Electroweak precision tests at future e+e- colliders

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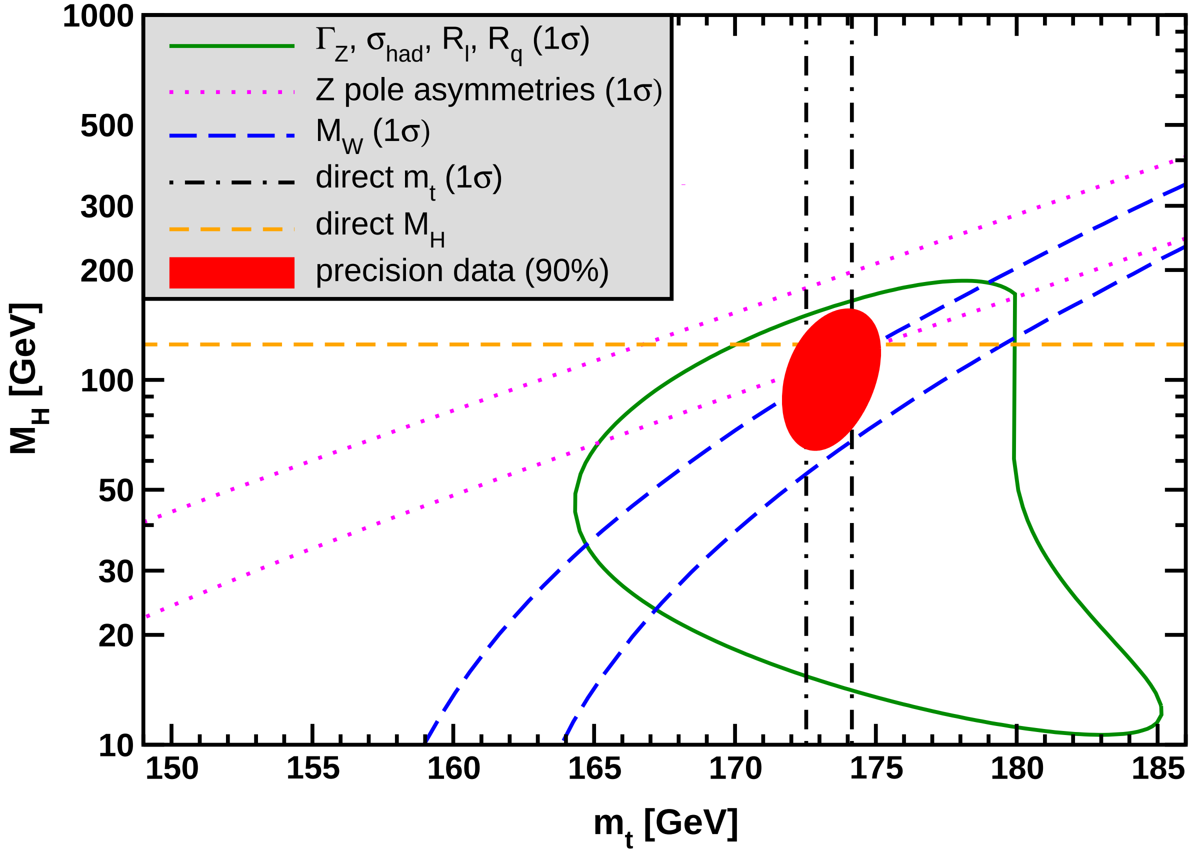
1. Overview of electroweak precision tests
2. Constraints on new physics
3. Current status of SM loop results
4. Future projections
5. Theory challenges

Current status of electroweak precision tests

Standard Model after Higgs discovery:

- Good agreement between measured mass and indirect prediction
- Very good agreement over large number of observables

Erler '16



Direct measurements:

$M_{\mathrm{H}} = 125.09 \pm 0.24$ GeV

$m_{\mathrm{t}} = 173.34 \pm 0.81$ GeV

Indirect prediction:

$M_{\mathrm{H}} = 126.1 \pm 1.9$ GeV (with LHC BRs)

$M_{\mathrm{H}} = 96^{+22}_{-19}$ GeV (w/o LHC data)

$m_{\mathrm{t}} = 176.7 \pm 2.1$ GeV

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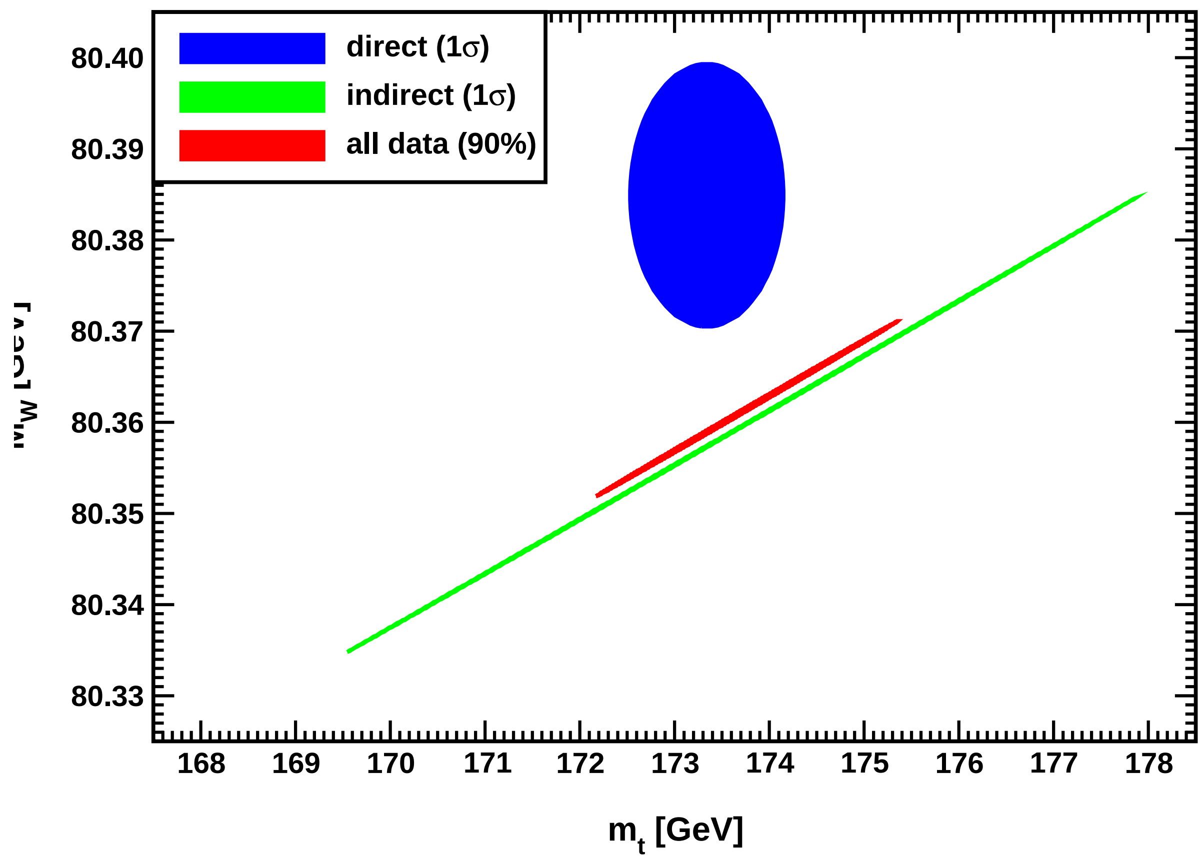


