{2}|b(\tau)|^2(|b(\tau)|^2 - 1) - \varphi b^*(\tau) - \varphi^* b(\tau)$$

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Integrating out boson field

$$S[\varphi^*,\varphi] = S[0] + \int_0^\beta d\tau \int d^3\mathbf{r} \left[u\varphi^*\partial_\tau\varphi + v|\partial_\tau\varphi|^2 + w|\nabla\varphi|^2 - a|\varphi|^2 + b|\varphi|^4 + \ldots\right]$$

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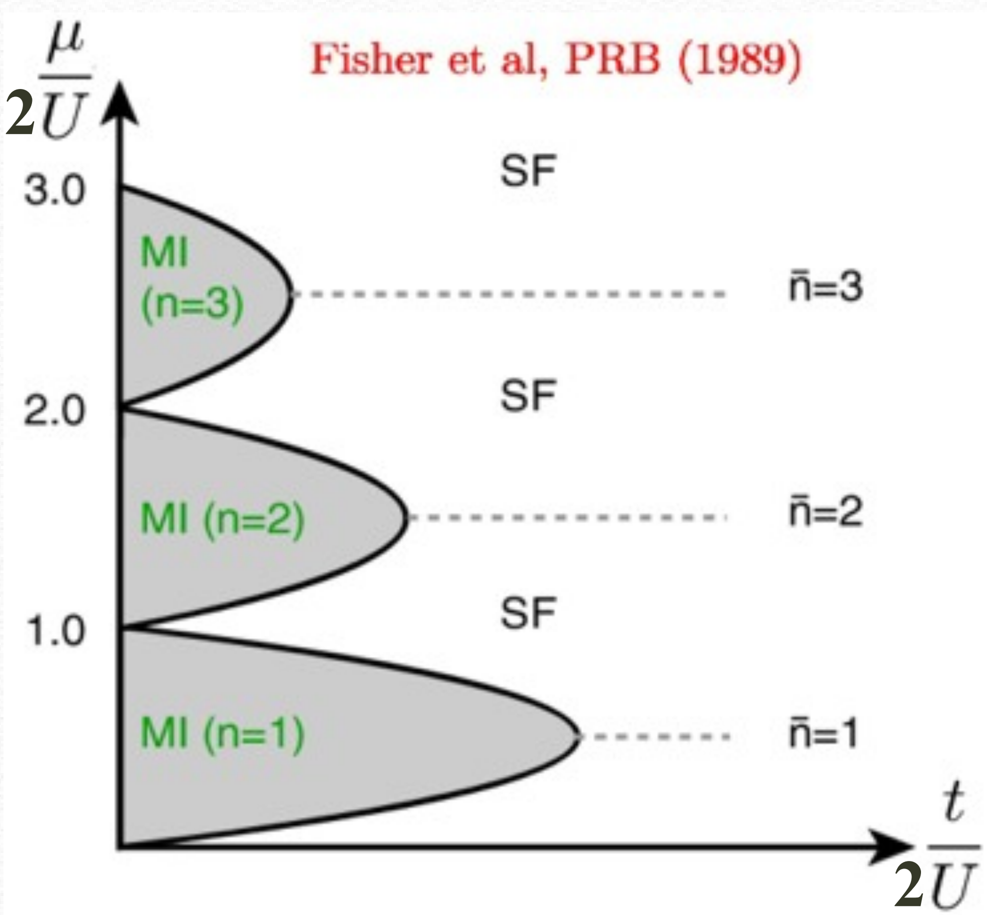
$$a = -\frac{1}{t} + \frac{n_0 + 1}{2n_0 U - \mu} + \frac{n_0}{\mu - 2U(n_0 - 1)}.$$



$a > 0$ Superfluid

$a < 0$ Mott insulator

$$\frac{t_c}{2U} = \frac{\left(n_0 - \frac{\mu}{2U}\right)\left(\frac{\mu}{2U} - (n_0 - 1)\right)}{\frac{\mu}{2U} + 1}$$

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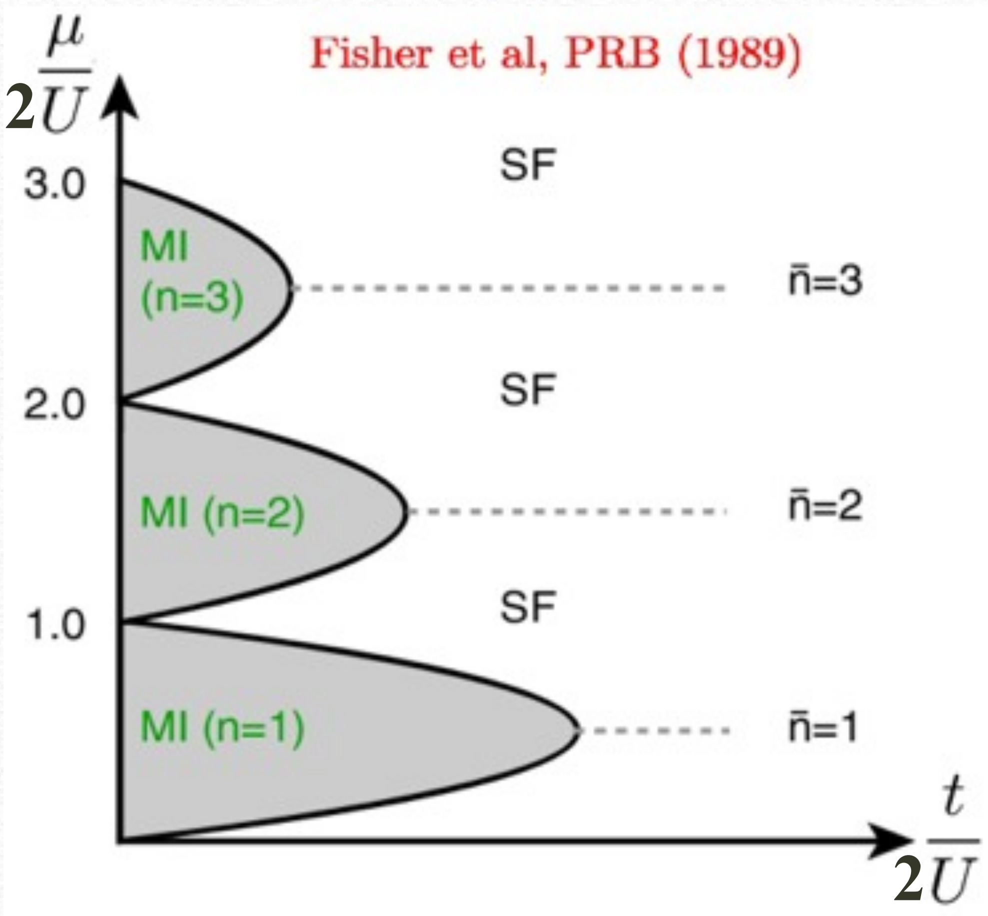
Gauge Symmetry Derivation

$$b(\tau)\to b(\tau)e^{i\theta(\tau)},\quad b^*(\tau)\to b^*(\tau)e^{-i\theta(\tau)}$$

$$\varphi(\tau)\to\varphi(\tau)e^{i\theta(\tau)},\quad \varphi^*(\tau)\to b(\tau)e^{i\theta(\tau)},\quad \mu\to\mu+i\partial_\tau\theta$$



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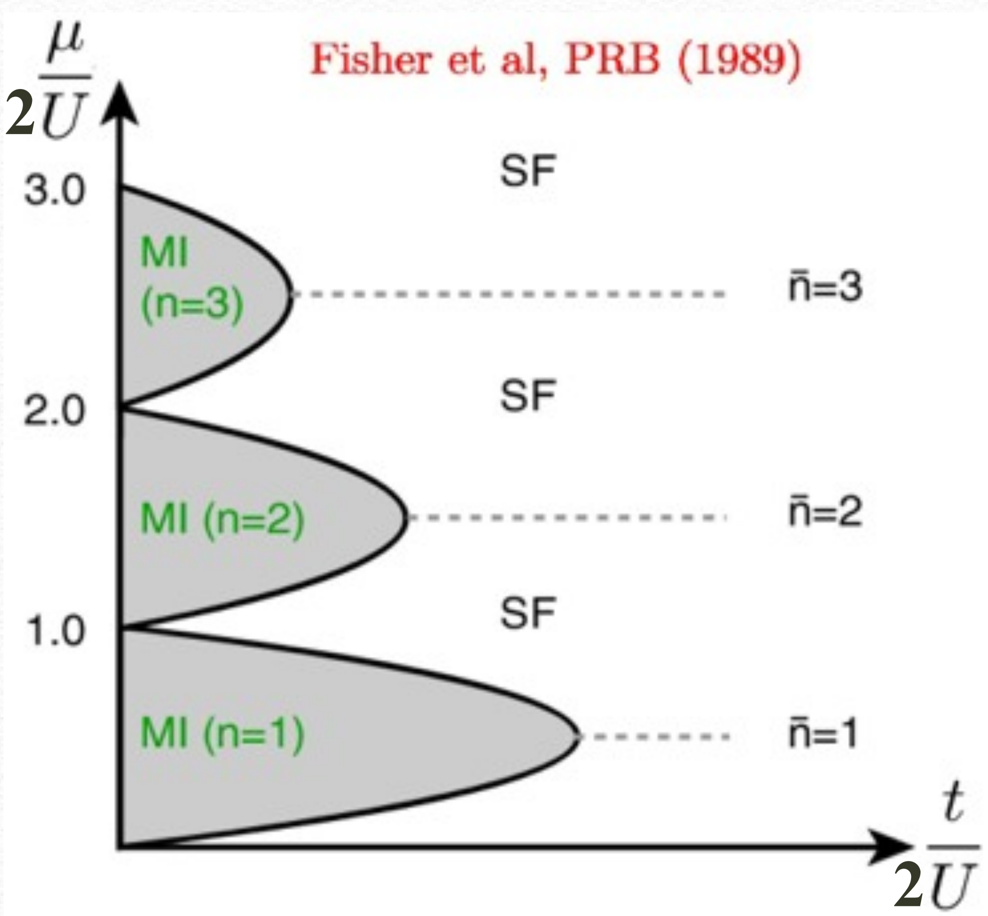

