# **Higgs Modes in Cold Atoms and Superconductor**

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# **Higgs Mode** ! <sup>=</sup> <sup>p</sup>*ck*<sup>2</sup> <sup>+</sup> *<sup>|</sup>*↵*<sup>|</sup>* (24)



 $\phi = \sqrt{\rho + \delta \rho} e^{i\theta}$ 

# ! <sup>=</sup> <sup>p</sup>*ck*<sup>2</sup> <sup>+</sup> *<sup>|</sup>*↵*<sup>|</sup>* (24) **Higgs Mode**  $\mathcal{U}$   $\mathcal{U}$   $\rightarrow$  1  $\bigodot$ **A** Higgs mode mode  $Im(\psi)$  $Re(\psi)$

# **Phase fluctuation**   $\phi = \sqrt{\rho + \delta \rho} e^{i\theta}$  **Phase fluctuation**

# **Higgs Mode** ! <sup>=</sup> <sup>p</sup>*ck*<sup>2</sup> <sup>+</sup> *<sup>|</sup>*↵*<sup>|</sup>* (24)



# **Experiments about Higgs modes** riments about Higgs modes

# **Superconducting charge density wave compound**

 $\mathrm{NbSe}_2$  and was later in the Higgs amplitude mode.



# **Framan scattering experiment**

Sooryakumar and Klein, PRL, 45, 660 (1980); Littlewood and Varma, PRB, 26, 4883 (1992); PRL, 47, 811 (1981).

### **Experiments about Higgs modes** vated to the plasma frequency as a result of the  $\alpha$  coupling to the gauge boson (photon field), which field  $s \circ \text{A}$  superconductor  $\text{N}$ thickness grown on an MgO substrate (24) with  $s_{\rm eff}$ superconducting critical temperature (T $\sim$  $\sim$   $\sim$   $\sim$   $\sim$ sensitively monitored through the change of Experiments about higgs modes International Control  $\mathbf{I}^{\bullet}$ **10** ante ahout Higge modes International Property (arb. units) Fig. 3. The initial spectroscopy. Spectroscopy. (A)  $\mathbb{R}$  $\overline{\mathbf{v}}$

low-temperature limit, which is 1.3 THz (Fig. 1C); this implies that the pump pulse does not gen-

15.5 Kilom

#### (8, 9). The Higgs amplitude mode in superconductors has been also studied theoretically  $\mathcal{O}$ porconducting comp perconducting comp  $t_{\rm in}$  denote this change as  $t_{\rm in}$ **details. Superconducting compound** E pump (kV/cm) iperconducting compound erconuut

 $\mathbf{N}$   $\mathbf{N}$ a variable time delay. In general, we can detect  $NbN$ 



pass filters are used to generate the different

order parameter and twice the parameter and twice the twice the twice the twice the twice the twice the twice t

2

0.6 THz

Goldstone (NG) mode] and (ii) gapped amplitude modes (Higgs mode) (5–7). In charged-particle modes (1–7). In charged-particle modes (1–7). In charged-particle

15.5 K

 $^{\rm n}$ 

pump (arb. units)



7 88 optical gate pulse such that, in the absence of the pump, Eprobe at this timing monotonically

time delay of another optical gate pulse and

using the electrooptic (EO) sampling method. In

this experiment, we fixed the time  $\eta$ 

Eprobe relative to its value in the absence of the

pump as a function of the pump-probe delay

time, the time  $\mathcal{U}$ 

dEprobe. For details, see (25). In the present case,

we investigated the order parameter dynamics of  $\eta$ 

#### $\mathbf{p}_{\mathbf{m}}$  $\overline{\phantom{a}}$  $T$  unip-1 tope ex  $m \cdot t$ **Pump-Probe experiment**

 $\frac{1}{\sqrt{2}}$  $\Omega$ <sub>r</sub>  $\Omega$ <sub>i</sub> 13 K  $\mathbf{a}$ in the presence of coherently oscillating multicycle pump fields. The temporal waveform of the  $\sigma$  or  $\sigma$  internal eq. 5 ventional regimes. The present scheme using  $51145(2014)$  $\epsilon$ , and  $\epsilon$  and  $\epsilon$ Tokyo group, Science, 345, 1145 (2014)

12 K

 $\mathcal{L}$ pnictide, which would provide new insight about

pulse (Fig. 1B). There are in fact various possible

as a function of the pump THz field strength  $\mathcal{H}_\mathbf{f}$ 

transmitted pump The transmitted pump T $\mathcal{H}$  $(100 \times 10^{-15})$  and above  $(150 \times 15)$  Tc  $= 15$ (D) Power spectra of the transmitted pump THz transmitted pump THz  $\mathcal{H}_\mathbf{H}$ pulse at various temperatures. (E) THG intensity

1.2 1.4 1.6 1.8 2.0 2.2 2.4

vealed here originates from resonance of ac fields to the collective amplitude mode of the collective  $\mathcal{H}$ order parameter, which leads to the strong THG

order harmonics will provide a unique avenue for probing ultrafast dynamics of the order parameter in out-of-equilibrium superconducto the intriguing tors. It is highly intriguing the  $\mathcal{U}$ tum trajectories of the pseudospins of the pseudospins of the pseudospins on  $\mathbb{R}$ sphere in the nonperturbative light-matter interaction regime with much higher THz fields, which would result in a dynamics of superconducting order parameter  $\mathcal{M}_{\text{eff}}$ 

δ

and shown in Fig. 4C agrees qualitatively with experiment in Fig. 4B. We conclude that the

Generally, collective modes in ordered phases

arising from spontaneous symmetry breaking are

classified into (i) gapless phase modes [Nambu-

Goldstone (NG) mode] and (ii) gapped amplitude

modes (Higgs mode) (5–7). In charged-particle

systems such as superconductors with long-

range Coulomb interactions, the gapless NG

mode becomes massive; that is, its energy is ele-

vated to the plasma frequency as a result of the

 $c$  the gauge boson  $\mathbb{R}^n$ 

is referred to as the Anderson-Higgs mechanism

 $\overline{\phantom{a}}$ 

ductors has been also studied theoretically

(6, 10–15); because it is not accompanied by

charge fluctuations, it does not couple directly to

electromagnetic fields in the linear response re-

# **Experiments about Higgs modes**

# **Cold atom system: bosons in optical lattices** 3

,50 experimental runs. Error bars, s.e.m.



agreement with an alumnum and the Munich group. Nature, 4 and dashed line, respectively; see text). Horizontal and vertical error bars Munich group, Nature, 487, 454 (2012) h



Registration

commensurate filling (solid line):

hns Fiffi

critical point in the same way as the gap frequency.

 $\mathcal{A}$ 

**14** 

Here h denotes Planck's constant. This value is based on an analysis of variations around a mean-field state7,16 (throughout the manuscript, we have rescaled the theoretical calculations to match the value of  $\epsilon$ jc<0:06 obtained from quantum Monte Carlo simulations26).