Direct measurements:

$M_W = 80385 \pm 15$ MeV

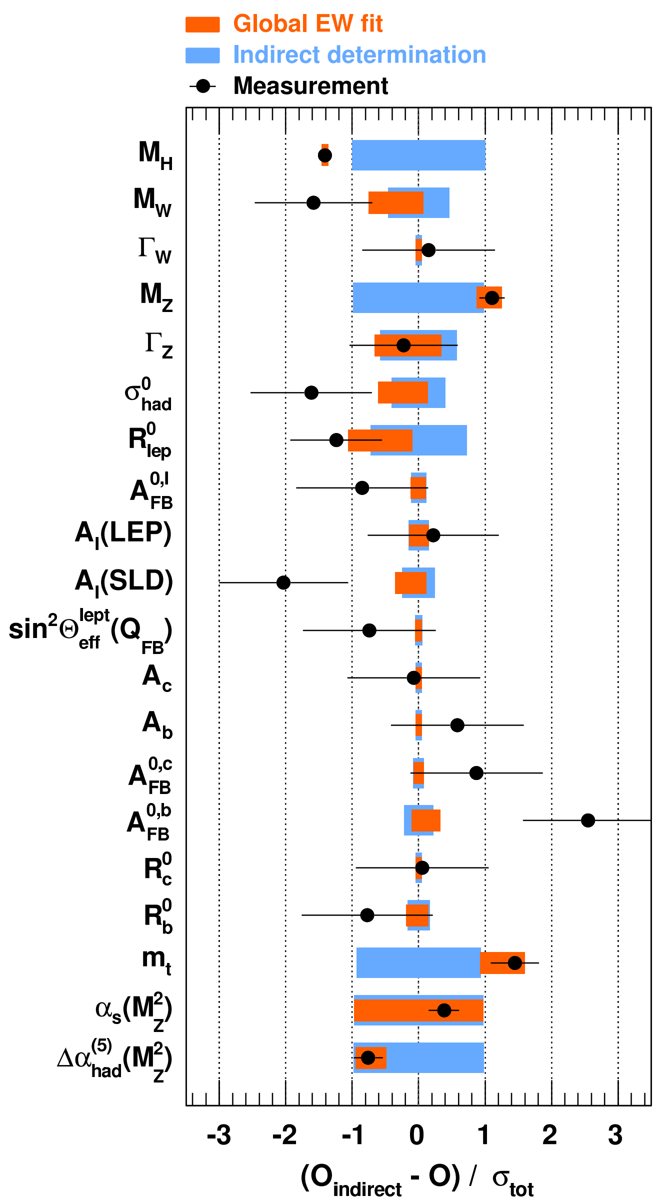
$m_t = 173.34 \pm 0.81$ GeV

Indirect prediction:

$M_W = 80357 \pm 6$ MeV

$m_t = 176.7 \pm 2.1$ GeV

Current status of electroweak precision tests



Surprisingly good agreement:
$\chi^2/\mathrm{d.o.f.} = 18.1/14$ ($p = 20\%$)

Most quantities measured with 1%–0.1% precision

GFitter coll. ’14

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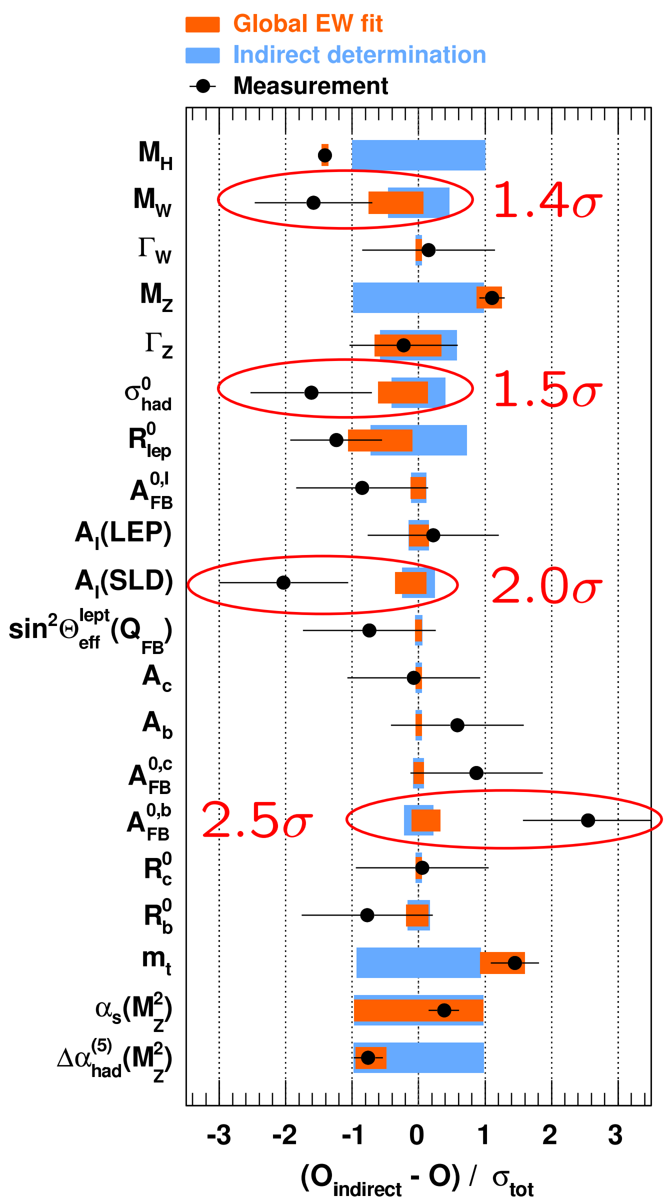

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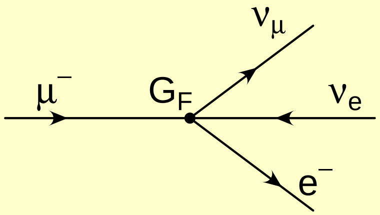
A few interesting deviations:

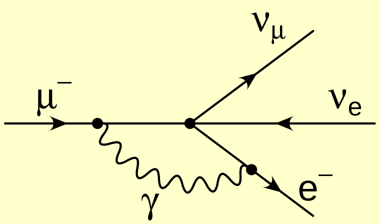
M_W	$(\sim 1.4\sigma)$
σ^0_{had}	$(\sim 1.5\sigma)$
A_ℓ(SLD)	$(\sim 2\sigma)$
A^b_{FB}	$(\sim 2.5\sigma)$
$(g_\mu - 2)$	$(\sim 3\sigma)$

GFitter coll. '14

W mass

μ decay in Fermi Model



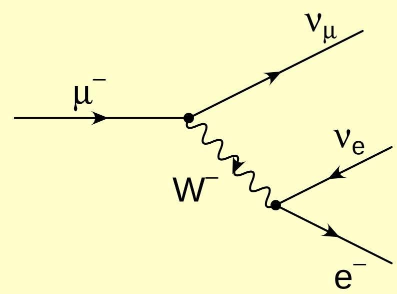


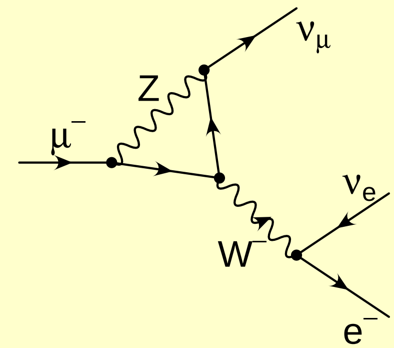
← QED corr. (2-loop)

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\, F\left(\frac{m_e^2}{m_\mu^2}\right)(1+\Delta q)$$

Ritbergen, Stuart ’98
Pak, Czarnecki ’08

μ decay in Standard Model





$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1+\Delta r)$$

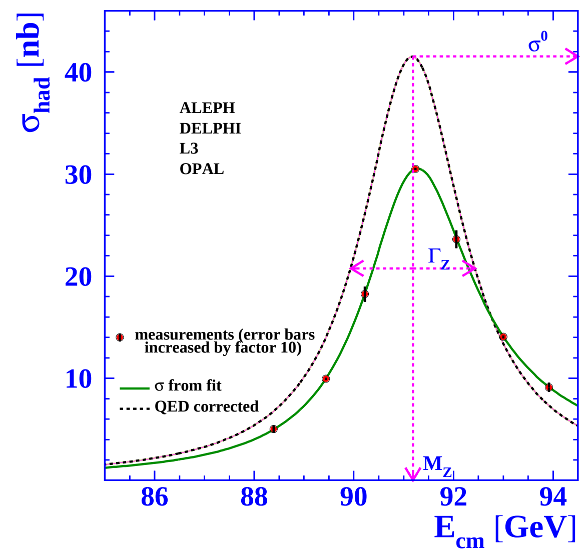
electroweak corrections

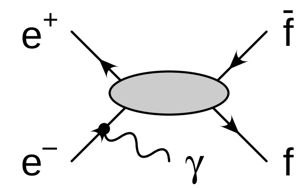
- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin ’85
Berends, Burgers, v. Neerven ’88
Kniehl, Krawczyk, Kühn, Stuart ’88
Beenakker, Berends, v. Neerven ’89
Skrzypek ’92
Montagna, Nicrosini, Piccinini ’97

LEP EWWG ’05





Z-pole observables

- Deconvolution of initial-state QED radiation:

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- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_{\text{Z}} + \sigma_\gamma + \sigma_{\gamma\text{Z}} + \sigma_{\text{box}}$$

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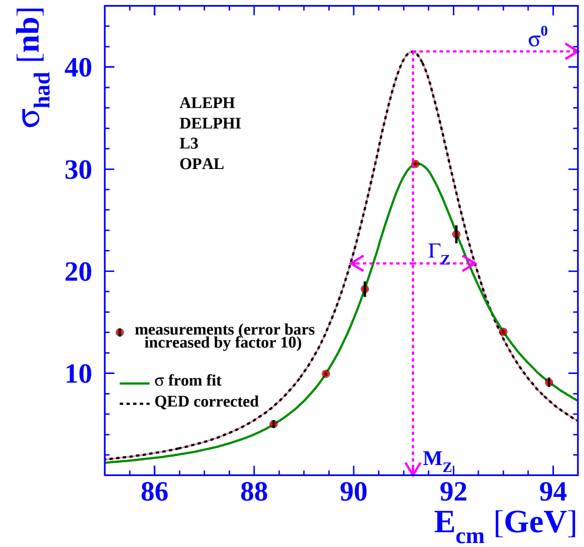

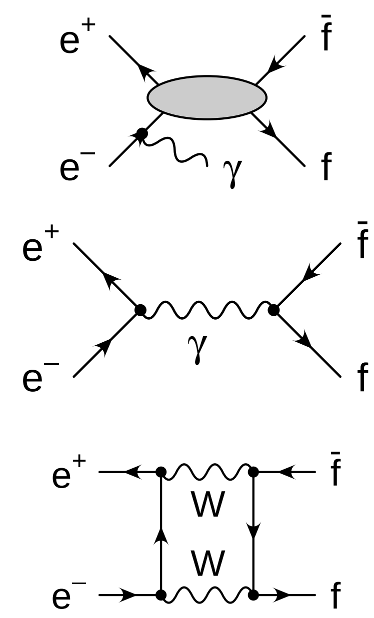

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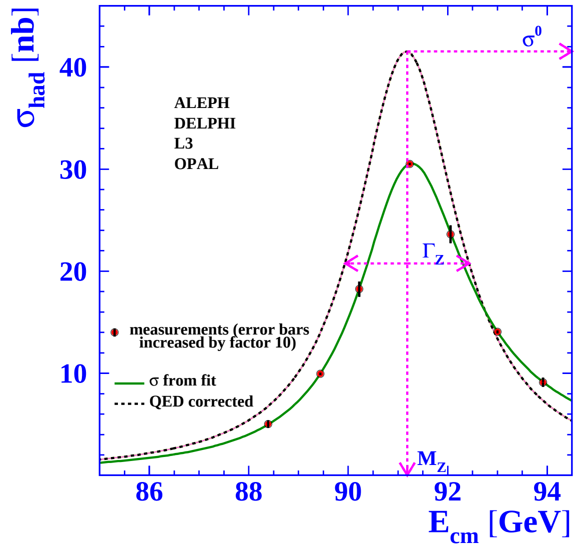
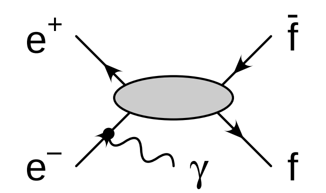
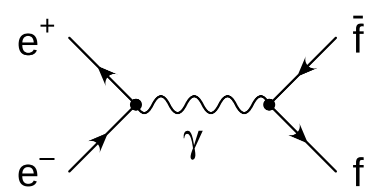
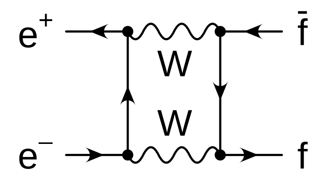
$$\sigma_{\text{Z}} = \frac{R}{(s-\overline{M}_{\text{Z}}^2)^2 + \overline{M}_{\text{Z}}^2\overline{\Gamma}_{\text{Z}}^2} + \sigma_{\text{non-res}}$$