$$\frac{\partial a}{\partial \mu} - u = 0$$

$$\frac{\partial \mu}{\partial \mu} + 2v = 0$$

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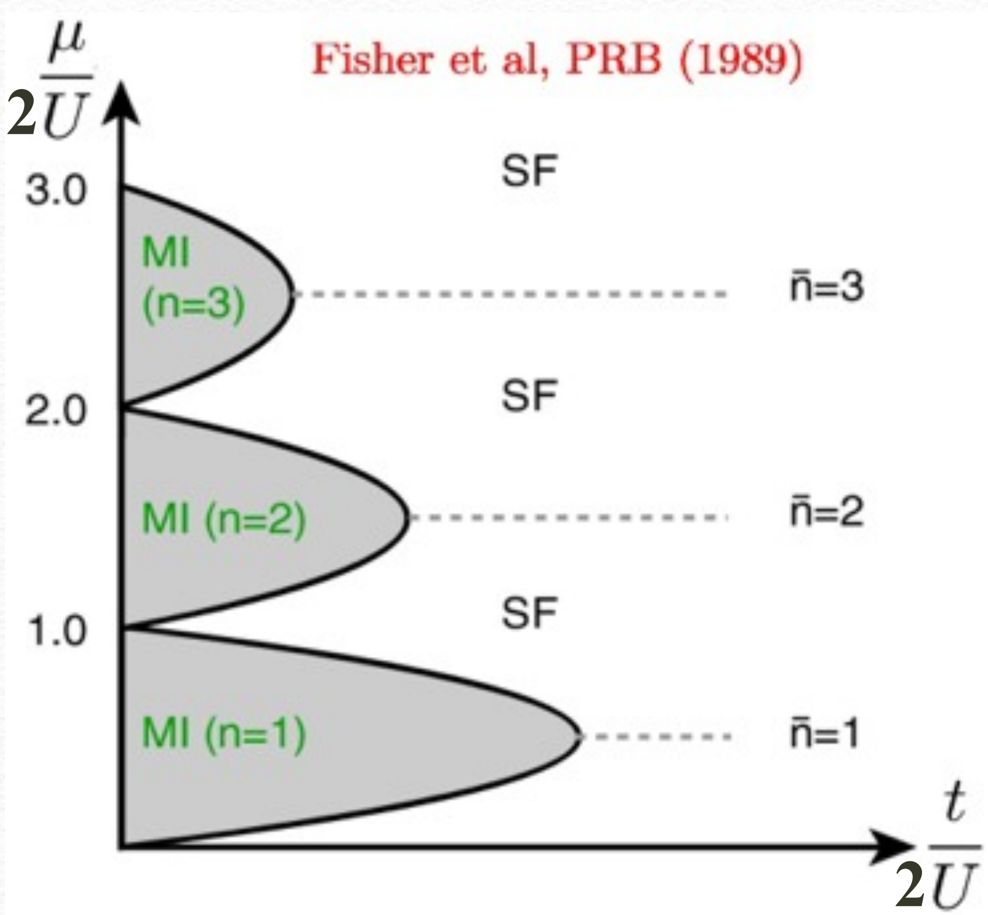

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$$\frac{\partial \mu}{\partial \mu} + 2v = 0$$

$$u = \frac{n_0 + 1}{(2n_0U - \mu)^2} - \frac{n_0}{(\mu - 2U(n_0-1))^2}$$

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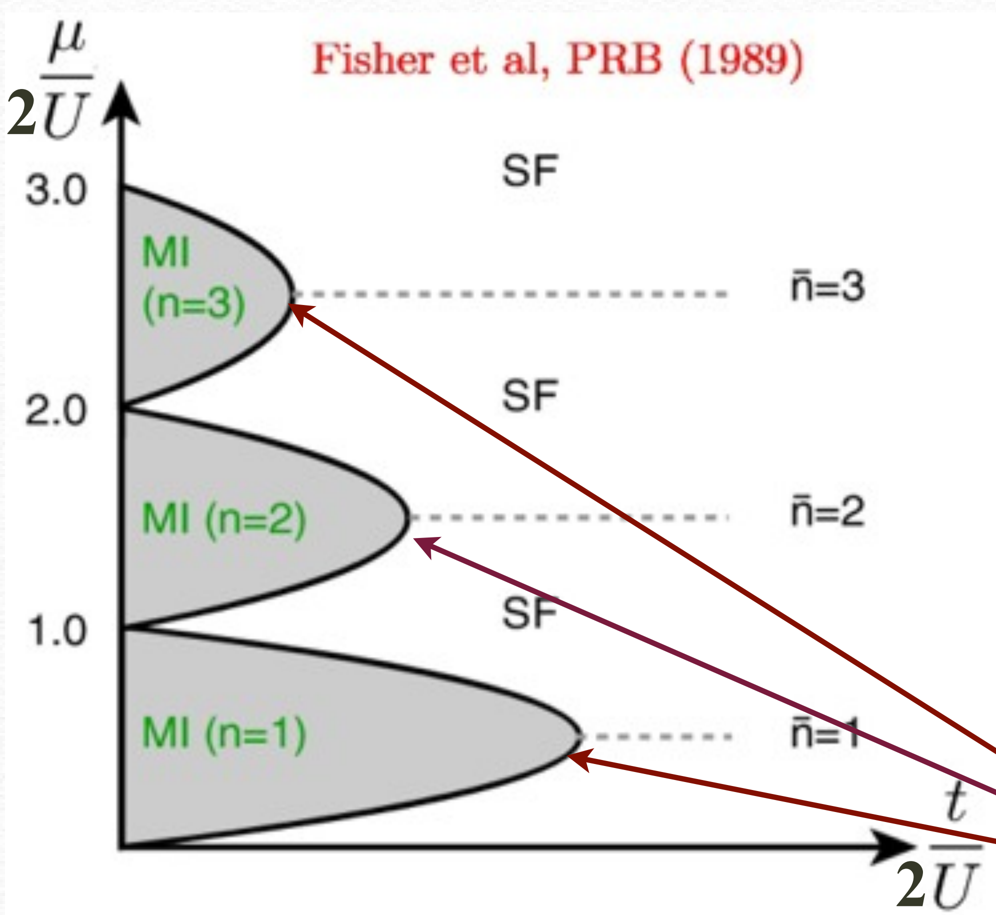


$$\frac{\partial a}{\partial \mu} - u = 0$$

$$\frac{\partial \mu}{\partial \mu} + 2v = 0$$

$$u = \frac{n_0 + 1}{(2n_0U - \mu)^2} - \frac{n_0}{(\mu - 2U(n_0 - 1))^2}$$

Emergent of Lorentz Invariance

Higgs mode

$$\mathcal{S}_\phi = \int d^d\mathbf{r} \int_0^\beta d\tau \left\{ (\partial_\tau \varphi)^2 + c^2 (\nabla\varphi)^2 - \alpha|\varphi|^2 + b|\phi|^4 \right\}$$

Mott insulator | **Superfluid**

α

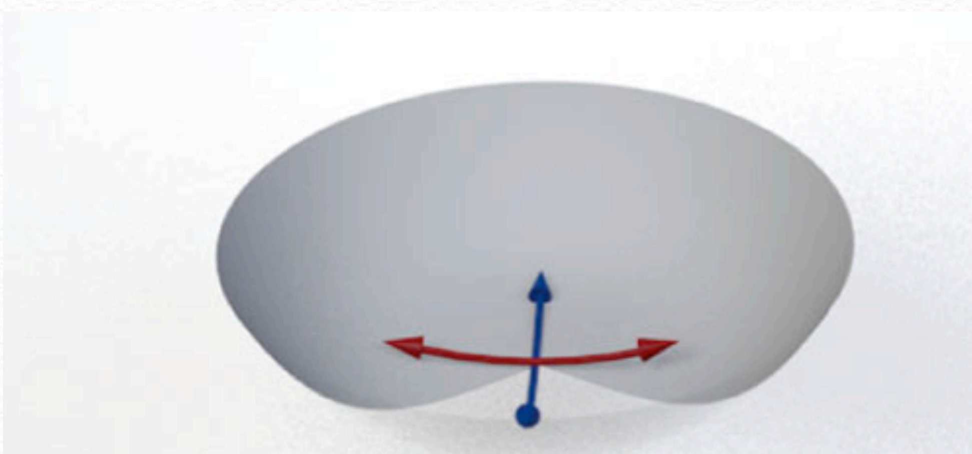
Mott insulator:

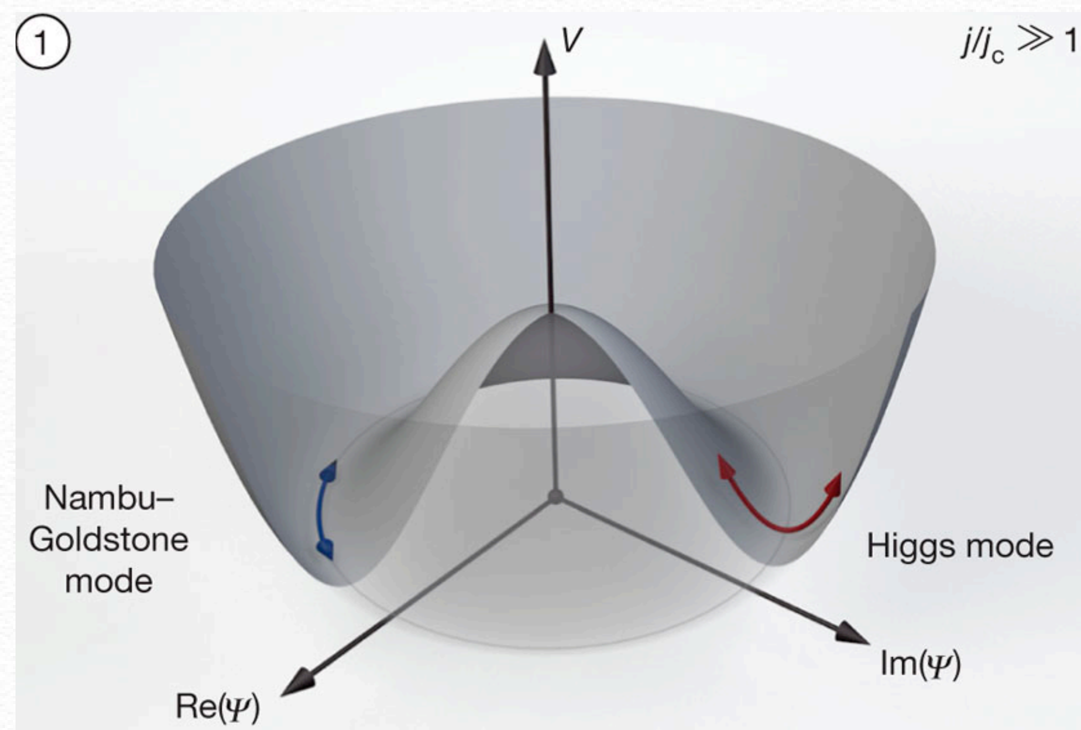
$$\omega = \sqrt{ck^2 + |\alpha|}$$

Superfluid:

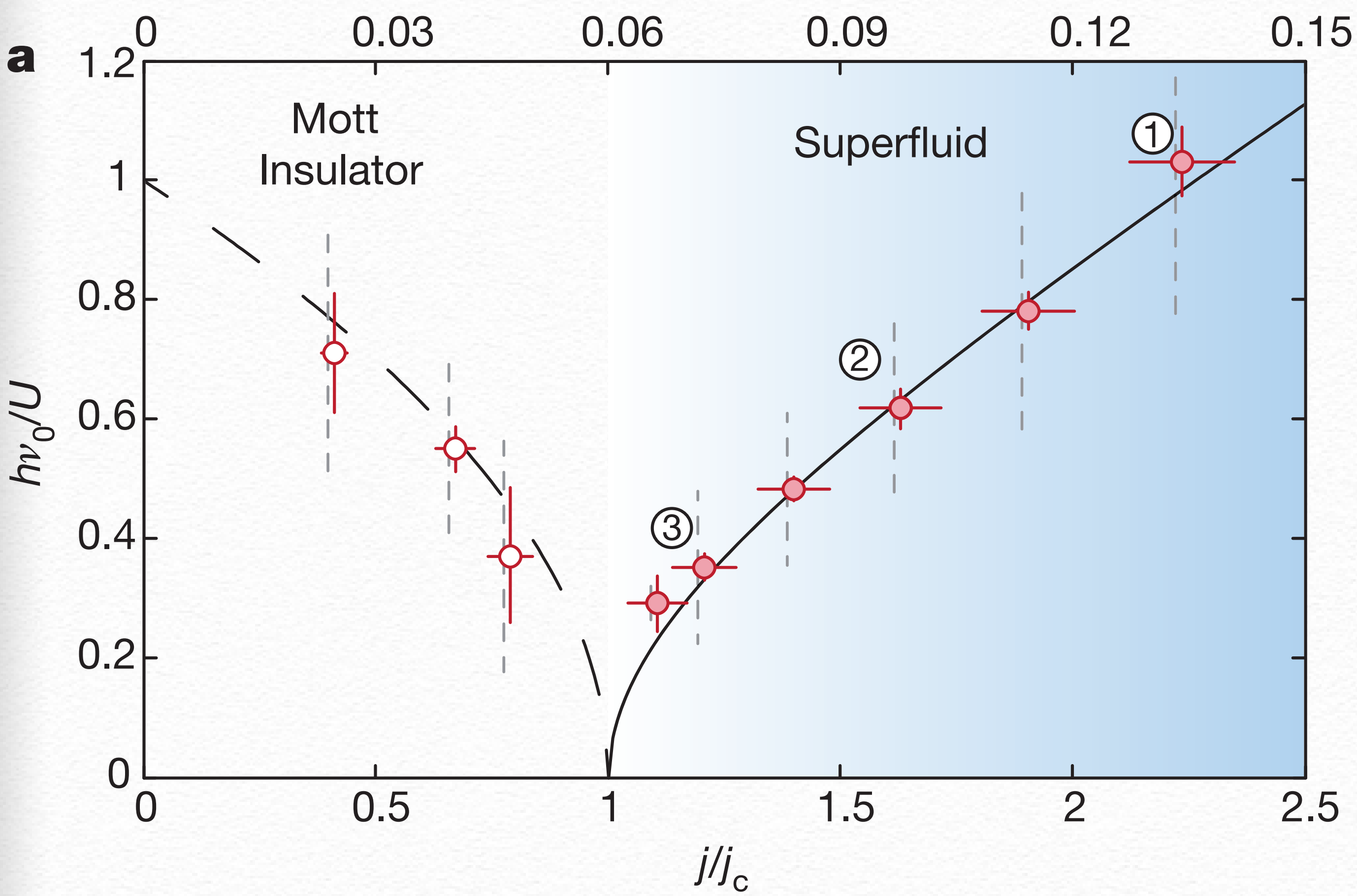
$$\omega = \sqrt{ck^2 + |\alpha|}$$

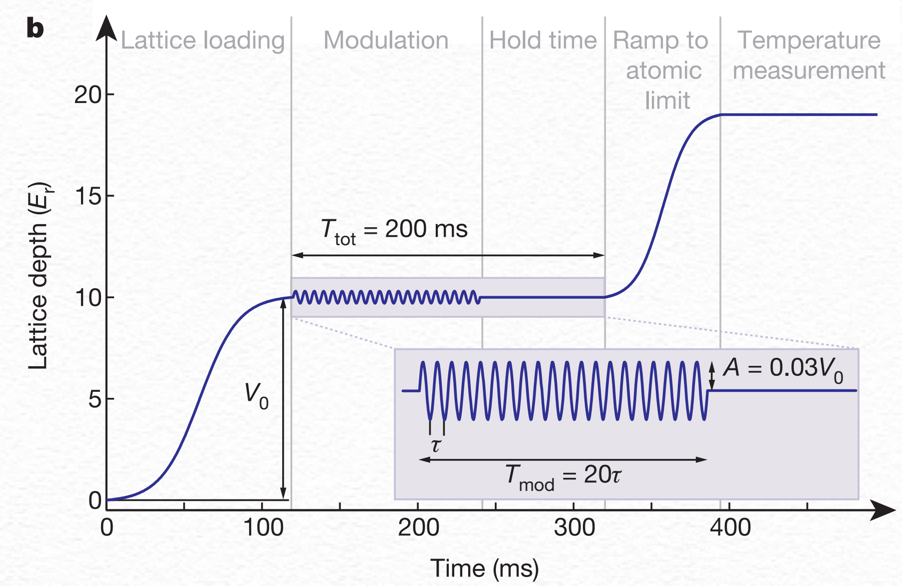
$$\omega = ck$$



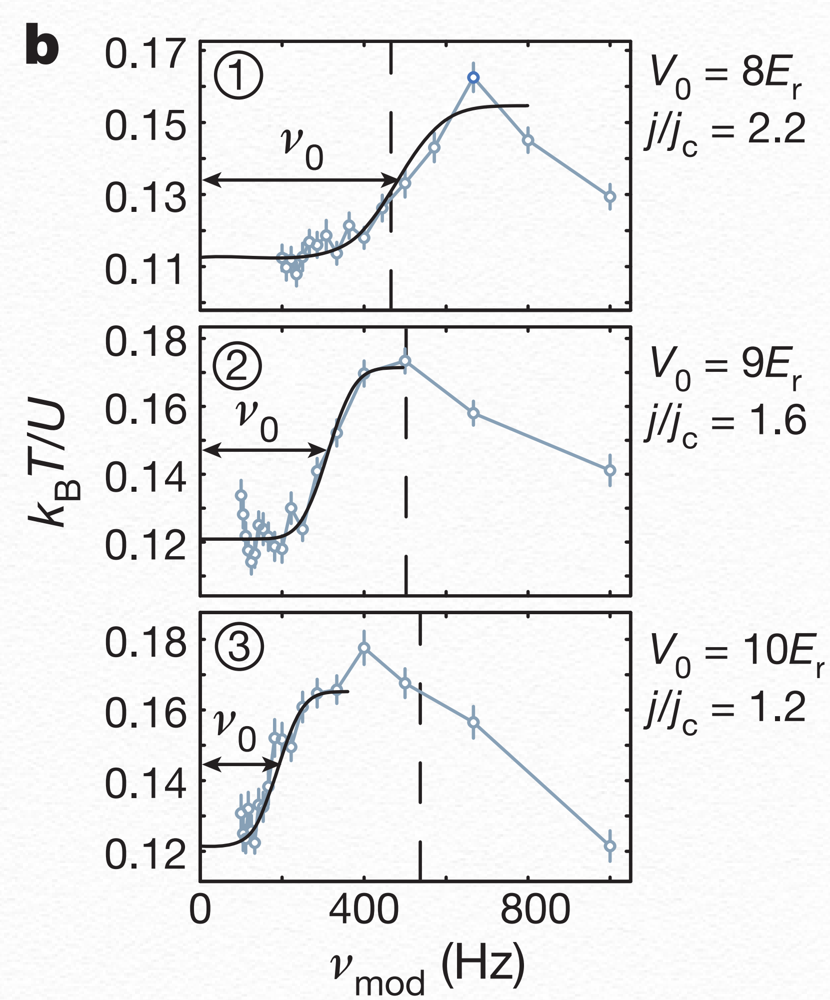


Higgs Mode Detection





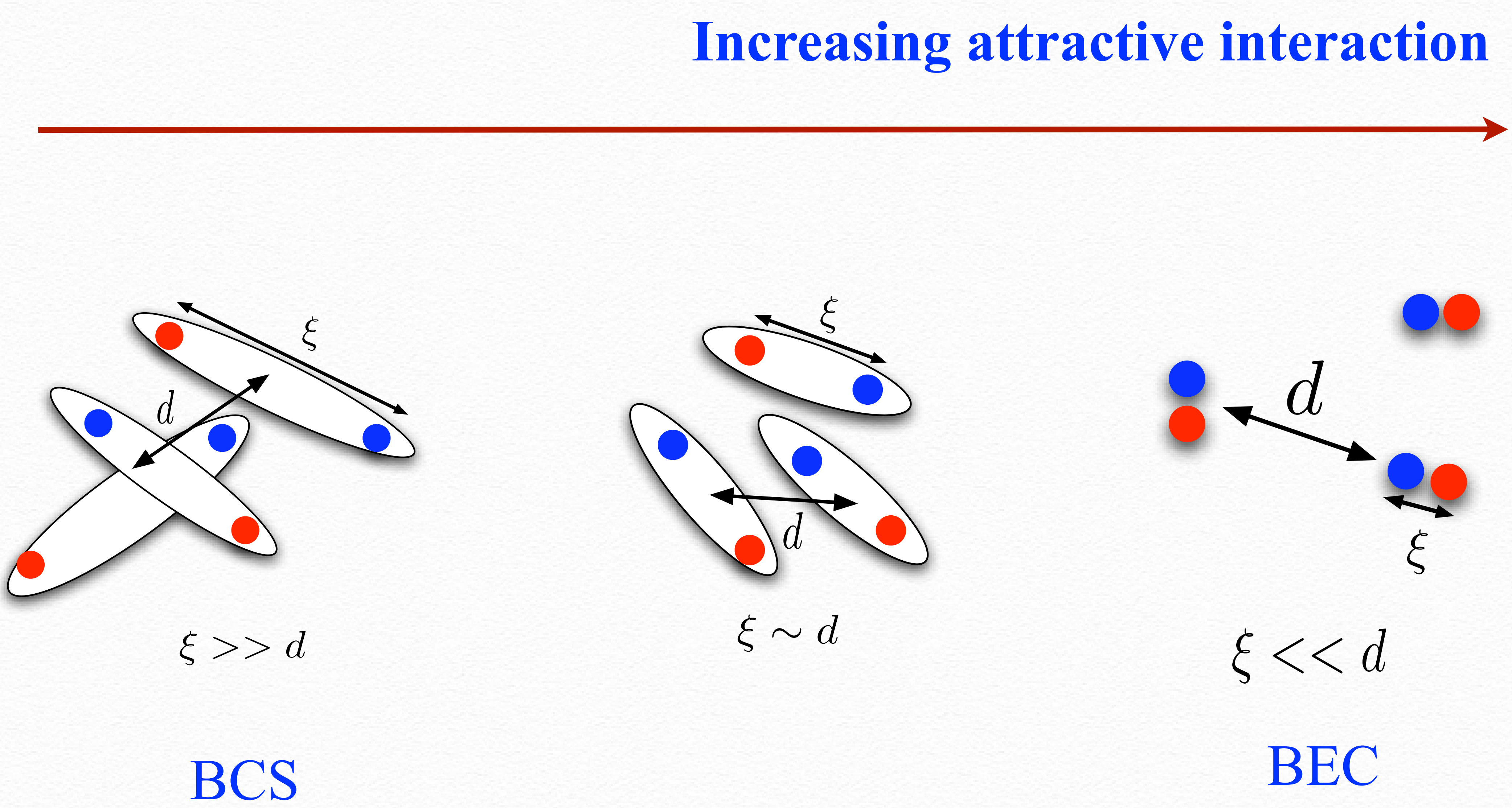
Lattice modulation spectroscopy



Munich group, Nature, 487, 454 (2012)

Generally, a condensed matter and cold atom system will not have Lorentz invariance, what is the fate to Higgs mode if without Lorentz invariance ?

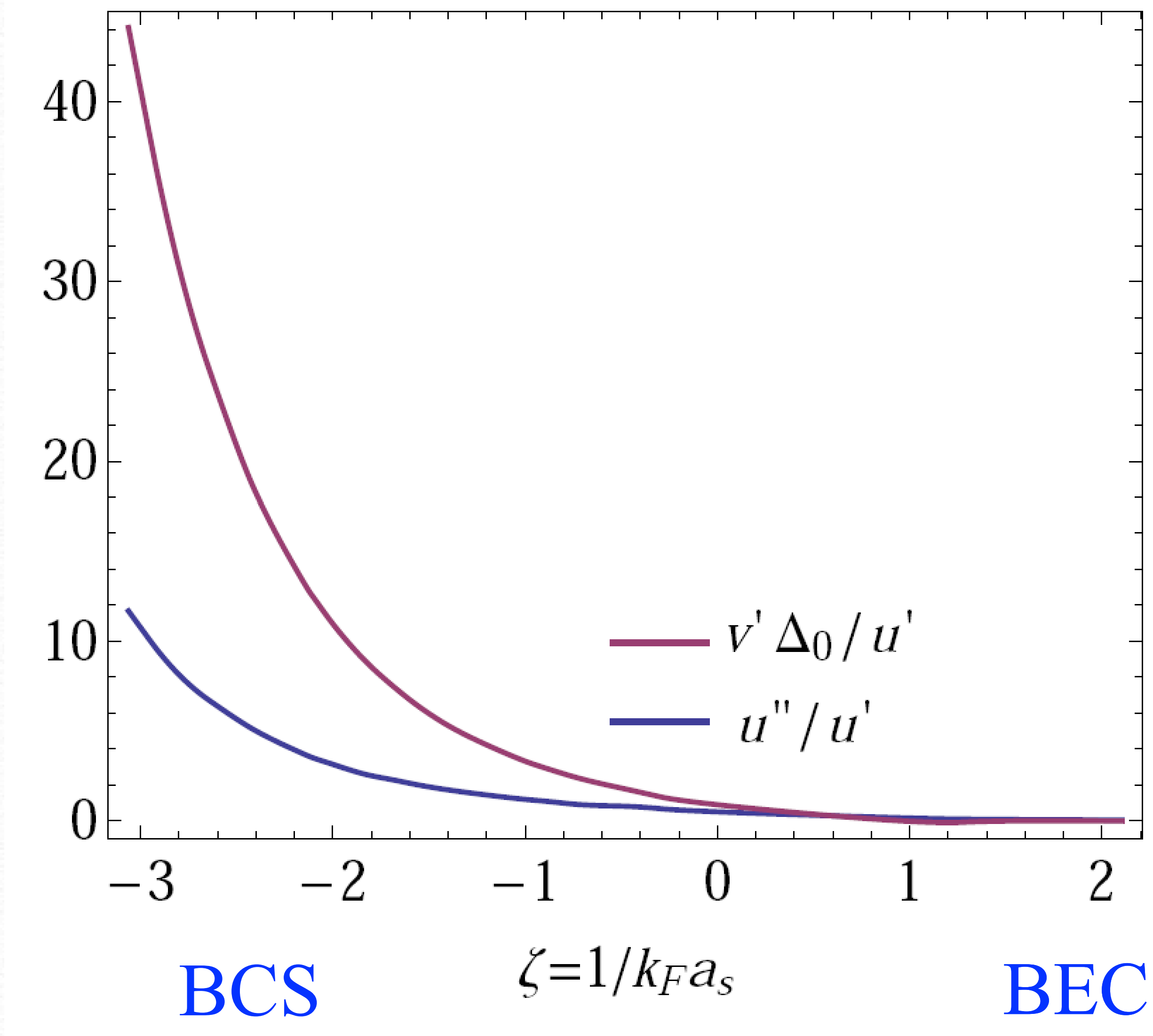
BEC-BCS Crossover



Ginburg-Landau Theory