The sharpness of the spectral onset can be quantified by the width of the fitted error function, which is shown as vertical dashed lines in Fig. 2a. Approaching the critical point, the spectral onset becomes sharper, and the width normalized to the centre frequency n<sup>0</sup> remains constant (Supplementary Fig. 3). The constancy of this ratio indicates that the width of the spectral onset scales with the distance to the

We observe similar gapped responses in the Mott insulating regime  $\mathbb{R}$  , with the gap closing continuously when approaching the critical point (Fig. 2a, open circles).  $W_{\rm eff}$  interpret this as a result of combined particle and hole excitations  $W_{\rm eff}$ with a frequency given by the Mott excitation given by the Mott excitation gap the Mott excitation gap that closes at the Mott excitation gap the Mott excitation gap that closes at the Mott excitation gap the Mott excitati transition point16. The fitted gaps are consistent with the Mott gap

\$ %1=<sup>2</sup> <sup>1</sup>{j=<sup>j</sup> ð Þ<sup>c</sup> <sup>1</sup>=<sup>2</sup>

18 IU

where no mean-field the MI is the MI

The observed softening of the onset of spectral response in the superfluid regime has led to an identification of the experimental signal with a response from collective excitations of Higgs type. To gain further insight into the full in-trap response, we calculated the eigenspectrum of the system in a Gutzwiller approach16,22 (Methods and Supplementary Information). The result is a series of discrete eigenfrequencies (Fig. 3a), and the corresponding eigenmodes show in-trap superfluid density distributions, which are reminiscent of the

**M** 

#### vibrational modes of a drum (Fig. 3b). The frequency of the lowestlying amplitude-like eigenmode n0,G closely follows the long-wave- $\mathcal{L}$  $\mathbf{I} = \mathbf{I} \cdot \mathbf{A}$ a, Classical energy density V as a function of the order parameter parameter  $\mathbf{v}$ **Lattice modulation spectroscopy**



denote the experimental uncertainty of the lattice depths and the fit error for the centre frequency of the error function, respectively (Methods). Vertical dashed

# **Higgs Mode**

 $\phi = \sqrt{\rho + \delta \rho} e^{i\theta}$ 



*A*(*k* ⇡ 0*,* !) (26)

## **Higgs Mode**  $\sum_{i=1}^{n} a_i$

*<sup>t</sup>* <sup>r</sup><sup>2</sup>

<sup>2</sup>*<sup>m</sup> <sup>r</sup>*) <sup>+</sup>

$$
\phi = \sqrt{\rho + \delta \rho} e^{i\theta}
$$



*A*(*k* ⇡ 0*,* !) (26)

where all the parameters *u*, *v*, *m*⇤

crossover, as shown in Fig. 1:

that *v*<sup>0</sup>

expressed in terms of *µ*, *T* and interaction parameter

⇣ = 1*/*(*k*F*a*s) [23]. The coecients of time derivative

terms *u* = *u*<sup>0</sup> + *iu*<sup>00</sup> and *v* = *v*<sup>0</sup> + *iv*<sup>00</sup> are complex in gen-

eral. The real parts *u*<sup>0</sup> and *v*<sup>0</sup> describe the propagating

behavior of the cooper pairs, while the imaginary parts

*u*<sup>00</sup> and *v*<sup>00</sup> indicate a damping process of the Cooper pairs

due to the coupling to the fermionic quasi-particles. Here

we first discuss the behaviors of *u* and *v* at BCS to BEC

than unity at the extreme BCS limit. This is because,

in the extreme BCS limit, the asymptotic behaviors of

*u*<sup>0</sup> can be derived as *u*<sup>0</sup> ! 0, which is a consequence of

particle-hole symmetry of the excitation spectrum of a

20

*||*

**Non-relativistic theory:** 

$$
S = \int dt d^3 \mathbf{x} \left\{ \phi^*(-i\partial_t - \frac{\nabla^2}{2m} - r)\phi + \frac{b}{2}|\phi|^4 \right\}
$$

$$
\omega=\sqrt{k^2/2m(k^2/2m+2r)}
$$



## **Higgs Mode**  $\mathbf{H}^2 \sim \mathbf{A} \mathbf{A} \mathbf{A}$ The existance of Higgs mode requires Lorentz invariance

pump part pump probe spectrum probe spectrum probe spectrum probe spectrum in a non-diabatic excitation and pr<br>Probe

$$
\phi = \sqrt{\rho + \delta \rho} e^{i\theta}
$$

The existance of Higgs mode requires Lorentz invariance



 $\mathbb{S}$ 

**Zoome Company And Company** 

*A*(*k* ⇡ 0*,* !) (26)

 $\mathbf{M}$ 

di↵erent superconducting materials have di↵erent pairing

 $\mathbb{Z}$ 

**Relativistic theory:** 

$$
S = \int dt d^3 \mathbf{x} \left\{ \phi^*(\partial_t^2 - \frac{\nabla^2}{2m} - r)\phi + \frac{b}{2} |\phi|^4 \right\}
$$

⇤(*i*@*<sup>t</sup>* <sup>r</sup><sup>2</sup>

**Gapless Numbu-Goldstone mode:** Canless Numbu-Coldstone mode: (a)

⇢

$$
\omega = \frac{k}{\sqrt{2m}}
$$

The existence of Higgs model is the existence of Higgs model in the existence of Higgs model in  $\mathcal{L}_\mathbf{r}$ 

c<sup>1</sup>  $\sqrt{12}$ 

Z

$$
\omega = \sqrt{\frac{k2}{2m} + 2r}
$$



.<br>Manazarta

*b*

One gapless Numbu-Goldstone mode:

Relativistic theory:

Symmetry breaking

One gapped Higgs amplitude mode:

One gapped Higgs amplitude mode:

## **Now we shall discuss how this general Model i** a seature resonance, **k** 0.000 **1, in this case interaction energy with seature interaction energy will be seen to band gap**  $\mathbf{r}$  $\blacksquare$

*iji*0*j*0

*m,i*



$$
\hat{H}_{\rm BH} = -t \sum_{\langle ij \rangle} \hat{b}_i^{\dagger} \hat{b}_j + U \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i
$$

Nearest neighboring On-site Chemical the filling of fermions is always smaller than unity,  $hopping$ ighboring On-site Chemical

h*ij*i*,*

*ijmn*

On-site interaction Chemical potential

*i,m*

Now we shall discuss how the shall discuss how this general Hamiltonian is simplified. Hamiltonian is simplified

*<sup>i</sup>.* (7)

With similar analysis, for spin-1*/*2 fermions, we can also reach a single-band Fermi Hubbard model, provided that

*H* ˆ

*m,i*

### **Quantum Phases in Bose-Hubbard Model** to *the international* complete interaction term interaction term except for  $\mathbf{p}_{\text{mean}}$  interaction  $\mathbf{p}_{\text{mean}}$  is ignored. WITH THESE SIN DOSE-HUDBARD MODEL AND BOSE HUBBARD MODEL AND BOSE TRANSITIONS OF REACH AND AND AND A II. **II. Cuantum Phases in Bose-Hubbard Model II. Hubbard Model Model Model Constitute Phases in Rose Hubbard Model**  $\frac{1}{1000}$  and  $\frac{1}{100}$   $\frac{1}{100}$   $\frac{1}{100}$   $\frac{1}{100}$   $\frac{1}{100}$   $\frac{1}{100}$   $\frac{1}{100}$

comparing to the on-site term. Besides, the nearest neighboring site interaction *Uiiij* or *Uiijj* is also smaller comparing

understand the mechanism of quantum materials better.

understand the mechanism of the mec

$$
\hat{H}_{\rm BH} = -t \sum_{\langle ij \rangle} \hat{b}_i^{\dagger} \hat{b}_j + U \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i
$$

$$
U = 0 \qquad \qquad |\Psi\rangle = \left(\frac{1}{\sqrt{M}} \sum_{i=1}^m \hat{b}_i^{\dagger}\right)^N |0\rangle
$$

material. For instance, using high-Tc superconductivity as an example, it is widely believed for  $\mathcal{W}$ 

*i*<sup>t</sup>e particle number fluctuation V te particle **!**  On-site particle number fluctuation  $\blacksquare$  $\frac{1}{1}$  $\mathbf{r}$ ,  $\sqrt{2}$  on site particle number matteaution The state particle number internation which site *i* obey a distribution, if  $\blacksquare$ 

$$
P[n_i] = e^{-\bar{n}} \frac{\bar{n}_i^n}{n_i!} \qquad \bar{n} = \langle n_i \rangle \quad \boxed{\langle \delta n_i^2 \rangle = \bar{n}}
$$

*i*

*<sup>i</sup>* = *Un*0(*n*<sup>0</sup> 1) *µn*0*,* (10)

*<sup>i</sup>* i = ¯*n*. ii) This state has a

*<sup>n</sup>*<sup>0</sup> *<sup>|</sup>*0i*.* (11)

*<sup>i</sup>.* (7)

*|* i =

!*<sup>N</sup>*

*<sup>i</sup>* i = ¯*n*. ii) This state has a

and to minimize the energy we have *n*<sup>0</sup> 1 *< µ/*(2*U*) *< n*0. The ground state is

excitation has a gapless phonon mode and is a superfluid when finite interaction is turned on.