LEP EWWG '05









Z-pole observables

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$$\sigma_{\rm hard} = \sigma_{\rm Z} + \sigma_\gamma + \sigma_{\gamma \rm Z} + \sigma_{\rm box}$$

- Z-pole contribution:

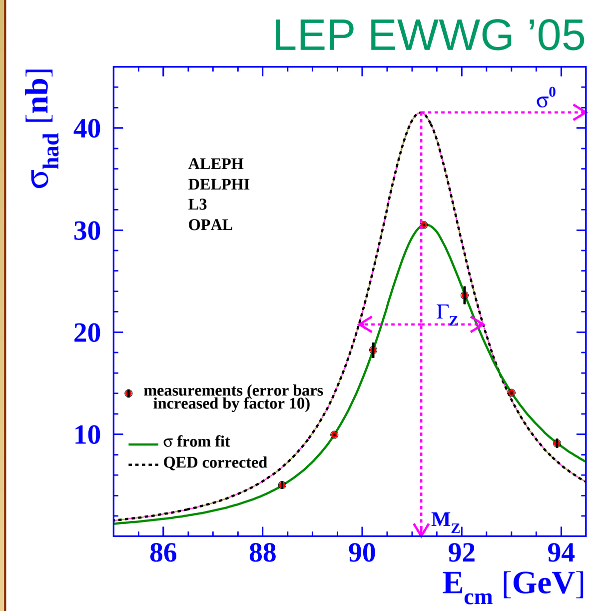
$$\sigma_{\rm Z} = \frac{R}{(s - \overline{M}_{\rm Z}^2)^2 + \overline{M}_{\rm Z}^2 \overline{\Gamma}_{\rm Z}^2} + \sigma_{\rm non-res}$$

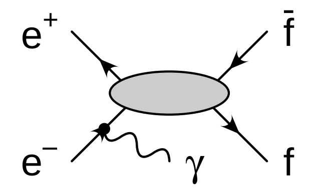
- In experimental analyses:

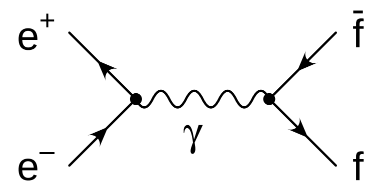
$$\sigma \sim \frac{1}{(s - M_{\rm Z}^2)^2 + s^2\Gamma_{\rm Z}^2/M_{\rm Z}^2}$$

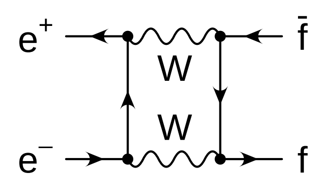
$$\overline{M}_{\rm Z} = M_{\rm Z} \Big/ \sqrt{1 + \Gamma_{\rm Z}^2/M_{\rm Z}^2} \approx M_{\rm Z} - 34 \text{ MeV}$$

$$\overline{\Gamma}_{\rm Z} = \Gamma_{\rm Z} \Big/ \sqrt{1 + \Gamma_{\rm Z}^2/M_{\rm Z}^2} \approx \Gamma_{\rm Z} - 0.9 \text{ MeV}$$









Z-pole observables

Z width:

$$\overline{\Gamma}_Z = \frac{1}{\overline{M}_Z} \mathrm{Im}\, \Sigma_Z(s_0).$$

Optical theorem:

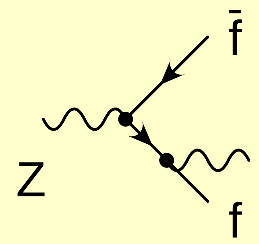
$$\overline{\Gamma}_Z = \sum_f \overline{\Gamma}_f, \qquad \overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi} \left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2 \right) \frac{1}{1 + \mathrm{Re}\, \Sigma_Z'} \right]_{s=\overline{M}_Z^2}$$

$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;
known to $\mathcal{O}(\alpha_s^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$

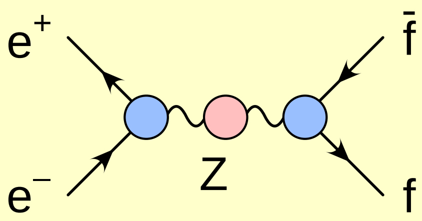
Kataev '92

Chetyrkin, Kühn, Kwiatkowski '96

Baikov, Chetyrkin, Kühn, Rittinger '12



g_V^f, g_A^f, Σ_Z': Electroweak corrections



Z-pole observables

Peak cross section:

$$\sigma^0_{\text{had}} = \sigma_Z(s = \overline{M}^2_Z)$$

(agrees with result from running-width BW with $s = M^2_Z$)

Explicit calculation:

$$\sigma^0_{\text{had}} = \frac{12\pi}{\overline{M}^2_Z} \sum_q \frac{\overline{\Gamma}_e \overline{\Gamma}_q}{\overline{\Gamma}^2_Z} (1 + \delta X)$$

Correction term first at NNLO:

Grassi, Kniehl, Sirlin '01
Freitas '13

$$\delta X_{(2)} = -(\text{Im}\,\Sigma'_{Z(1)})^2 - 2\overline{\Gamma}_Z \overline{M}_Z\, \text{Im}\,\Sigma''_{Z(1)}$$

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Branching ratios:

$R_q = \Gamma_q/\Gamma_{\text{had}}$ ($q = b, c$, probes heavy quark generations)

$R_\ell = \Gamma_{\text{had}}/\Gamma_\ell$ ($\ell = e, \mu, \tau$)

Z-pole observables

Effective weak mixing angle:

Z-pole asymmetries:

$$A_{\mathsf{FB}}^{f} \equiv \frac{\sigma(\theta < \frac{\pi}{2}) - \sigma(\theta > \frac{\pi}{2})}{\sigma(\theta < \frac{\pi}{2}) + \sigma(\theta > \frac{\pi}{2})} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

$$A_{\mathsf{LR}} \equiv \frac{\sigma(\mathcal{P}_e > 0) - \sigma(\mathcal{P}_e < 0)}{\sigma(\mathcal{P}_e > 0) + \sigma(\mathcal{P}_e < 0)} = \mathcal{A}_e$$

$$\mathcal{A}_f = 2\frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2} = \frac{1 - 4|Q_f|\sin^2\theta^f_{\mathsf{eff}}}{1 - 4|Q_f|\sin^2\theta^f_{\mathsf{eff}} + 8(|Q_f|\sin^2\theta^f_{\mathsf{eff}})^2}$$