$$F[\bar{\Delta},\Delta] = \int dt d^3\mathbf{x}\{\bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta\}$$

$$u = u' + iu''$$



Increasing attractive interaction

Ginburg-Landau Theory

$$F[\bar{\Delta},\Delta] = \int dt d^3\mathbf{x}\{\bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta\}$$

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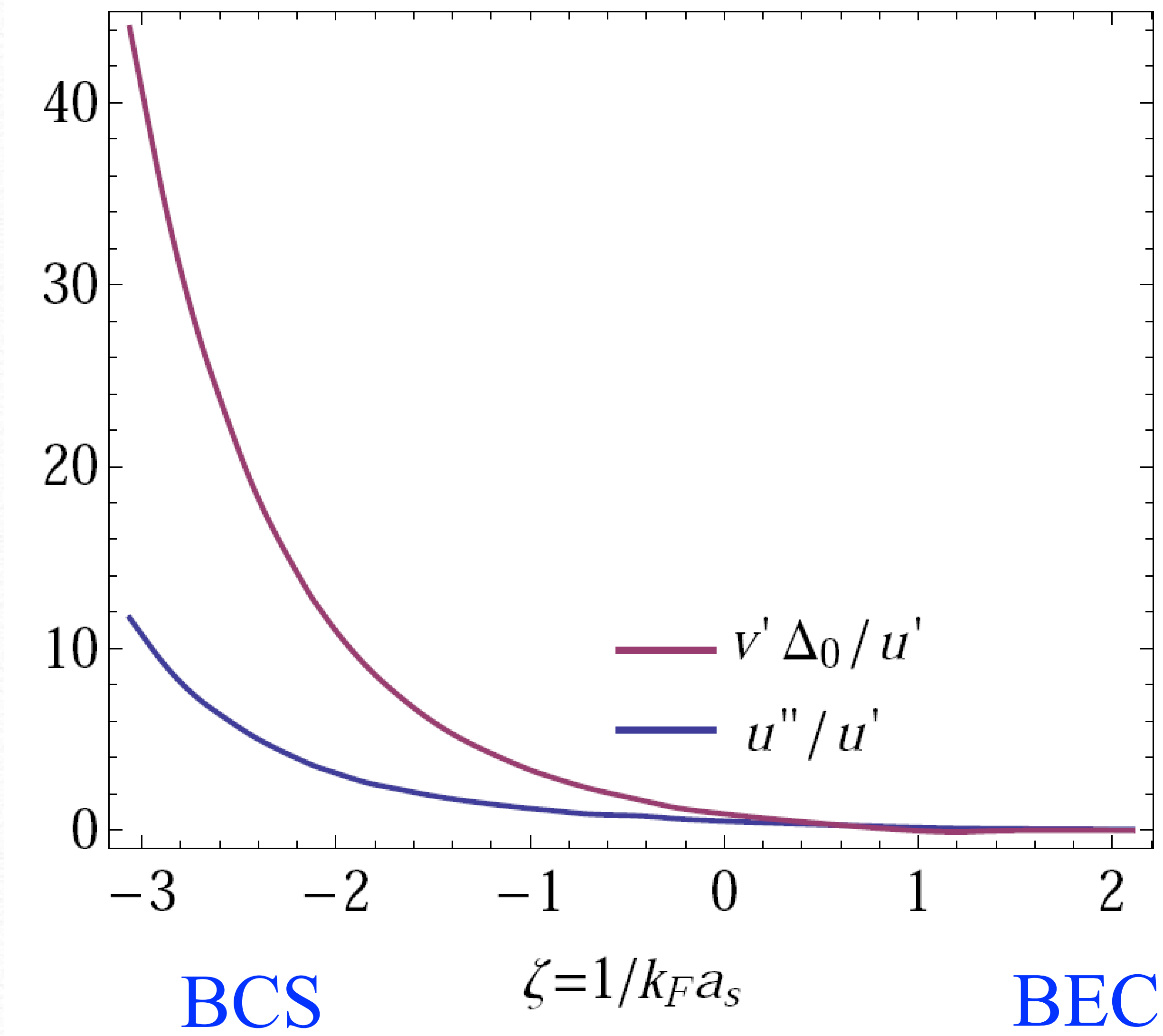

Increasing attractive interaction

Ginburg-Landau Theory

$$F[\bar{\Delta}, \Delta] = \int dt d^3\mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta \}$$