*<sup>i</sup>* i = ¯*n*. ii) This state has a

*<sup>i</sup>* = *Un*0(*n*<sup>0</sup> 1) *µn*0*,* (10)

*<sup>i</sup>* = *Un*0(*n*<sup>0</sup> 1) *µn*0*,* (10)

*<sup>H</sup>*ˆ*<sup>i</sup>* <sup>=</sup> *Un*0(*n*<sup>0</sup> 1) *µn*0*,* (10)

*|*0i*.* (8)

*,* (9)

*,* (9)

 $\nabla$  Long-range correlations,  $U(1)$  symmetry breaking **This is state in the particle number at each site is particle number at each site** *i* **observations,**  $U(1)$  **symmetry breaking** *P*<sub>*Fanless* Golds</sub>  $\nabla$  Long-range correlations, U(1) symmetry breaking *b* is in *C* and *C* as *i* is a Bose condensed point in the set of the Bose condensed phase, as we discussed before, the set of the  $\langle \hat{b}_i^{\dagger} \hat{b}_j \rangle \rightarrow C \qquad |i-j| \rightarrow \infty.$ In the other limit *t* = 0, each site becomes independent, and for each site *n<sup>i</sup>* is a fixed number denoted by *n*<sup>0</sup>  $\hat{b}$  i  $\hat{b}$  i  $\hat{b}$   $\hat{c}$  is a  $\hat{c}$  condensed phase,  $\hat{c}$  is a  $\hat{c}$  $\langle b_i^{\dagger} b_j \rangle \rightarrow C \qquad |i-j| \rightarrow \infty$  $\blacksquare$ In the other limit of the other limits in the site of the site  $\mathbf{r}$  is a finite of the site  $\mathbf{r}$  is a fixed by  $\mathbf{r}$  is *i*  $\frac{1}{2}$   $\frac{1}{2}$ Long-range correlations, U(1) symmetry breaking w Long-range correlations,  $U(1)$  symmetry breaking<br>where  $\frac{1}{2}$ long-range order h  $\hat{b}$  $b_i^{\intercal}$ *i*  $\langle \hat{b}_i^\dagger \hat{b}_j \rangle \rightarrow \mathcal{C}$   $|i-j| \rightarrow \infty$ excitation has a gapless phonon mode and is a superfluid when finite interaction is turned on.  $\hat{h}$  $\hat{b}_j$ *i*  $\sqrt{2}$  $\phi \rightarrow C$  as  $|i-j| \rightarrow \infty$ excitation has a gapless phonon mode and is a superfluid when finite interaction is a superfluid when finite i<br>The contraction is turned on the contraction is turned on. It is turned on the contraction is turned on. It is

> ( ˆ *b † i* )

oldstone mod  $\int$   $\cos \theta$   $\cos \theta$ *<sup>i</sup>* = *Un*0(*n*<sup>0</sup> 1) *µn*0*,* (10) In the other limit *t* = 0, each site becomes independent, and for each site *n<sup>i</sup>* is a fixed number denoted by *n*<sup>0</sup> and the energy we have the energy we have  $\frac{1}{2}$ Gapless Goldstone mode

*<sup>|</sup>* <sup>i</sup> <sup>=</sup> <sup>Y</sup>

of fermion Hubbard model, for instance, for instance, experimentally determining whether the ground state of fermion Hubbard model, and in the ground state of fermion Hubbard model when the ground state of fermion Hubbard

model is superconducting or not. By understanding these strongly correlated models, it will eventually help us to the strongly correlated models, it will eventually help us to the strongly help us to the strongly help us t

understand the mechanism of quantum materials better.

*|* i =

ˆ *b*

long-range order h

### **Quantum Phases in Bose-Hubbard Model** to *the international* complete interaction term interaction term except for  $\mathbf{p}_{\text{mean}}$  interaction  $\mathbf{p}_{\text{mean}}$  is ignored. WITH THESE SIN DOSE-HUDBATH MODEL *P[<i>n***i**] = *n***<sup>***n***</sup>]** *e***<sup>***n***</sup>] =** *n***<sup>***n***</sup>] =** *n***<sup>***n***</sup>** *i ni*! , and  $\alpha$  (9)  $\alpha$  (9)  $\alpha$ **In the other contains in the other limit is a fixed number of** *n***<sub>i</sub> is a fixed number of** *n***<sub>i</sub> is a fixed by** *n***<sup>1</sup> is a fixed by** *n***<sup>1**</sup> <sup>ˆ</sup>*b<sup>j</sup>* i ! *<sup>C</sup>* as *<sup>|</sup><sup>i</sup> <sup>j</sup><sup>|</sup>* ! 1. (iii) Because this is a Bose condensed phase, as we discussed before, the excited thas a gaple in the superfluid when finite interaction is turned when  $\mathbf{u}$

comparing to the on-site term. Besides, the nearest neighboring site interaction *Uiiij* or *Uiijj* is also smaller comparing

where  $\alpha$  is the formulation at each site  $\alpha$  is the fluctuation at each site  $\alpha$ 

excitation has a gapless phonon model when finite interaction is a superfluid when finite interaction is turned on.

⇣

'

ˆ *b †*

*<sup>i</sup>* '⇤

ˆ

*i*

*b<sup>i</sup>* + *Un*

ˆ

*<sup>i</sup>*(ˆ*n<sup>i</sup>* 1) *µn*

ˆ

⌘

*.* (12)

~<sup>2</sup>*a<sup>s</sup>*

*ma*<sup>3</sup> (5)

*i* is in the state of  $\frac{1}{2}$ 

*b<sup>j</sup>* i ! *C* as *|i j|* ! 1. (iii) Because this is a Bose condensed phase, as we discussed before, the

$$
\hat{H}_{\text{BH}} = -t \sum_{\langle ij \rangle} \hat{b}_i^{\dagger} \hat{b}_j + U \sum_i \hat{n}_i (\hat{n}_i - 1) - \mu \sum_i \hat{n}_i
$$
\n
$$
t = 0 \qquad |\Psi\rangle = \prod_i (\hat{b}_i^{\dagger})^{n_0} |0\rangle \qquad \qquad \boxed{\qquad \qquad }
$$
\n
$$
\xrightarrow{\mu}
$$
\n
$$
\xrightarrow{\text{Feyl}, \text{Deyn's 5, 14}}
$$
\n
$$
\xrightarrow{\text{Feyl}, \text{Deyn's 5, 14}}
$$
\n
$$
\xrightarrow{\text{Feyl}, \text{Deyn's 5, 14}}
$$

ˆ

long-range order <sup>h</sup>ˆ*<sup>b</sup>*

long-range order h

transition.