Most precisely measured for $f = \ell$ (also $f = b, c$)

Current uncertainties

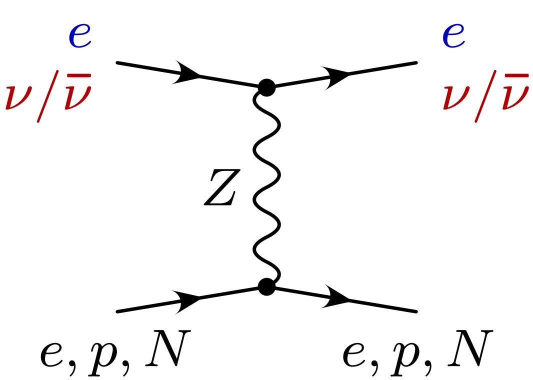
	Experiment	Theory error	Main source
M_{W}	80385 ± 15 MeV	4 MeV	α^3, $\alpha^2\alpha_{\text{s}}$
Γ_Z	2495.2 ± 2.3 MeV	0.5 MeV	α^2_{bos}, α^3, $\alpha^2\alpha_{\text{s}}$, $\alpha\alpha^2_{\text{s}}$
σ^0_{had}	41540 ± 37 pb	6 pb	α^2_{bos}, α^3, $\alpha^2\alpha_{\text{s}}$
R_b	0.21629 ± 0.00066	0.00015	α^2_{bos}, α^3, $\alpha^2\alpha_{\text{s}}$
$\sin^2\theta^{\ell}_{\text{eff}}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_{\text{s}}$

Low-energy observables

Test of running $\overline{\text{MS}}$ weak mixing angle $\sin^2\bar{\theta}(\mu)$

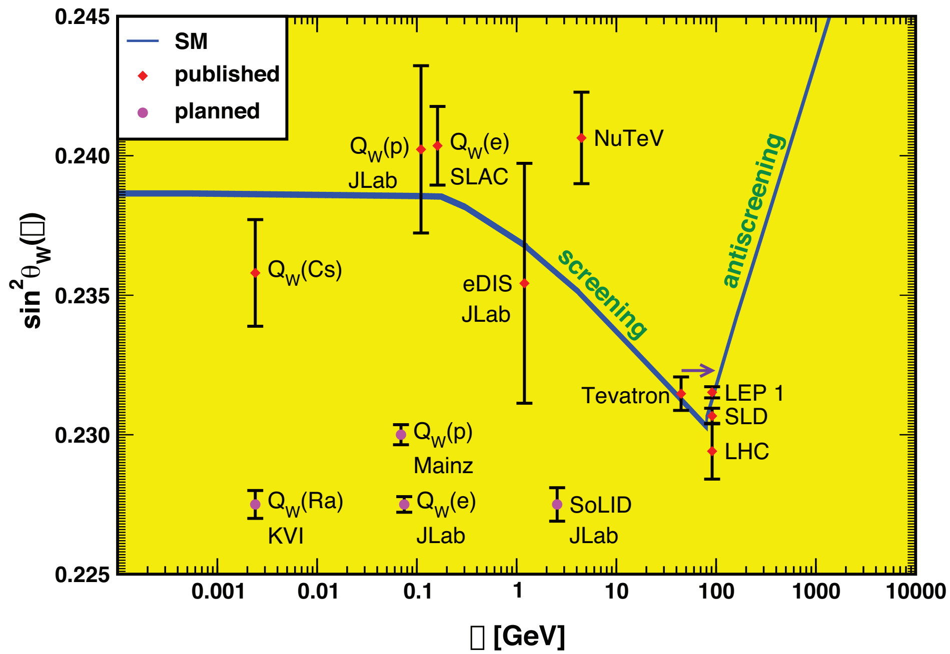
- Polarized ee, ep, ed scattering
  ($Q_W(e)$, $Q_W(p)$, eDIS)
  E158 '05; Qweak '13; JLab Hall A '13

- $\nu N/\bar{\nu} N$ scattering NuTeV '02



- Atomic parity violation
  ($Q_W(^{133}$Cs)) Wood et al. '97
  Guéna, Lintz, Bouchiat '05

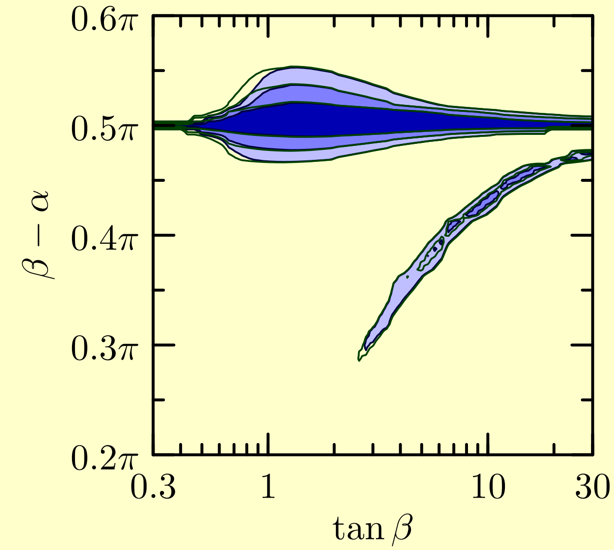
Erler '14



Constraints on new physics

Two-Higgs-Doublet Model:

Eberhardt, Nierste, Wiebusch '13



Constraints on couplings of SM-like Higgs

$$\left|\frac{g_{hVV}^{\text{THDM}}}{g_{hVV}^{\text{SM}}}\right| = \sin(\beta-\alpha),$$