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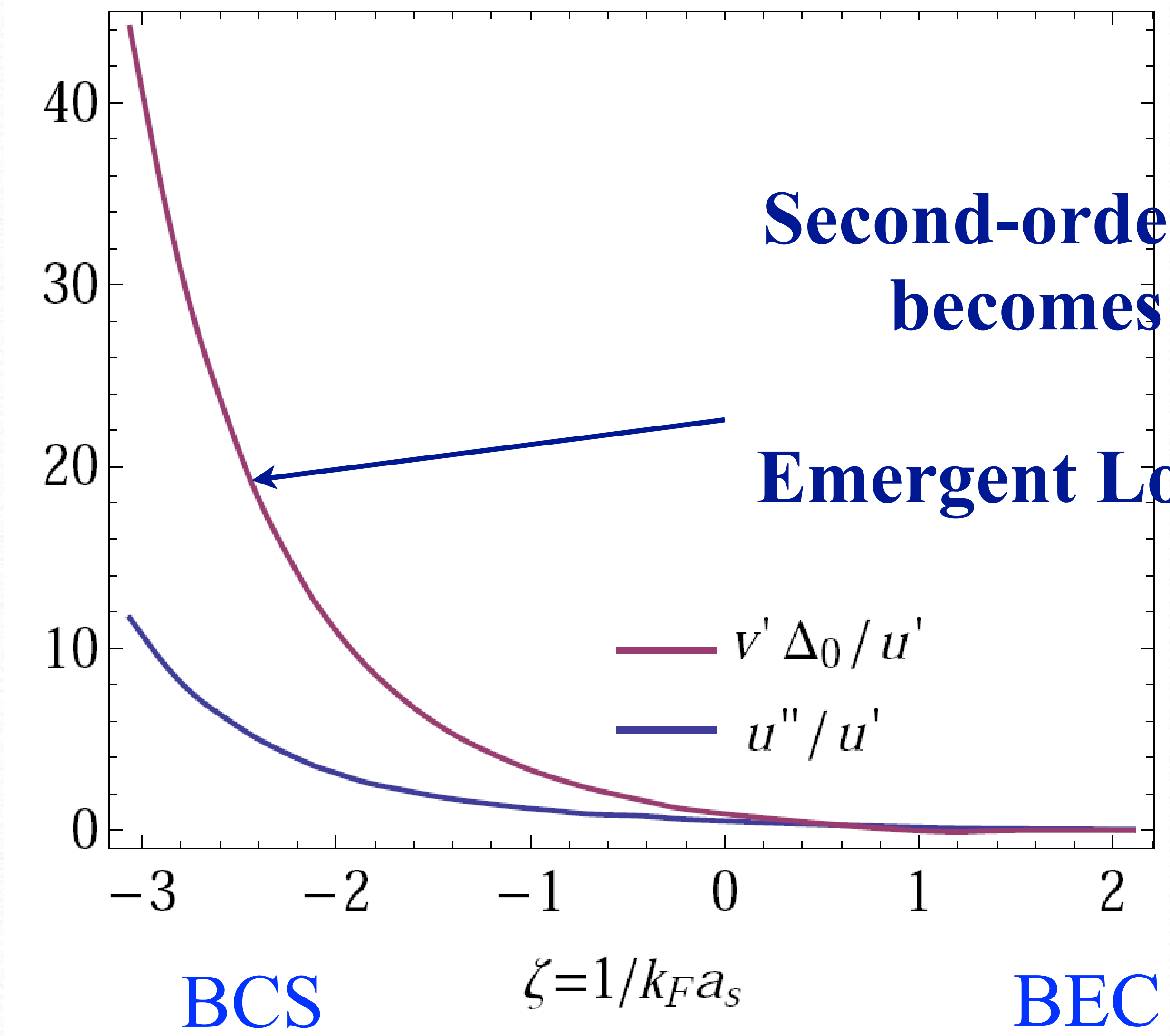

Second-order time derivative becomes much larger

Emergent Lorentz invariance

Increasing attractive interaction

Ginburg-Landau Theory

$$F[\bar{\Delta}, \Delta] = \int dt d^3\mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta \}$$

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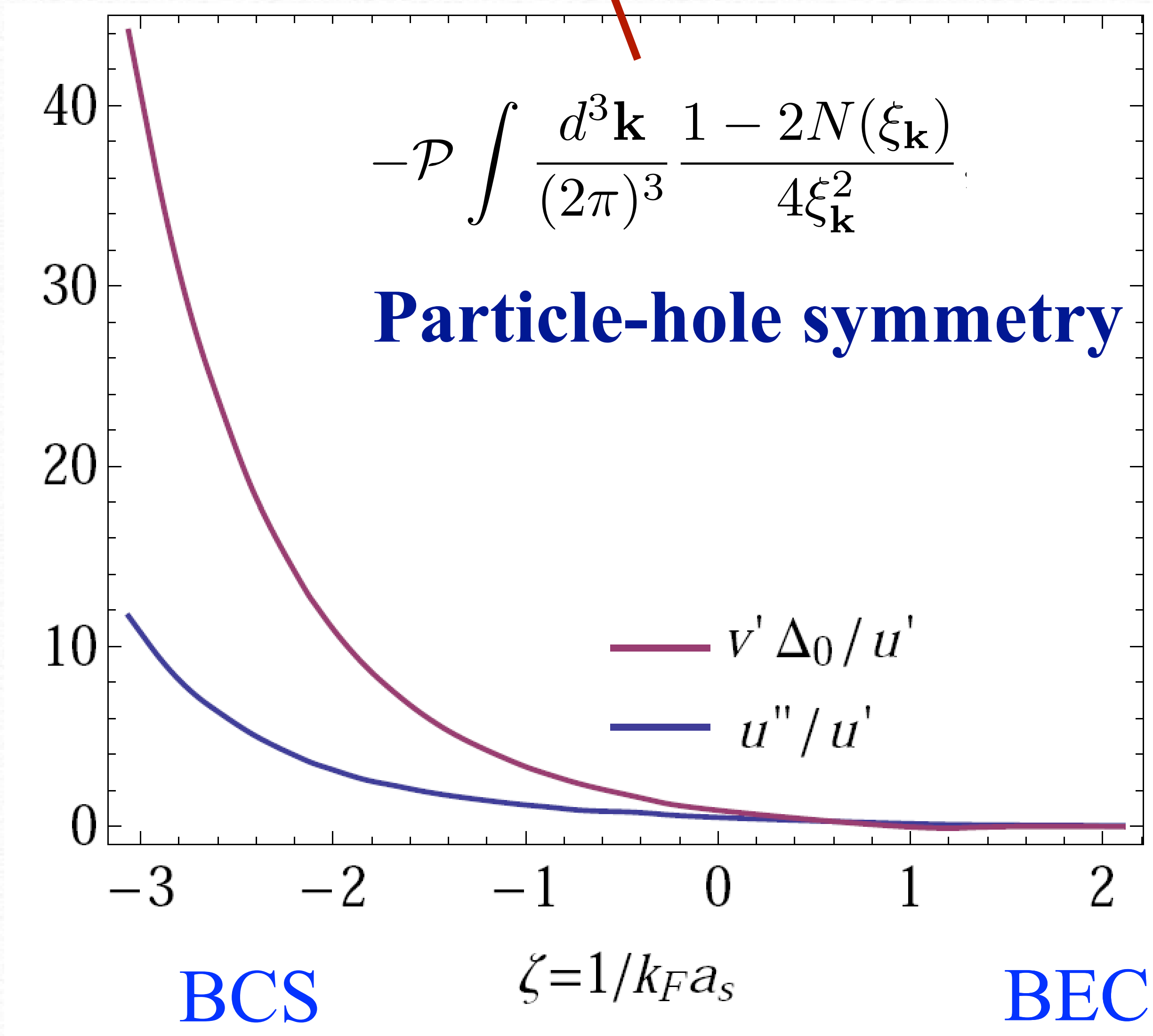


Increasing attractive interaction

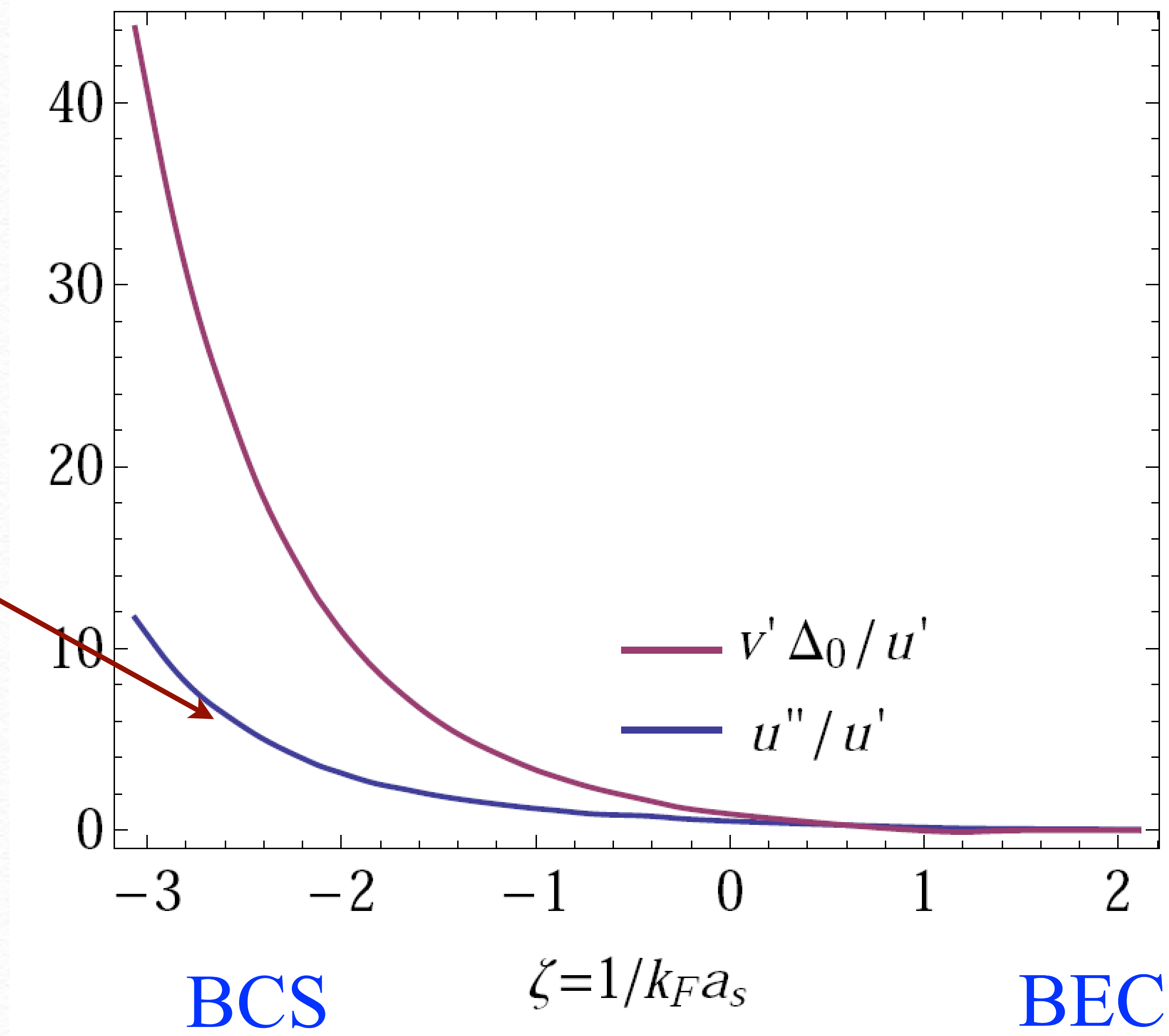
Ginburg-Landau Theory

$$F[\bar{\Delta}, \Delta] = \int dt d^3\mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta \}$$