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comparing to the on-site term. Besides, the nearest neighboring site interaction *Uiiij* or *Uiijj* is also smaller comparing

where  $\alpha$  is the formulation at each site  $\alpha$  is the fluctuation at each site  $\alpha$ 

*b<sup>j</sup>* i ! *C* as *|i j|* ! 1. (iii) Because this is a Bose condensed phase, as we discussed before, the



This state has very di↵erent properties comparing to state at *U* = 0. i) At each site the number fluctuation vanishes.

(*Un*0(*n*<sup>0</sup> 1) *µn*0)) = *µ* + 2*U* 2*Un*<sup>0</sup> *>* 0. Thus, both excitation are gapped, and the compressibility is zero. The

*<sup>P</sup>*[*ni*] = *<sup>e</sup>n*¯ *<sup>n</sup>*¯*<sup>n</sup>*

*i*

1) (*Un*0(*n*<sup>0</sup> 1) *µn*0)=2*Un*<sup>0</sup> *µ >* 0, or taking a particle way with *E* = *U*(*n*<sup>0</sup> 1)(*n*<sup>0</sup> 2) *µ*(*n*<sup>0</sup> 1)

ˆ

Therefore, we conclude a quantum phase transition must take place in between. This is supprefluid to  $\mathcal{W}$ 

⌘

*<sup>i</sup>*(ˆ*n<sup>i</sup>* 1) *µn*

*b<sup>j</sup>* i = 0; and iii) The excitation is either adding a particle, *E* = *U*(*n*<sup>0</sup> + 1)*n*<sup>0</sup> *µ*(*n*<sup>0</sup> +

~<sup>2</sup>*a<sup>s</sup>*

*ma*<sup>3</sup> (5)

*i* is in the state of  $\frac{1}{2}$ 

*,* (9)

*.* (12)

*.* (12)

**The Form instance in Superconduction**  $U(1)$  symmetry breaking  $U(1)$  symme  $\mathbf{e}$ tr: *b † i* ) *<sup>n</sup>*<sup>0</sup> *<sup>|</sup>*0i*.* (11) *i No* Long-range correlations, and no U(1) symmetry breaking and the comparison are gapped as  $\mathbb{Z}$ *V* No Long-range correlations, and no U(1) symmetry breaking system is in a *Mott insulator* phase. The Long range conclusions, and no  $U(1)$  by inner fluctuations.

$$
\langle \hat{b}^{\dagger}_i \hat{b}_j \rangle = 0
$$

Mean-field Theory. In this case one invents a mean-field Hamiltonian as

*<sup>i</sup>* '⇤

ˆ

*i*

*b<sup>i</sup>* + *Un*

ˆ

excitation has a gapless phonon model when finite interaction is a superfluid when finite interaction is turned on.

Mean-field Theory. In this case one invention theory. In this case one invention  $\mathcal{A}$ 

system is in a *Mott insulator* phase.

⇣

'

ˆ *b †*

ii) There is no ODLRO, i.e. h

transition.

transition.

ˆ

long-range order <sup>h</sup>ˆ*<sup>b</sup>*

long-range order h

ii) There is no ODLRO, i.e. h

system is in a *Mott insulator* phase.

### **Quantum Phases in Bose-Hubbard Model** to *the international* complete interaction term interaction term except for  $\mathbf{p}_{\text{mean}}$  interaction  $\mathbf{p}_{\text{mean}}$  is ignored. WITH THESE SIN DOSE-HUDBATH MODEL *P[<i>n***i**] = *n***<sup>***n***</sup>]** *e***<sup>***n***</sup>] =** *n***<sup>***n***</sup>] =** *n***<sup>***n***</sup>** *i ni*! , and  $\alpha$  (9)  $\alpha$  (9)  $\alpha$ **In the other contains in the other limit is a fixed number of** *n***<sub>i</sub> is a fixed number of** *n***<sub>i</sub> is a fixed by** *n***<sup>1</sup> is a fixed by** *n***<sup>1**</sup> <sup>ˆ</sup>*b<sup>j</sup>* i ! *<sup>C</sup>* as *<sup>|</sup><sup>i</sup> <sup>j</sup><sup>|</sup>* ! 1. (iii) Because this is a Bose condensed phase, as we discussed before, the excited thas a gaple in the superfluid when finite interaction is turned when  $\mathbf{u}$ *m* **Rose-Hubbard Model**

comparing to the on-site term. Besides, the nearest neighboring site interaction *Uiiij* or *Uiijj* is also smaller comparing

~2

where  $\alpha$  is the formulation at each site  $\alpha$  is the fluctuation at each site  $\alpha$ 

excitation has a gapless phonon model when finite interaction is a superfluid when finite interaction is turned on.

'

*<sup>i</sup>* '⇤

*b<sup>j</sup>* i ! *C* as *|i j|* ! 1. (iii) Because this is a Bose condensed phase, as we discussed before, the

~<sup>2</sup>*a<sup>s</sup>*

*ma*<sup>3</sup> (5)

*i* is in the state of  $\frac{1}{2}$ 



ˆ

transition.

long-range order <sup>h</sup>ˆ*<sup>b</sup>*

#### $\blacksquare$ ~<sup>2</sup>*a<sup>s</sup>* **Quantum Phases in Bose-Hubbard Model** to *the international* complete interaction term interaction term except for  $\mathbf{p}_{\text{mean}}$  interaction  $\mathbf{p}_{\text{mean}}$  is ignored. WITH THESE SIN DOSE-HUDBATH MODEL *P[<i>n***i**] = *n***<sup>***n***</sup>]** *e***<sup>***n***</sup>] =** *n***<sup>***n***</sup>] =** *n***<sup>***n***</sup>** *i ni*! , and  $\alpha$  (9)  $\alpha$  (9)  $\alpha$ **In the other contains in the other limit is a fixed number of** *n***<sub>i</sub> is a fixed number of** *n***<sub>i</sub> is a fixed by** *n***<sup>1</sup> is a fixed by** *n***<sup>1**</sup> <sup>ˆ</sup>*b<sup>j</sup>* i ! *<sup>C</sup>* as *<sup>|</sup><sup>i</sup> <sup>j</sup><sup>|</sup>* ! 1. (iii) Because this is a Bose condensed phase, as we discussed before, the excited thas a gaple in the superfluid when finite interaction is turned when  $\mathbf{u}$

comparing to the on-site term. Besides, the nearest neighboring site interaction *Uiiij* or *Uiijj* is also smaller comparing

where  $\alpha$  is the formulation at each site  $\alpha$  is the fluctuation at each site  $\alpha$ 

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Thursday, December 25, 14 Thursday, December 25, 14

ˆ

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*b<sup>j</sup>* i ! *C* as *|i j|* ! 1. (iii) Because this is a Bose condensed phase, as we discussed before, the



This state has very di↵erent properties comparing to state at *U* = 0. i) At each site the number fluctuation vanishes.