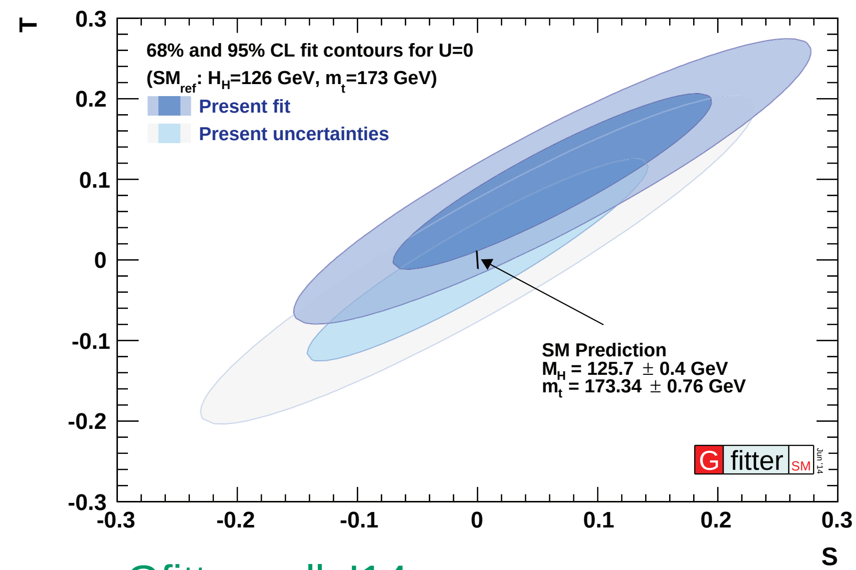
$$\left|\frac{g_{hff}^{\text{THDM}}}{g_{hff}^{\text{SM}}}\right| = \frac{\cos\alpha}{\sin\alpha} \quad \text{or} \quad \frac{\sin\alpha}{\cos\alpha}$$

Oblique parameters:

$$\alpha T = \frac{\Sigma_{WW}(0)}{M_W} - \frac{\Sigma_{ZZ}(0)}{M_Z}$$

$$\frac{\alpha}{4s^2c^2} S = \frac{\Sigma_{ZZ}(M_Z^2) - \Sigma_{ZZ}(0)}{M_Z}$$

$$+ \frac{s^2 - c^2}{sc} \frac{\Sigma_{Z\gamma}(M_Z^2)}{M_Z} - \frac{\Sigma_{\gamma\gamma}(M_Z^2)}{M_Z}$$



Gfitter coll. '14

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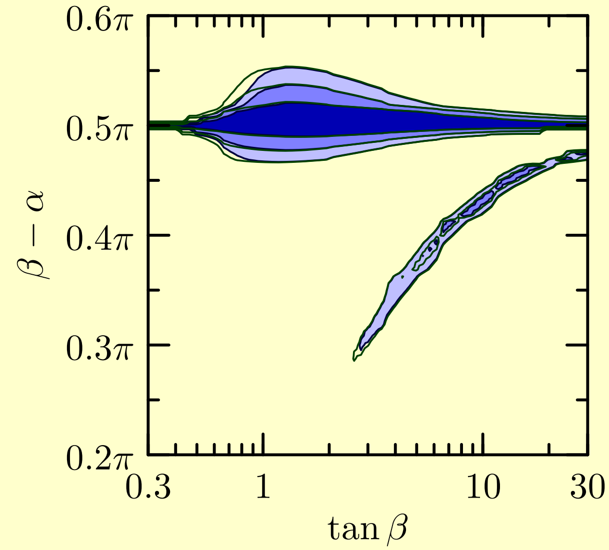

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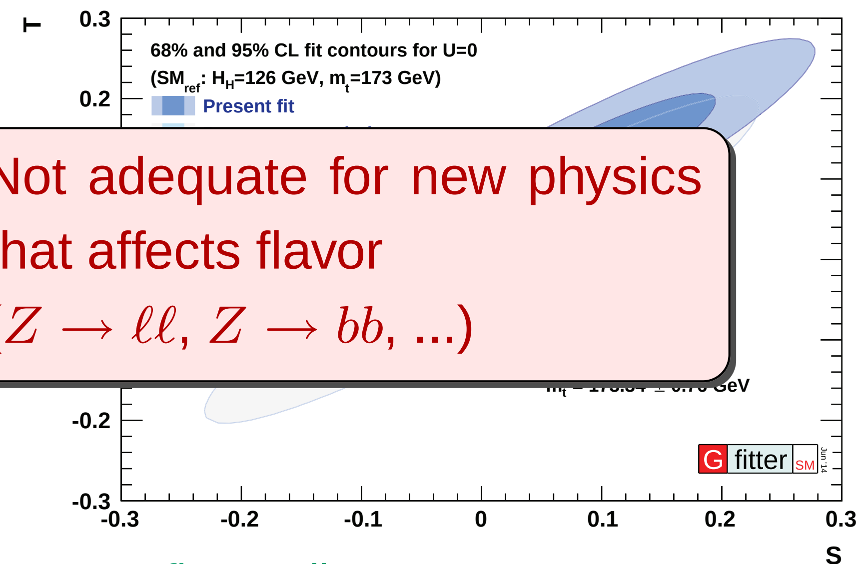

Not adequate for new physics that affects flavor ($Z \to \ell\ell$, $Z \to bb$, ...)

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Parametrization of new physics

More general setup: Use pseudo-observables

M_W, Γ_Z, σ^0_had, R_b, R_ℓ, A_ℓ, A_b, A_c $(\ell = e, \mu, \tau)$ $\longrightarrow$ 12 quantities

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Effective field theory: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \qquad (\Lambda \gg M_{\mathrm{Z}})$

$$\mathcal{O}_{\phi 1} = (D_\mu \Phi)^\dagger \Phi\, \Phi^\dagger (D^\mu \Phi) \qquad \alpha \Delta T = -\frac{v^2}{2}\frac{c_{\phi 1}}{\Lambda^2}$$

$$\mathcal{O}_{\mathrm{BW}} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi \qquad \alpha \Delta S = -e^2 v^2 \frac{c_{\mathrm{BW}}}{\Lambda^2}$$

$$O^{(3)e}_{\mathrm{LL}} = (\bar{L}^e_{\mathrm{L}} \sigma^a \gamma_\mu L^e_{\mathrm{L}})(\bar{L}^e_{\mathrm{L}} \sigma^a \gamma^\mu L^e_{\mathrm{L}}) \qquad \Delta G_F = -\sqrt{2}\frac{c^{(3)e}_{\mathrm{LL}}}{\Lambda^2}$$

$$O^f_{\mathrm{R}} = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_{\mathrm{R}} \gamma^\mu f_{\mathrm{R}}) \qquad f = e, \mu\, \tau, b, lq$$

$$O^F_{\mathrm{L}} = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_{\mathrm{L}} \gamma^\mu F_{\mathrm{L}}) \qquad F = \begin{pmatrix}\nu_e \\ e\end{pmatrix}, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}, \begin{pmatrix}u, c \\ d, s\end{pmatrix}, \begin{pmatrix}t \\ b\end{pmatrix}$$