$$u = u' + iu''$$

Damping term increases

-- because of small gap



Increasing attractive interaction

(A) Ignoring damping term

Ginburg-Landau Theory

$$F[\bar{\Delta}, \Delta] = \int dt d^3\mathbf{x}\{\bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta\}$$

$$u = u' + iu''$$

Ginburg-Landau Theory

$$F[\bar{\Delta}, \Delta] = \int dt d^3\mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta \}$$

$$u = u'$$

Ginburg-Landau Theory

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$$u = u'$$

$$\omega^2 = \frac{1}{v'}(\frac{k^2}{2m^*} + r) + \frac{u'^2}{2v'^2} \pm \sqrt{\frac{u'^4}{4v'^4} + \frac{u'^2}{v'^3}(\frac{k^2}{2m^*} + r) + \frac{r^2}{v'^2}}.$$

Ginburg-Landau Theory

$$F[\bar{\Delta}, \Delta] = \int dt d^3\mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta \}$$

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$$v' \gg u'$$

Ginburg-Landau Theory

$$F[\bar{\Delta},\Delta] = \int dt d^3\mathbf{x}\{\bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta\}$$

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$$\omega^2 = \frac{1}{v'}(\frac{k^2}{2m^*} + r) + \quad \pm \sqrt{\quad + \quad + \frac{r^2}{v'^2}}.$$

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Ginburg-Landau Theory

$$F[\bar{\Delta}, \Delta] = \int dt d^3\mathbf{x}\{\bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta\}$$

$$u = u'$$

$$\omega^2 = \frac{1}{v'}(\frac{k^2}{2m^*} + r) + \quad \pm \sqrt{\quad + \quad + \frac{r^2}{v'^2}}.$$

$$v' \gg u'$$

$$\omega = k/\sqrt{2m^* v'}$$

$$\omega = \sqrt{(k^2/2m^* + 2r)/v'}$$

Ginburg-Landau Theory

$$F[\bar{\Delta}, \Delta] = \int dt d^3\mathbf{x}\{\bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta\}$$

$$u = u'$$

$$\omega^2 = \frac{1}{v'}(\frac{k^2}{2m^*} + r) + \frac{u'^2}{2v'^2} \pm \sqrt{\frac{u'^4}{4v'^4} + \frac{u'^2}{v'^3}(\frac{k^2}{2m^*} + r) + \frac{r^2}{v'^2}}.$$

$$v' \gg u'$$

$$\omega = k/\sqrt{2m^* v'}$$

$$\omega = \sqrt{(k^2/2m^* + 2r)/v'}$$

$$u' \gg v'$$

$$\omega = \sqrt{2r/(2mu^2)}k$$

$$\omega = \sqrt{\frac{k^2}{v'm^*} + \frac{2r}{v'} + \frac{u'^2}{v'^2}}$$

Spectral Weight Transfer

$$\Delta \rightarrow \sqrt{r/b} + \delta_a + i\delta_p$$

Spectral Weight Transfer

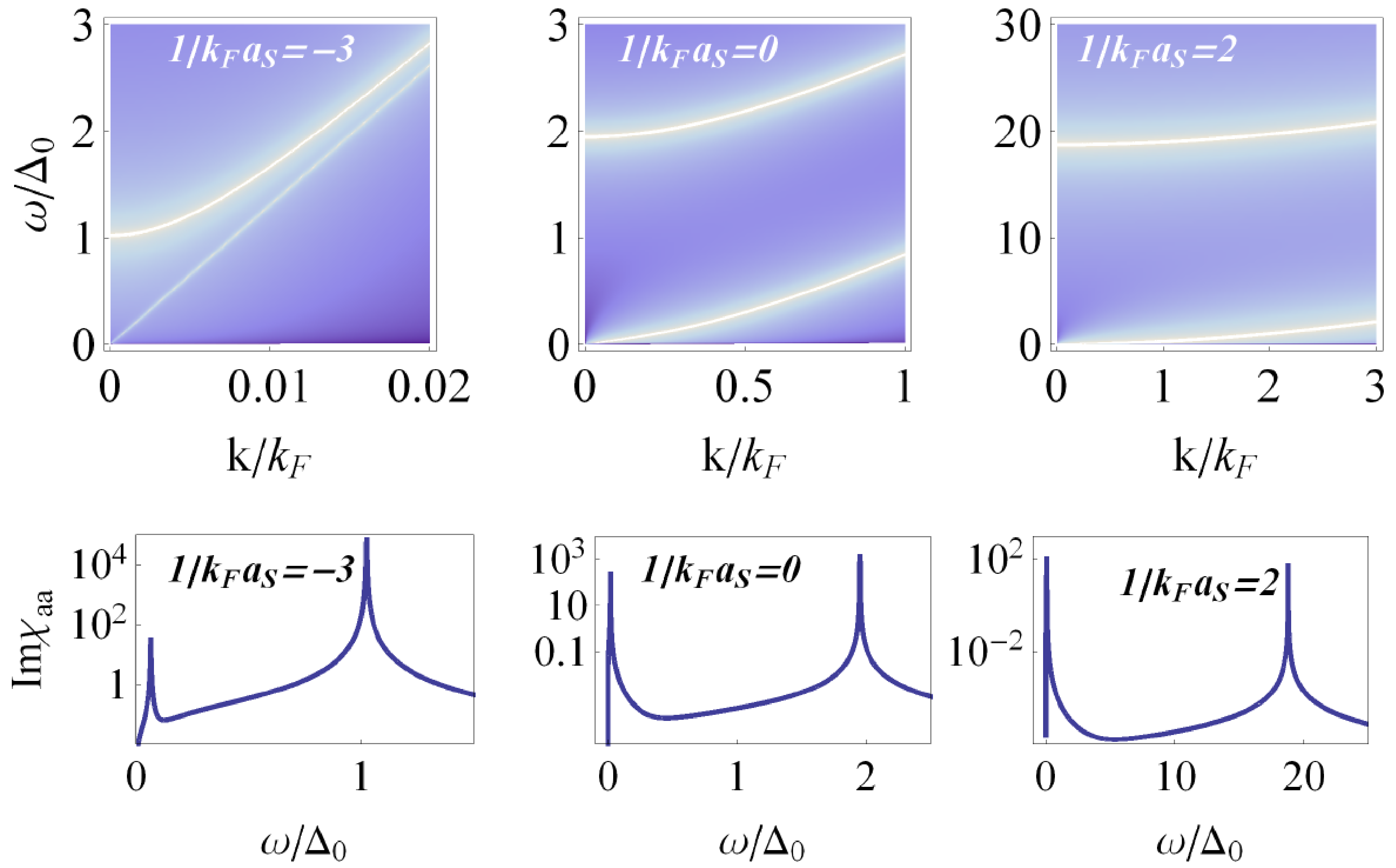
$$\Delta \rightarrow \sqrt{r/b} + \delta_a + i\delta_p$$

$$< \delta_a^*(\omega, \mathbf{k})\delta_a(\omega, \mathbf{k}) >= \frac{-v'\omega^2 + k^2/2m^*}{-u'^2\omega^2 + (-v'\omega^2 + k^2/2m^*)(-v'\omega^2 + k^2/2m^* + 2r)}$$

Spectral Weight Transfer

$$\Delta \rightarrow \sqrt{r/b} + \delta_a + i\delta_p$$

$$< \delta_a^*(\omega, \mathbf{k})\delta_a(\omega, \mathbf{k}) >= \frac{-v'\omega^2 + k^2/2m^*}{-u'^2\omega^2 + (-v'\omega^2 + k^2/2m^*)(-v'\omega^2 + k^2/2m^* + 2r)}$$



BCS **Increasing attractive interaction** BEC

(B) Including damping term

Ginburg-Landau Theory

$$F = \int dt d^3\mathbf{x} \left\{ \bar{\Delta}\left(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r\right)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta + \bar{\Delta}\xi + \Delta\xi^* \right\}$$