*<sup>P</sup>*[*ni*] = *<sup>e</sup>n*¯ *<sup>n</sup>*¯*<sup>n</sup>*

*i*

1) (*Un*0(*n*<sup>0</sup> 1) *µn*0)=2*Un*<sup>0</sup> *µ >* 0, or taking a particle way with *E* = *U*(*n*<sup>0</sup> 1)(*n*<sup>0</sup> 2) *µ*(*n*<sup>0</sup> 1)

ˆ

Therefore, we conclude a quantum phase transition must take place in between. This is supprefluid to  $\mathcal{W}$ 

⌘

*.* (12)

*.* (12)

~<sup>2</sup>*a<sup>s</sup>*

*ma*<sup>3</sup> (5)

*i* is in the state of  $\frac{1}{2}$ 

*,* (9)

*<sup>i</sup>*(ˆ*n<sup>i</sup>* 1) *µn*

**The Form instance in Superconduction**  $U(1)$  symmetry breaking  $U(1)$  symme  $\mathbf{e}$ tr: *b † i* ) *<sup>n</sup>*<sup>0</sup> *<sup>|</sup>*0i*.* (11) *i No* Long-range correlations, and no U(1) symmetry breaking and the comparison are gapped as  $\mathbb{Z}$ *V* No Long-range correlations, and no U(1) symmetry breaking system is in a *Mott insulator* phase. The Long range conclusions, and no  $U(1)$  by inner fluctuations.

$$
\langle \hat{b}^{\dagger}_i \hat{b}_j \rangle = 0
$$

*<sup>i</sup>* '⇤

ˆ

*i*

*b<sup>i</sup>* + *Un*

ˆ

### iii) There is no ODLRO, i.e. here is no optimal to the interest of the interes ns ↑ Pear-field Theory. In this case of the Mean-field Theory. In this case of the Mean-field Hamiltonian are gapped *b* is either and in the excitation is either adding a particle,  $\frac{1}{2}$ *Ⅰ I v Excitations are gapped* and the compression are gapped and the compression are compressible in the compression of the compression and the compression of the compression are compression of the compression of t system is the *Mottalions* are gapped Excitations are gapped

⇣

'

ˆ *b †*

Mean-field Theory. In this case one invention theory. In this case one invention  $\mathcal{A}$ 

excitation has a gapless phonon model when finite interaction is a superfluid when finite interaction is turned on.

(*Unidate 1995, 14*<br>(*Unidate December 25, 14* Thursday, December 25, 14 *H*<br>mber 25, 14 Thursday, December 25, 14

ˆ

long-range order <sup>h</sup>ˆ*<sup>b</sup>*

long-range order h

transition.

transition.

ii) There is no ODLRO, i.e. h

system is in a *Mott insulator* phase.

#### **Quantum Phases in Bose-Hubbard Model** to *the international* complete interaction term interaction term except for  $\mathbf{p}_{\text{mean}}$  interaction  $\mathbf{p}_{\text{mean}}$  is ignored. WITH THESE SIN DOSE-HUDBATH MODEL *P[<i>n***i**] = *n***<sup>***n***</sup>]** *e***<sup>***n***</sup>] =** *n***<sup>***n***</sup>] =** *n***<sup>***n***</sup>** *i ni*! , and  $\alpha$  (9)  $\alpha$  (9)  $\alpha$ **In the other contains in the other limit is a fixed number of** *n***<sub>i</sub> is a fixed number of** *n***<sub>i</sub> is a fixed by** *n***<sup>1</sup> is a fixed by** *n***<sup>1**</sup> <sup>ˆ</sup>*b<sup>j</sup>* i ! *<sup>C</sup>* as *<sup>|</sup><sup>i</sup> <sup>j</sup><sup>|</sup>* ! 1. (iii) Because this is a Bose condensed phase, as we discussed before, the excited that a gapless in the superfluid when  $\mathbf{u}$  and  $\mathbf{u}$  and  $\mathbf{u}$  is turned on. ~<sup>2</sup>*a<sup>s</sup>* **n Phases in Bose-Hubbard Mod**  $\overline{\mathbf{A}^{\mathbf{a}}$

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Thursday, December 25, 14 Thursday, December 25, 14

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long-range order <sup>h</sup>ˆ*<sup>b</sup>*

long-range order h

transition.

# **Phase Diagram**



## **diagram of BHM. Inside a harmonic trap, local density and BHM. Institute a contract of BHM. Institute a wedding case of BH** one can read out in the interest of  $\bf{r}$  read  $\bf{r}$  in the interest of  $\bf{r}$

| implies the self-consistent Hamiltonian can only be solved numerically be solved numerically determine ', and

we need to minimizing energy h MF*|H|* MFi for each *t/U* and *µ/U*. The energy minimization gives rise to a phase

Path integral representation

$$
\mathcal{Z} = \int \prod_i \mathcal{D}b_i^*(\tau)\mathcal{D}b_i(\tau) \exp \left\{ \int_0^\beta \left[ \sum_i b_i^*(\tau)\partial_\tau b_i(\tau) - H_{\text{BH}}(b^*(\tau), b(\tau)) \right] \right\}
$$

**i** (  $\frac{1}{2}$  ). Introducing another complex field  $\frac{1}{2}$ 

*,* (14)

where *<sup>H</sup>*BH(*b*⇤(⌧ )*, b*(⌧ )) is given by replacing *<sup>H</sup>*ˆBH with <sup>ˆ</sup>*b<sup>i</sup>* ! *<sup>b</sup>i*(⌧ ) and <sup>ˆ</sup>*<sup>b</sup>*

' to decouple the hopping term by Hubbard-Stratonovich term by Hubbard-Stratonovich transformation, we reach t<br>The hopping term by Hubbard-Stratonovich transformation, we reach the hopping term in the stratonovich term in

function is given by

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### Introducing an auxiliary field to coupling hopping <u>z</u> *z*  $\frac{1}{2}$ <u>uxili</u> Introducing an auxiliary field to coupling hopping  $\sigma$  (14)  $\mu$  (14)  $\sigma$ where *<sup>H</sup>*BH(*b*⇤(⌧ )*, b*(⌧ )) is given by replacing *<sup>H</sup>*ˆBH with <sup>ˆ</sup>*b<sup>i</sup>* ! *<sup>b</sup>i*(⌧ ) and <sup>ˆ</sup>*<sup>b</sup> <sup>i</sup>* ! *b*⇤ **i** (  $\frac{1}{2}$  ). Introducing another complex field  $\frac{1}{2}$  $\blacksquare$  to decouple the to decouple the hopping term by Hubbard-Stratonovich transformation term by Theorem and an animal transformation to reach the hopping term in the Hubbard-Stratonovich transformation, we reach the for

$$
\mathcal{Z} = \int \prod_{i} \mathcal{D}\varphi_i^* \mathcal{D}\varphi_i(\tau) e^{-S[\varphi^*, \varphi]}
$$

$$
S[\varphi^*, \varphi] = \int_0^\beta d\tau \sum_{ij} \varphi_i^* \frac{1}{t} \varphi_j - \sum_i \ln \int \mathcal{D}b^*(\tau) \mathcal{D}b(\tau) e^{-\int_0^\beta d\tau \mathcal{L}[b, \varphi_i]}
$$

$$
\mathcal{L} = -b^*(\tau) \partial_\tau b(\tau) - \mu |b(\tau)|^2 + \frac{U}{2} |b(\tau)|^2 (|b(\tau)|^2 - 1) - \varphi b^*(\tau) - \varphi^* b(\tau)
$$

*L* is a local on-site Lagrangian so we can drop the site index. Nearby the phase transition where *|*'*|* is small, *b*-field

*L* is a local on-site Lagrangian so we can drop the site index. Nearby the phase transition where *|*'*|* is small, *b*-field

can be integrated out and the action can be expanded in powers of the long-wave length limit, it becomes a second

<sup>2</sup> 1) '*b*⇤(⌧ ) '⇤*b*(⌧ ) (16)

<sup>4</sup> + *...* ⇤ (17)