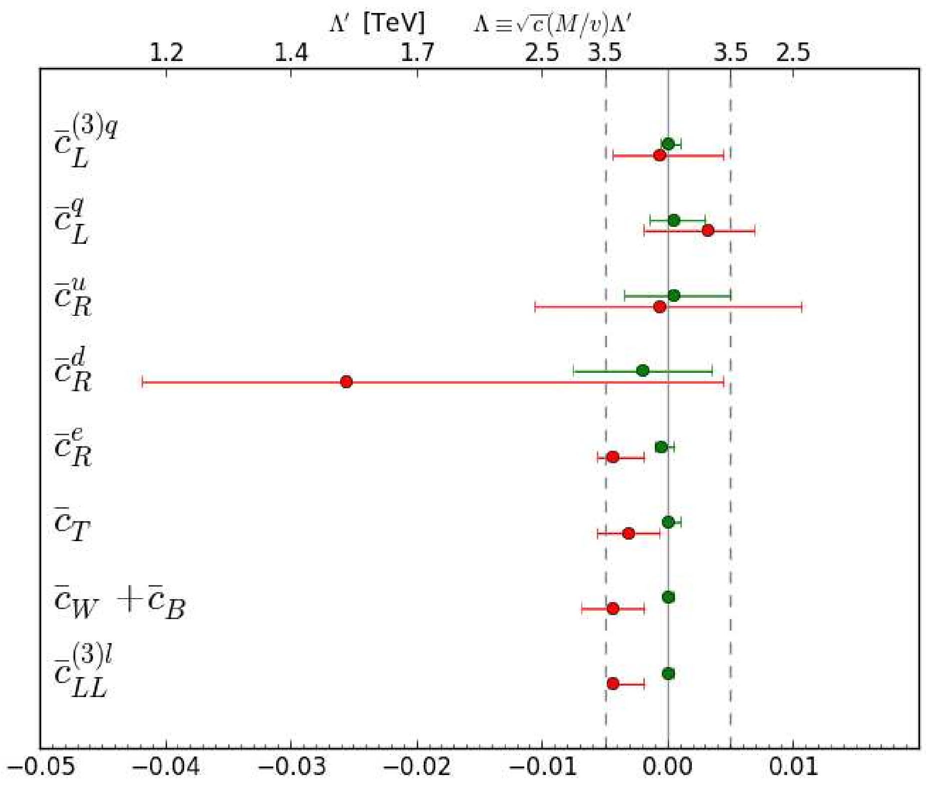
$$O^{(3)F}_{\mathrm{L}} = i(\Phi^\dagger \overleftrightarrow{D}^a_\mu \Phi)(\bar{F}_{\mathrm{L}} \sigma_a \gamma^\mu F_{\mathrm{L}})$$

More operators than EWPOs

$\longrightarrow$ Some can be constrained by $W \to \ell\nu$, had., $e^+e^- \to W^+W^-$

Current constraints on some dim-6 operators

Assuming flavor universality:

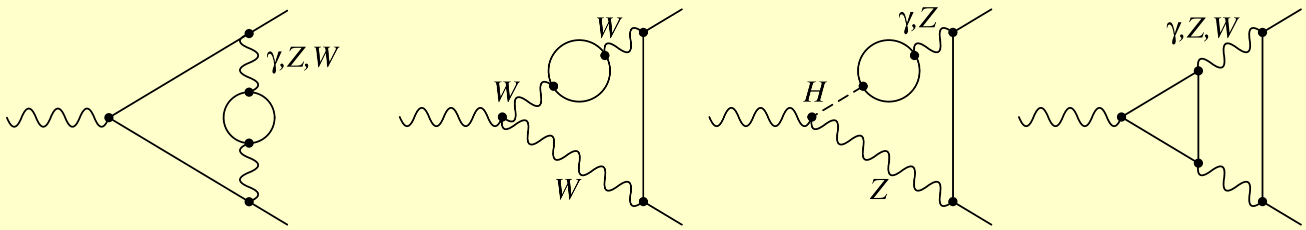


Significant correlation/ degeneracy between different operators

Pomaral, Riva ’13
Ellis, Sanz, You ’14

Current status of SM loop results

Known corrections to Δr, $\sin^2\theta^f_{\rm eff}$, g_{Vf}, g_{Af}:



- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\rm eff}$) Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02; Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- "Fermionic" NNLO corrections (g_{Vf}, g_{Af}) Czarnecki, Kühn '96
  Harlander, Seidensticker, Steinhauser '98
  Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_s^3)$ Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05; Chetyrkin et al. '06
  Boughezal, Czakon '06

  $(\alpha_t \equiv \frac{y_t^2}{4\pi})$

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σ^0_{had}	41540 ± 37 pb	6 pb	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$R_b \equiv \Gamma^b_{\mathrm{Z}}/\Gamma^{\mathrm{had}}_{\mathrm{Z}}$	0.21629 ± 0.00066	0.00015	α^2_{bos}, α^3, $\alpha^2\alpha_{\mathrm{s}}$
$\sin^2\theta^{\ell}_{\mathrm{eff}}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_{\mathrm{s}}$

Methods for theory error estimates:

- Parametric factors, $i.e.$ factors of α, N_c, N_f, ...
- Geometric progression, $e.g.$ $\frac{\mathcal{O}(\alpha^3)}{\mathcal{O}(\alpha^2)} \sim \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}$
- Renormalization scale dependence (often underestimates error)
- Renormalization scheme dependence (may underestimate error)

Renormalization scheme dependence

Use of $\overline{\text{MS}}$ renormalization for m_t reduces h.o. QCD corrections of $\mathcal{O}(\alpha_\text{t}\alpha_\text{s}^n)$:

loops $(n+1)$	$\Delta\rho^{\overline{\text{MS}}}_{(n)} \Big/ \left(\frac{3G_F\overline{m}_\text{t}^2}{8\sqrt{2}\pi^2}\right)$	$\Delta\rho^{\text{OS}}_{(n)} \Big/ \left(\frac{3G_F m_\text{t}^2}{8\sqrt{2}\pi^2}\right)$	
2	$-0.193\left(\frac{\alpha_\text{s}}{\pi}\right)$	$-3.970\left(\frac{\alpha_\text{s}}{\pi}\right)$	Djouadi, Verzegnassi '87 Kniehl '90
3	$-2.860\left(\frac{\alpha_\text{s}}{\pi}\right)^2$	$-14.59\left(\frac{\alpha_\text{s}}{\pi}\right)^2$	Avdeev, Fleischer, et al. '94 Chetyrkin, Kühn, Steinhauser '95
4	$-1.680\left(\frac{\alpha_\text{s}}{\pi}\right)^3$	$-93.15\left(\frac{\alpha_\text{s}}{\pi}\right)^3$	Schröder, Steinhauser '05 Chetyrkin, Faisst, Kühn, et al. '06 Boughezal, Czakon '06