$$u = u' + iu''$$

Langevin force

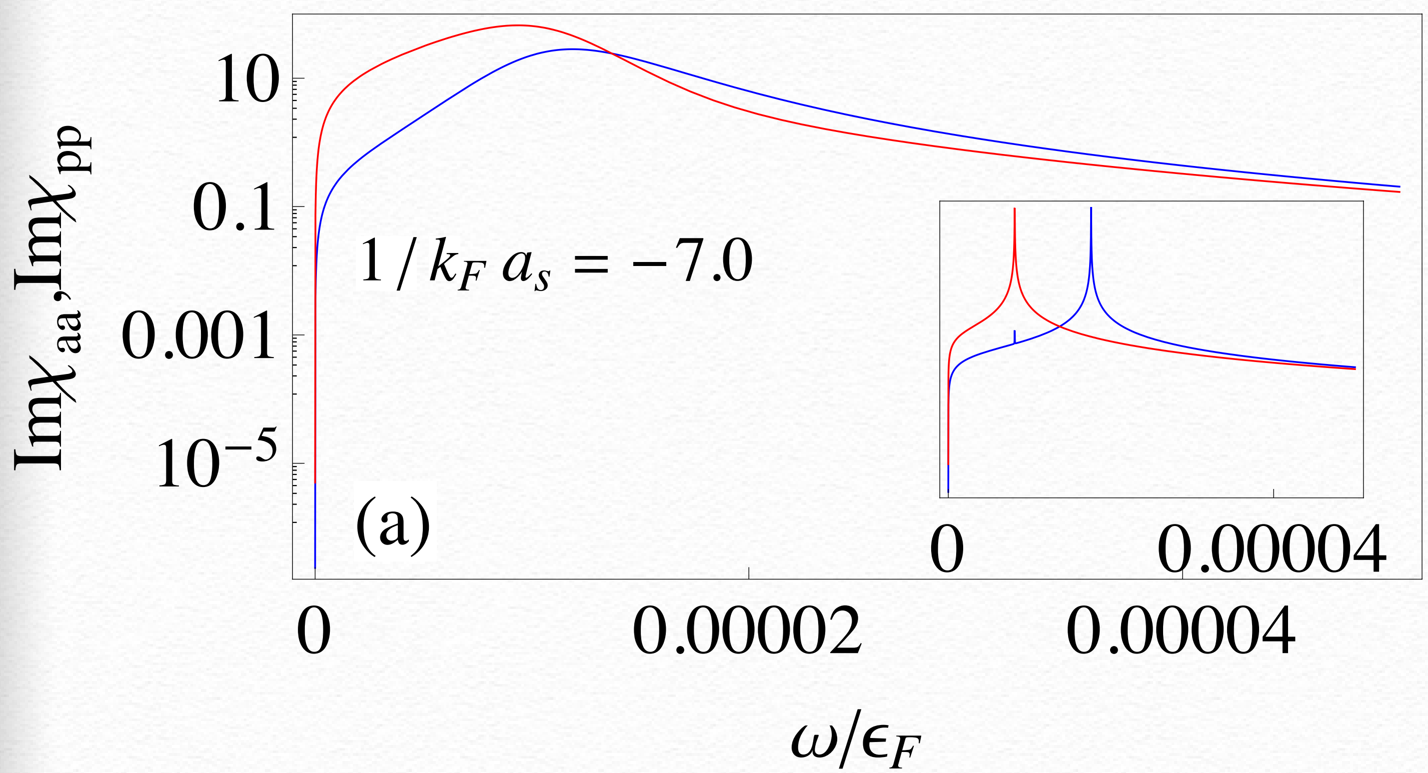
$$< \xi(t', \mathbf{x}')\xi(t, \mathbf{x}) >=< \xi^*(t', \mathbf{x}')\xi^*(t, \mathbf{x}) >= 0$$

$$< \xi^*(t', \mathbf{x}')\xi(t, \mathbf{x}) >= N\delta(t - t')\delta(\mathbf{x} - \mathbf{x}')$$

Ginburg-Landau Theory

$$F = \int dt d^3\mathbf{x}\left\{\bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta + \bar{\Delta}\xi + \Delta\xi^*\right\}$$

$$u = u' + iu''$$



Strong damping term leads to hybridization between amplitude and phase mode when gap is small

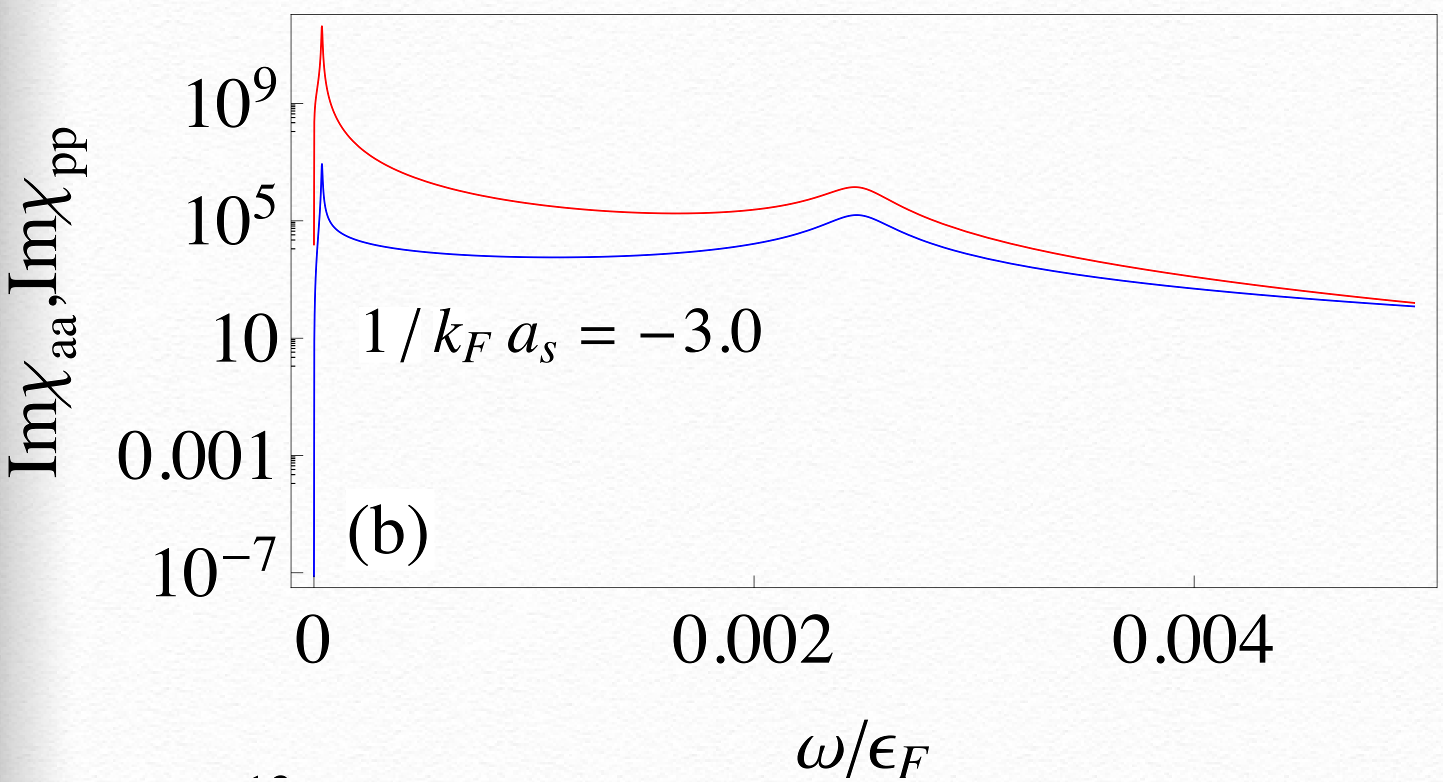
$$\Delta_0/E_F \simeq 2\times 10^{-5}$$

BCS — Increasing attractive interaction — BEC

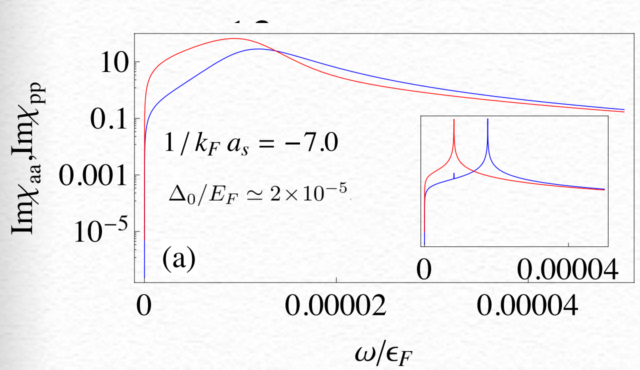
Ginburg-Landau Theory

$$F = \int dt d^3\mathbf{x} \left\{ \bar{\Delta}\left(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r\right)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta + \bar{\Delta}\xi + \Delta\xi^* \right\}$$