*t*

function is given by

where *w* = 1*/*(2*t*) and

 $\overline{\phantom{a}}$ 

0

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$$

### $\frac{15}{1}$  $L_{\text{eff}}$  is a local on-site index. Nearby the site index.  $\mathcal{C}$  is small,  $\mathcal{C}$  i Integrating out boson field *Latermating suit becam dial* can be integrating out boson field in and the action can be expanded in powers of  $\mathbb{R}$

$$
S[\varphi^*, \varphi] = S[0] + \int_0^\beta d\tau \int d^3 \mathbf{r} \left[ u \varphi^* \partial_\tau \varphi + v |\partial_\tau \varphi|^2 + w |\nabla \varphi|^2 - a |\varphi|^2 + b |\varphi|^4 + \dots \right]
$$

*t*

function is given by

where *w* = 1*/*(2*t*) and

where *w* = 1*/*(2*t*) and

 $\overline{\phantom{a}}$ 

0

### **Local on-site Lagrangian so we can description** where *li* is small, *b*-field **Theory Description** can be integrated out and the action can be expanded in powers of the long-wave length limit, it becomes length  $\mathcal{U}$ can be integrated out and the action can be expanded in powers of  $\mathbb{R}^n$ ✓  $e$ <sup>*Id*</sup> Theory Descriptio <sup>2</sup>*m*⇤ *<sup>|</sup>*<sup>r</sup> *<sup>|</sup>*  $\overline{\phantom{a}}$ *Field Theory Descrip U*  $\dot{\mathbf{p}}$

*L* is a local on-site Lagrangian so we can drop the site index. Nearby the phase transition where *|*'*|* is small, *b*-field

<sup>2</sup>(*|b*(⌧ )*<sup>|</sup>*

*i*

*t*

<sup>2</sup> 1) '*b*⇤(⌧ ) '⇤*b*(⌧ ) (16)

<sup>2</sup> <sup>+</sup> *<sup>r</sup><sup>|</sup> <sup>|</sup>*

<sup>2</sup> *<sup>a</sup>|*'*<sup>|</sup>*

*, b*⇤(⌧ ) ! *<sup>b</sup>*⇤(⌧ )*ei*✓(⌧) (20)

 $\frac{1}{2}$ 

<sup>2</sup> 1) '*b*⇤(⌧ ) '⇤*b*(⌧ ) (16)

<sup>2</sup> *<sup>|</sup>b*(⌧ )*<sup>|</sup>*

*ij*

<sup>2</sup> +

iiiii

*L* = *b*⇤(⌧ )@⌧ *b*(⌧ ) *µ|b*(⌧ )*|*

za za zamiela<br>Za za za za zamiela<br>Za za za za zamiela

*L* =

Z

Z

$$
S[\varphi^*, \varphi] = S[0] + \int_0^{\beta} d\tau \int d^3 \mathbf{r} \left[ u\varphi^* \partial_{\tau} \varphi + v | \partial_{\tau} \varphi|^2 + w |\nabla \varphi|^2 - a |\varphi|^2 + b |\varphi|^4 + \dots \right]
$$
  
\n
$$
a = -\frac{1}{t} + \frac{n_0 + 1}{2n_0 U - \mu} + \frac{n_0}{\mu - 2U(n_0 - 1)}.
$$
  
\n
$$
\frac{\mu}{2U}
$$
  
\n
$$
a > 0
$$
  
\nSuperfluid  
\n3.0  
\n3.0  
\n
$$
a > 0
$$
  
\n
$$
a < 0
$$
  
\nMott insulator  
\n5.0  
\n
$$
t_c = \frac{(n_0 - \frac{\mu}{2U})(\frac{\mu}{2U} - (n_0 - 1))}{\frac{\mu}{2U} + 1}
$$

*<sup>b</sup>*(⌧ ) ! *<sup>b</sup>*(⌧ )*e<sup>i</sup>*✓(⌧)

## **Field Theory Description** can be integrated out and the action can be expanded in powers of the long-wave length limit, it becomes length  $\mathcal{U}$ *<sup>µ</sup>* <sup>2</sup>*U*(*n*<sup>0</sup> 1)*.* (18) superfluid. *a* = 0 determines the critical condition, i.e *t*c*/*(2*U*) as a function of *µ/*(2*U*), i. e.

+

<sup>2</sup>(*|b*(⌧ )*<sup>|</sup>*

*n*0

<sup>2</sup> 1) '*b*⇤(⌧ ) '⇤*b*(⌧ ) (16)

<sup>2</sup>*<sup>U</sup>* + 1 (19)

+ *u* = 0 (22)

(*<sup>µ</sup>* <sup>2</sup>*U*(*n*<sup>0</sup> 1))<sup>2</sup> *,* (24)

+ 2*v* = 0 (23)

+ *u* = 0 (22)

, *µ | i*@

<sup>2</sup> *<sup>|</sup>b*(⌧ )*<sup>|</sup>*

<sup>2</sup> +

*n*<sup>0</sup> + 1

$$
\left|S[\varphi^*,\varphi]=S[0]+\int_0^\beta d\tau\int d^3{\bf r}\left[u\varphi^*\partial_\tau\varphi+v|\partial_\tau\varphi|^2+w|\nabla\varphi|^2-a|\varphi|^2+b|\varphi|^4+\ldots\right]\right|
$$

### Gauge Symmetry Derivation There are di↵erent ways to deduce *u* and *v*. Here we introduce a method using"gauge symmetry". Note that the original Lagrangian Eq. 16 has a global symmetry as  $Gauge$

$$
b(\tau) \to b(\tau)e^{i\theta(\tau)}, \quad b^*(\tau) \to b^*(\tau)e^{-i\theta(\tau)}
$$
  

$$
\varphi(\tau) \to \varphi(\tau)e^{i\theta(\tau)}, \quad \varphi^*(\tau) \to b(\tau)e^{i\theta(\tau)}, \quad \mu \to \mu + i\partial_\tau\theta
$$

@*µ*



*L* = *b*⇤(⌧ )@⌧ *b*(⌧ ) *µ|b*(⌧ )*|*

*<sup>a</sup>* <sup>=</sup> <sup>1</sup>

*t* +

where *w* = 1*/*(2*t*) and

## **Field Theory Description** *<sup>µ</sup>* <sup>2</sup>*U*(*n*<sup>0</sup> 1)*.* (18) *<u>Poserintion</u>* <sup>2</sup>*<sup>U</sup>* + 1 (19)

+

<sup>2</sup>(*|b*(⌧ )*<sup>|</sup>*

iiiii

*n*0

*<sup>n</sup>*<sup>0</sup> *<sup>µ</sup>*

2*U*

*µµµµµµµ* 

<sup>2</sup> 1) '*b*⇤(⌧ ) '⇤*b*(⌧ ) (16)

<sup>2</sup>*<sup>U</sup>* (*n*<sup>0</sup> 1)

<sup>2</sup> *<sup>|</sup>b*(⌧ )*<sup>|</sup>*

*t*c

<sup>2</sup> +

*n*<sup>0</sup> + 1

*L* = *b*⇤(⌧ )@⌧ *b*(⌧ ) *µ|b*(⌧ )*|*

*<sup>a</sup>* <sup>=</sup> <sup>1</sup>

*t* +



### **Field Theory Description** *<sup>µ</sup>* <sup>2</sup>*U*(*n*<sup>0</sup> 1)*.* (18) *<u>Pescription</u>* <sup>2</sup>*<sup>U</sup>* + 1 (19) 2*U*  $\epsilon$  *<sup>n</sup>*<sup>0</sup> *<sup>µ</sup>* 2*U µ*

+

<sup>2</sup>(*|b*(⌧ )*<sup>|</sup>*

iiiii

superfluid. *a* = 0 determines the critical condition, i.e *t*c*/*(2*U*) as a function of *µ/*(2*U*), i. e.

*n*0

*<sup>n</sup>*<sup>0</sup> *<sup>µ</sup>*

2*U*

*µµµµµµµ* 

<sup>2</sup> 1) '*b*⇤(⌧ ) '⇤*b*(⌧ ) (16)

<sup>2</sup>*<sup>U</sup>* (*n*<sup>0</sup> 1)

<sup>2</sup> *<sup>|</sup>b*(⌧ )*<sup>|</sup>*

*t*c

<sup>2</sup> +

*n*<sup>0</sup> + 1

*L* = *b*⇤(⌧ )@⌧ *b*(⌧ ) *µ|b*(⌧ )*|*

*<sup>a</sup>* <sup>=</sup> <sup>1</sup>

*t* +



 $\overline{\phantom{a}}$  Thursday, December 25, Thursday, December 25, 14

### **Field Theory Description** *<sup>µ</sup>* <sup>2</sup>*U*(*n*<sup>0</sup> 1)*.* (18) *<u>Pescription</u>* <sup>2</sup>*<sup>U</sup>* + 1 (19) 2*U*  $\epsilon$  *<sup>n</sup>*<sup>0</sup> *<sup>µ</sup>* 2*U µ*

+

<sup>2</sup>(*|b*(⌧ )*<sup>|</sup>*

iiiii

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*n*0

*<sup>n</sup>*<sup>0</sup> *<sup>µ</sup>*

2*U*

*µµµµµµµ* 

<sup>2</sup> 1) '*b*⇤(⌧ ) '⇤*b*(⌧ ) (16)

<sup>2</sup>*<sup>U</sup>* (*n*<sup>0</sup> 1)

<sup>2</sup> *<sup>|</sup>b*(⌧ )*<sup>|</sup>*

*t*c

<sup>2</sup> +

*n*<sup>0</sup> + 1

*L* = *b*⇤(⌧ )@⌧ *b*(⌧ ) *µ|b*(⌧ )*|*

*<sup>a</sup>* <sup>=</sup> <sup>1</sup>

*t* +



### *a >* 0 *a <* 0 (21) **Higgs mode** *S* =  $\overline{\mathbf{Q}}$ *dd* r mc *d*⌧

*u* = *u*<sup>0</sup> + *iu*<sup>00</sup> (20)

$$
\mathcal{S}_{\phi} = \int d^{d} \mathbf{r} \int_{0}^{\beta} d\tau \left\{ (\partial_{\tau} \varphi)^{2} + c^{2} (\nabla \varphi)^{2} - \alpha |\varphi|^{2} + b |\phi|^{4} \right\}
$$

*d*⌧

## **Mott insulator Superfluid** Mott insulator <sup>2</sup> ↵*|*'*<sup>|</sup>*

# Superfluid

$$
\omega = \sqrt{ck^2 + |\alpha|}
$$



(r')

<sup>2</sup> ↵*|*'*<sup>|</sup>*

<sup>2</sup> <sup>+</sup> *<sup>b</sup>|<sup>|</sup>*

(22)

4





*S* =

# **Higgs Mode Detection**

Fig. 2a. Approaching the critical point, the spectral onset becomes sharper, and the width normalized to the centre frequency n<sup>0</sup> remains constant (Supplementary Fig. 3). The constancy of this ratio indicates that the width of the spectral onset scales with the distance to the

**We observe since the Motor in th** 

critical point in the same way as the gap frequency.



Figure 2 and Higgs mode. And the Higgs model gap values has been added. As a set of the fitted gap values of the fitted ga

RESEARCH LETTER

# **Generally, a condensed matter and cold atom system will not have Lorentz invariance, what is the fate to Higgs mode if without Lorentz invariance ?**

# **BEC-BCS Crossover**

# **Increasing attractive interaction**









### **Time-dependent Ginburg-Landau Theory** 1 We expand the action up to the second order time derivative, the we have a TDGL theory as  $Cinkmax$  I and an Theory possible. Including both the spatial and time derivatives (after Wick rotation) and retaining the parameter up to

is the Gor'kov Green function.





Thursday, December 25, 14

# **(A) Ignoring damping term**

◆

$$
F[\bar{\Delta}, \Delta] = \int dt d^3x \{\bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta\}
$$
  

$$
u = u' + i u''
$$

◆

$$
F[\bar{\Delta}, \Delta] = \int dt d^3 \mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2} \bar{\Delta} \bar{\Delta} \Delta \Delta \}
$$

$$
u = u' \boxed{\mathbf{a} \mathbf{b} \mathbf{b} \mathbf{b} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{d} \mathbf{d} \mathbf{d} \mathbf{d} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf
$$

in general and . The contract of the contract o<br>The contract of the contract o

◆

$$
F[\bar{\Delta}, \Delta] = \int dt d^3 \mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2} \bar{\Delta} \bar{\Delta} \Delta \Delta \}
$$

$$
u = u' \boxed{\mathbf{a} \mathbf{b} \mathbf{b} \mathbf{b} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{d} \mathbf{d} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf
$$

$$
\omega^2 = \frac{1}{v'}(\frac{k^2}{2m^*} + r) + \frac{u'^2}{2v'^2} \pm \sqrt{\frac{u'^4}{4v'^4} + \frac{u'^2}{v'^3}(\frac{k^2}{2m^*} + r) + \frac{r^2}{v'^2}}.
$$

◆

$$
F[\bar{\Delta}, \Delta] = \int dt d^3 \mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2} \bar{\Delta} \bar{\Delta} \Delta \Delta \}
$$

$$
u = u' \boxed{\mathbf{a} \mathbf{b} \mathbf{b} \mathbf{b} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{d} \mathbf{d} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf
$$

$$
\omega^{2} = \frac{1}{v'}(\frac{k^{2}}{2m^{*}} + r) + \frac{u'^{2}}{2v'^{2}} \pm \sqrt{\frac{u'^{4}}{4v'^{4}} + \frac{u'^{2}}{v'^{3}}(\frac{k^{2}}{2m^{*}} + r) + \frac{r^{2}}{v'^{2}}}.
$$
  

$$
v' \gg u'
$$

◆

$$
F[\bar{\Delta}, \Delta] = \int dt d^3 \mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2} \bar{\Delta} \bar{\Delta} \Delta \Delta \}
$$

$$
u = u' \boxed{\mathbf{a} \mathbf{b} \mathbf{b} \mathbf{b} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{d} \mathbf{d} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf{c} \mathbf{c} \mathbf{d} \mathbf
$$

$$
\omega^2 = \frac{1}{v'}(\frac{k^2}{2m^*} + r) + \boxed{\phantom{0}} + \sqrt{\phantom{0}} + \frac{r^2}{v'^2}.
$$

◆

$$
F[\bar{\Delta}, \Delta] = \int dt d^3 \mathbf{x} \{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2} \bar{\Delta} \bar{\Delta} \Delta \Delta \}
$$

$$
\omega^{2} = \frac{1}{v'}(\frac{k^{2}}{2m^{*}} + r) + \left(\frac{k^{2}}{2m^{*}} + \frac{1}{v'^{2}}\right) + \frac{r^{2}}{v'^{2}}.
$$

$$
\omega = k/\sqrt{2m^*v'}
$$

$$
\omega = \sqrt{(k^2/2m^* + 2r)/v'}
$$

### **Ginburg-Landau Theory**  $\overline{\mathbf{I}}$ We expand the expanding the second theory and theory and the second order time derivative, the west of the west of the second order time derivative, the second order time derivative, the second of the west second order to