$$u = u' + iu''$$



A Higgs mode can be well identified



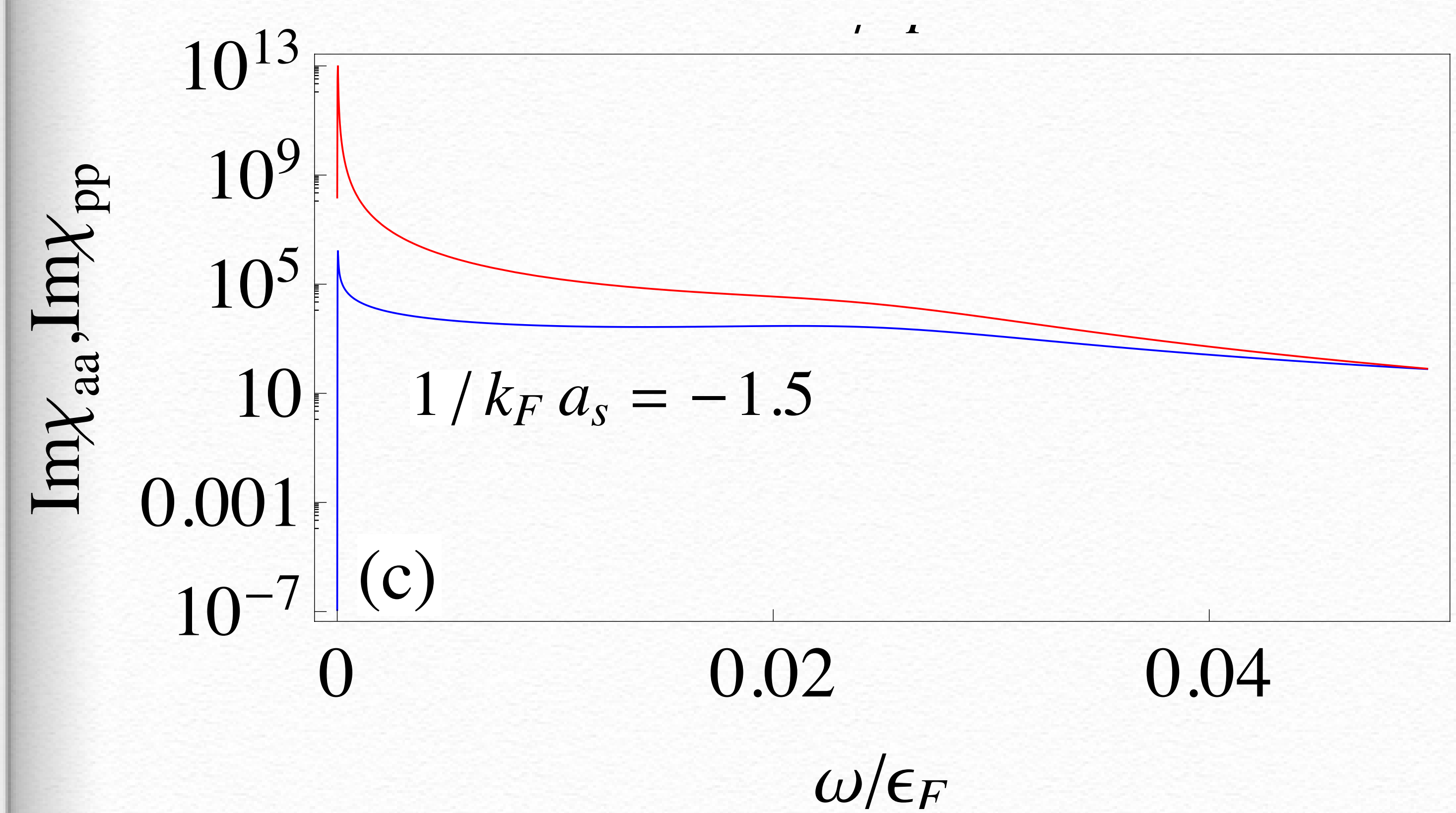
$$\Delta_0/E_F \simeq 0.01$$

BCS **Increasing attractive interaction** BEC

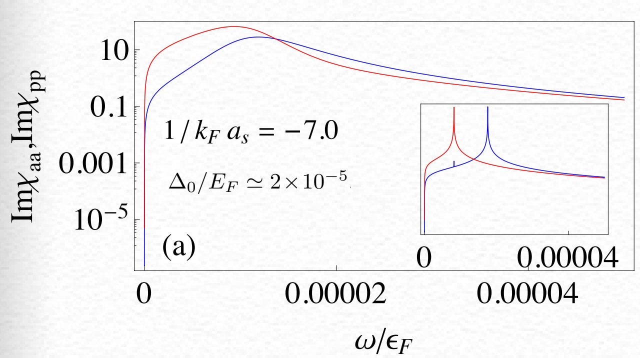
Ginburg-Landau Theory

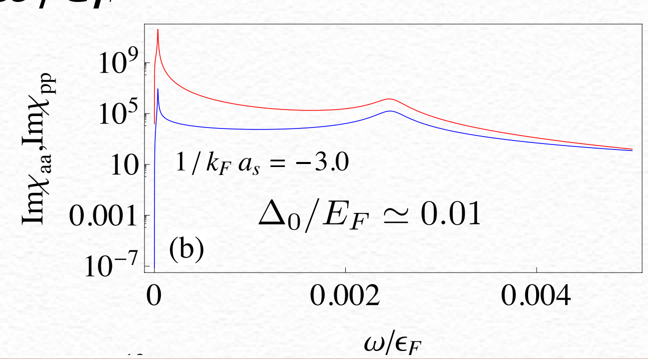
$$F = \int dt d^3\mathbf{x} \left\{ \bar{\Delta}\left(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r\right)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta + \bar{\Delta}\xi + \Delta\xi^* \right\}$$

$$u = u' + iu''$$



Higgs mode disappears





$$\Delta_0/E_F \simeq 0.1$$

BCS **Increasing attractive interaction** BEC

(C) Superconductor:
including coupling to external electromagnetic field

Ginburg-Landau Theory

$$F = \int dt d^3\mathbf{x}\Big\{ -\frac{1}{8\pi}\phi\nabla^2\phi + \bar{\Delta}\Big(-iu(\partial_t - 2e\phi) + v(\partial_t - 2e\phi)^2 - \frac{\nabla^2}{2m^*} - r\Big)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta\Big\}$$

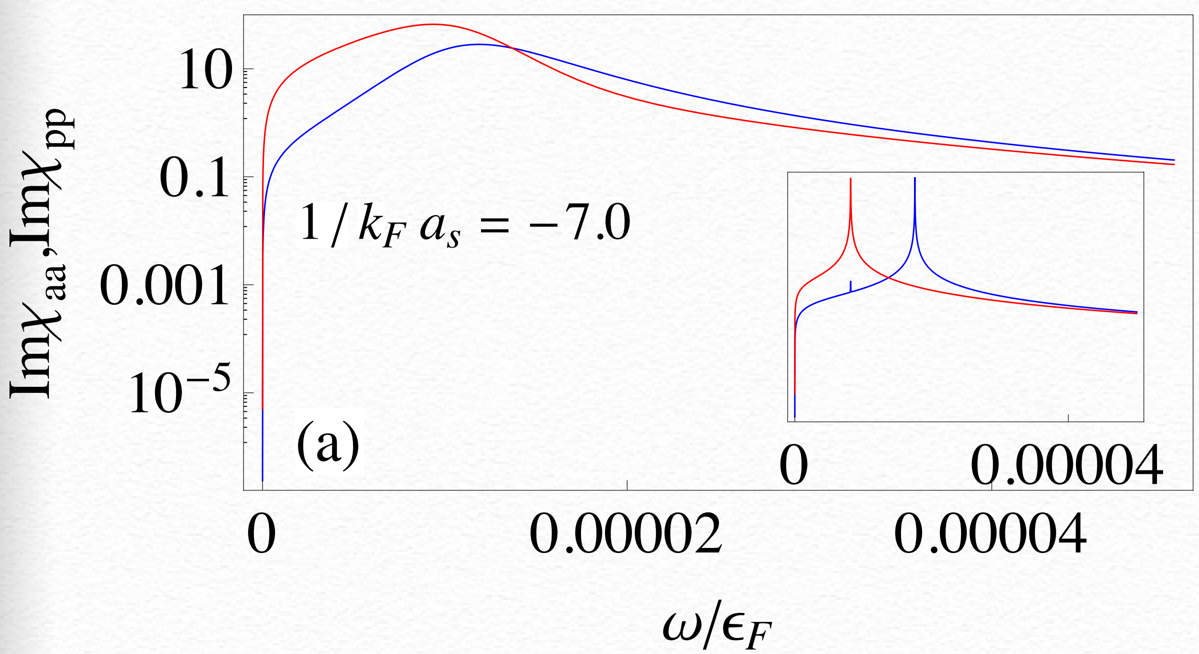
$$u = u' + iu''$$

Ginburg-Landau Theory

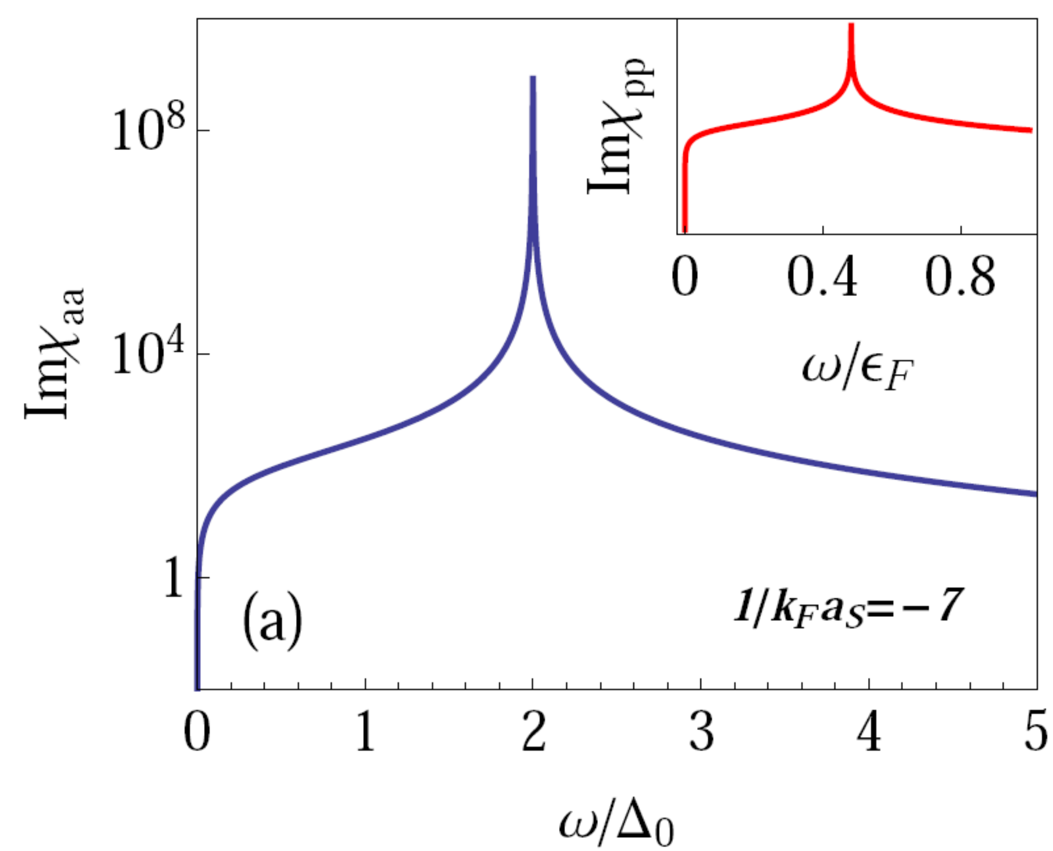
$$F = \int dt d^3\mathbf{x}\Big\{ -\frac{1}{8\pi}\phi\nabla^2\phi + \bar{\Delta}\Big(-iu(\partial_t - 2e\phi) + v(\partial_t - 2e\phi)^2 - \frac{\nabla^2}{2m^*} - r\Big)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta\Big\}$$

$$u = u' + iu''$$

BCS Regime: $\Delta_0/E_F \simeq 2\times 10^{-5}$



Neutral case



Charged case

Conclusion:

1. In the BCS regime, Anderson-Higgs mechanism plays an important role in measuring a well-defined Higgs mode

2. From BCS to BEC, Higgs mode is pushed to high energy and meanwhile, the spectral weight is transferred to Bogoliubov mode.

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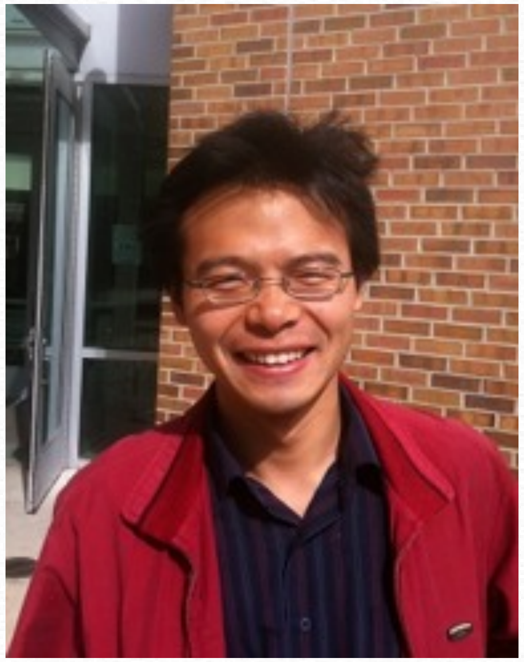
（清华高研院）



张世忠
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Thank you very much for your attention