◆

Ginburg-Landau Theory  
\n
$$
F[\bar{\Delta}, \Delta] = \int dt d^3x \left\{ \bar{\Delta}(-iu\partial_t + v\partial_t^2 - \frac{\nabla^2}{2m^*} - r)\Delta + \frac{b}{2} \bar{\Delta} \bar{\Delta} \Delta \Delta \right\}
$$
\n
$$
u = u'
$$
\n
$$
\omega^2 = \frac{1}{v'}(\frac{k^2}{2m^*} + r) + \frac{u'^2}{2v'^2} \pm \sqrt{\frac{u'^4}{4v'^4} + \frac{u'^2}{v'^3}(\frac{k^2}{2m^*} + r) + \frac{r^2}{v'^2}}.
$$
\n
$$
v' \gg u'
$$
\n
$$
\omega = k/\sqrt{2m^*v'}
$$
\n
$$
\omega = \sqrt{2r/(2mu^2)}k
$$
\n
$$
\omega = \sqrt{(k^2/2m^* + 2r)/v'}
$$
\n
$$
\omega = \sqrt{\frac{k^2}{v'm^*} + \frac{2r}{v'} + \frac{u'^2}{v'^2}}
$$

3

**Spectral Weight Transfer**

 $\Delta \rightarrow \sqrt{r/b} + \delta_a + i \delta_p$ 

# **Spectral Weight Transfer**

$$
\Delta \rightarrow \sqrt{r/b} + \delta_a + i \delta_p
$$

 $<\delta_a^*(\omega,{\bf k})\delta_a(\omega,{\bf k})>=\frac{-v'\omega^2+k^2/2m^*}{-u'^2\omega^2+(-v'\omega^2+k^2/2m^*)(-v'\omega^2+k^2/2m^*+2r)}$ 

# **Spectral Weight Transfer**

$$
\Delta \rightarrow \sqrt{r/b} + \delta_a + i \delta_p
$$

 $<\delta_a^*(\omega,{\bf k})\delta_a(\omega,{\bf k})>=\frac{-v'\omega^2+k^2/2m^*}{-u'^2\omega^2+(-v'\omega^2+k^2/2m^*)(-v'\omega^2+k^2/2m^*+2r)}$ 





BCS **Increasing attractive interaction** BEC

**(B) Including damping term**

## <u>*<u><sup>2</sup></u></sup></u>*  $\overline{\mathbf{I}}$ **In order to Cinburg-Landau Theory** In order to calculate the spectral functions we have to introduce a Langevin force to tack  $\mathbb{N}$

The Langevin force is a second control of the Langevin force in the Langevin force in the Langevin force in the

◆

$$
F = \int dt d^{3}x \left\{ \bar{\Delta}(-iu\partial_{t} + v\partial_{t}^{2} - \frac{\nabla^{2}}{2m^{*}} - r)\Delta + \frac{b}{2}\bar{\Delta}\bar{\Delta}\Delta\Delta + \bar{\Delta}\xi + \Delta\xi^{*} \right\}
$$

$$
u = u' + i u''
$$

## **Langevin force** The Langevin force is a white noise is a white noise is a white noise of  $\alpha$

$$
\langle \xi(t', \mathbf{x}') \xi(t, \mathbf{x}) \rangle = \langle \xi^*(t', \mathbf{x}') \xi^*(t, \mathbf{x}) \rangle = 0.
$$

$$
\langle \xi^*(t',\mathbf{x}')\xi(t,\mathbf{x}) \rangle = N\delta(t-t')\delta(\mathbf{x}-\mathbf{x}')
$$

 $\mathcal{T}_\text{max}=\frac{1}{2}$  and the fluctuation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-d

The parameter N can be fixed by the parameter  $\mathcal{L}_{\mathcal{A}}$  can be fixed by the fluctuation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipation-dissipati

*d*⌧*d*<sup>3</sup>

TDGL with damping terms.

r

# 1 **Ginburg-Landau Theory** In order to calculate the spectral functions we have to introduce a Langevin force to tack  $\mathbb{N}$

◆



Thursday, December 25, 14  $\frac{m}{2}$  is well defined.  $\frac{m}{2}$  is  $\frac{m}{2}$ 

*<sup>|</sup>* (*u*!)<sup>2</sup> + (*v*!<sup>2</sup> <sup>+</sup> *<sup>k</sup>*<sup>2</sup>

*<sup>|</sup>* (*u*!)<sup>2</sup> + (*v*!<sup>2</sup> <sup>+</sup> *<sup>k</sup>*<sup>2</sup>

<sup>2</sup> *· <sup>|</sup> <sup>v</sup>*!<sup>2</sup> <sup>+</sup> *<sup>k</sup>*<sup>2</sup>



bridization e↵ect of damping term *u*<sup>00</sup> becomes weaker.

*Thursday, December 25, 14* 

ory can be calculated as *b* ' 7<sup>2</sup>⇣(3)⌫(✏*<sup>F</sup>* )*/*(8⇡<sup>2</sup>) and



Thursday, December 25, 14

<sup>00</sup>*/v*<sup>0</sup>

ory can be calculated as *b* ' 7<sup>2</sup>⇣(3)⌫(✏*<sup>F</sup>* )*/*(8⇡<sup>2</sup>) and

# **(C) Superconductor: including coupling to external electromagnetic field**

## <u>*i*</u> <u>v</u><sup>1111</sup>*w* ii<sub></sub> *g*  $\frac{1}{2}$   $\frac{1}{2}$ 1 **Ginburg-Landau Theory** We can write down a TDGL with Coulomb interaction and the coulomb interaction as  $\sim$

The spectral functions of the spectral functions of the Higgs and phase mode for a Coulomb gas mode for a Coulomb gas

◆

*L* =  $F = \int dt d^3\mathbf{x} \Big\{ -\frac{1}{8\pi} \phi \nabla^2 \phi + \bar{\Delta} \Big( -iu(\partial_t - 2e\phi) + v(\partial_t - 2e\phi)^2 - \frac{\nabla^2}{2m^*} - r \Big) \Delta + \frac{b}{2} \bar{\Delta} \bar{\Delta} \Delta \Delta \Big\}$ <sup>2</sup> <sup>+</sup> *<sup>r</sup><sup>|</sup> <sup>|</sup>* <sup>2</sup> <sup>+</sup> ↵*<sup>|</sup> <sup>|</sup>*  $u = u' + i u''$ integrating out the electric field of  $\phi$ Symmetry breaking

*d*⌧*d*<sup>3</sup>

r

### $\frac{2}{\sqrt{2}}$ 1 **Ginburg-Landau Theory** We can write down a TDGL with Coulomb interaction and the coulomb interaction as  $\sim$ **In Fig. 2 we present the spectral functions of the spectral funct**

The spectral functions of the spectral functions of the Higgs and phase mode for a Coulomb gas mode for a Coulomb gas

◆



2*m*⇤*v*<sup>0</sup>

them separately:

# **Conclusion:**

**1. In the BCS regime, Anderson-Higgs mechanism plays an important role in measuring a well-defined Higgs mode** 

**2. From BCS to BEC, Higgs mode is pushed to high energy and meanwhile, the spectral weight is transferred to Bogoliubov mode.** 





刘波扬 (清华高研院)



# **Thank you very much for your attention**