No clear pattern of this kind known for $\mathcal{O}(\alpha^n)$

$\longrightarrow$ Only few results available that allow direct comparison

e.g. Faisst, Kühn, Seidensticker, Veretin '03

Parametrization: Resummation

Parametrization of perturbation series: $\boldsymbol{\alpha}$ **vs.** $\boldsymbol{G_F}$**?**

G_F can resum some leading one-loop terms

$$\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_\mathsf{Z})} \approx 0.059 \qquad\qquad \Delta\rho = \frac{3\alpha}{16\pi s^2 c^2}\frac{m_\mathsf{t}^2}{M_\mathsf{Z}^2}$$

But: Strong cancellations between $\Delta\alpha$ and $\Delta\rho$ terms beyond one-loop:

$$\begin{aligned}\Delta r^{(3)}_{\mathsf{res}} &= (\Delta\alpha)^3 \quad -3(\Delta\alpha)^2\left(\tfrac{c^2}{s^2}\Delta\rho\right) \quad +6(\Delta\alpha)\left(\tfrac{c^2}{s^2}\Delta\rho\right)^2 \quad -5\left(\tfrac{c^2}{s^2}\Delta\rho\right)^3 \\ &\approx (2.05 \quad -3.40 \quad +3.74 \quad -1.72) \times 10^{-4} \\ &= 0.68 \times 10^{-4}\end{aligned}$$

$\longrightarrow$ Not *the* numerically leading contribution anymore

Future projections

ILC: High-energy e^+e^- linear collider, running at $\sqrt{s} \approx M_Z$ with 30 fb^{-1}

CEPC: Circular e^+e^- collider, running at $\sqrt{s} \approx M_Z$ with 2×150 fb^{-1}

FCC-ee: Circular e^+e^- collider, running at $\sqrt{s} \approx M_Z$ with 4×3000 fb^{-1}

	Current exp.	ILC	CEPC	FCC-ee	Current perturb.
M_W [MeV]	15	3–4	3	1	4
Γ_Z [MeV]	2.3	0.8	0.5	0.1	0.5
R_b [10^{-5}]	66	14	17	6	15
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	16	1	2.3	0.6	4.5

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC, but not ILC/CEPC/FCC-ee!

Theory and parametric uncertainties

	ILC	CEPC	perturb. error with 3-loop†	Param. error ILC*	Param. error CEPC**
M_W [MeV]	3–4	3	1	2.6	2.1
Γ_Z [MeV]	0.8	0.5	$\lesssim 0.2$	0.5	0.15
R_b [10^{-5}]	14	17	5–10	< 1	< 1
$\sin^2\theta^\ell_\text{eff}$ [10^{-5}]	1	2.3	1.5	2	2

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_\text{s}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_\text{s})$, $\mathcal{O}(N_f^2\alpha^2\alpha_\text{s})$

(N_f^n = at least n closed fermion loops)

Parametric inputs:

*** ILC:** $\delta m_t = 100$ MeV, $\delta\alpha_\text{s} = 0.001$, $\delta M_Z = 2.1$ MeV

****CEPC:** $\delta m_t = 600$ MeV, $\delta\alpha_\text{s} = 0.0002$, $\delta M_Z = 0.5$ MeV

also: $\delta(\Delta\alpha) = 5\times 10^{-5}$

- Subtraction of QED radiation contributions

  $\longrightarrow$ Known to $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha^3 L^3)$ for **ISR**, $\mathcal{O}(\alpha^2)$ for **FSR** and $\mathcal{O}(\alpha^2 L^2)$ for **A$_{\mathbf{FB}}$**

  $(L = \log \frac{s}{m_e^2})$

  Berends, Burgers, v.Neerven '88
  Kniehl, Krawczyk, Kühn, Stuart '88
  Beenakker, Berends, v.Neerven '89
  Skrzypek '92; Montagna, Nicrosini, Piccinini '97

  $\longrightarrow$ $\mathcal{O}(0.1\%)$ uncertainty on σ_Z, A_{FB}

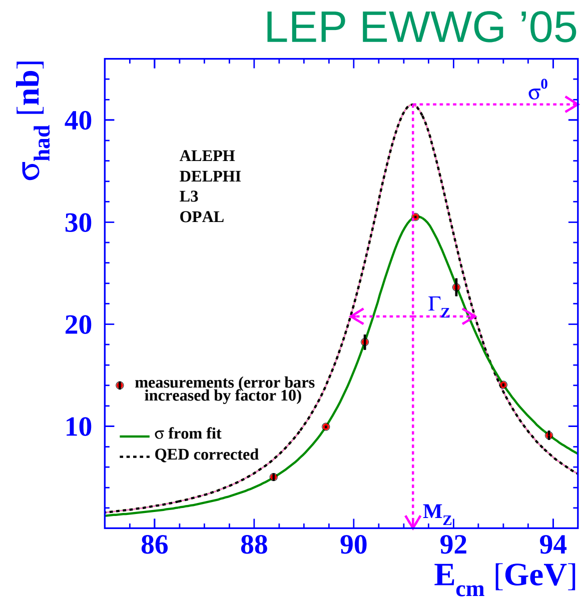
  $\longrightarrow$ Improvement needed for ILC/CEPC/FCC-ee

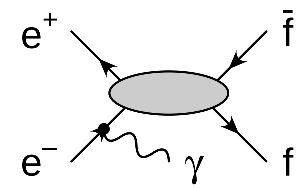
- Subtaction of non-resonant γ-exchange, γ–Z interf., box contributions, Bhabha scattering

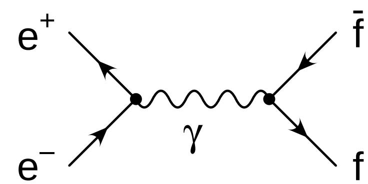
  see, e.g., Bardin, Grünewald, Passarino '99

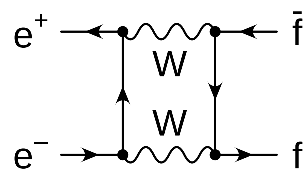
  $\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed)

  $\longrightarrow$ Sensitivity to some NP beyond EWPO





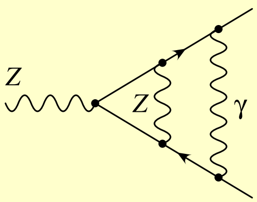




Non-factorizable contributions

Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi} \left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2 \right) \frac{1}{1 + \mathrm{Re}\,\Sigma'_Z} \right]_{s=\overline{M}_Z^2}$$



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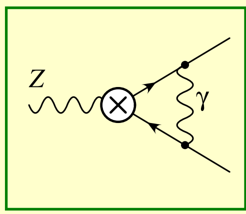
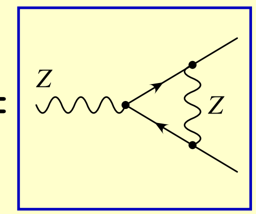
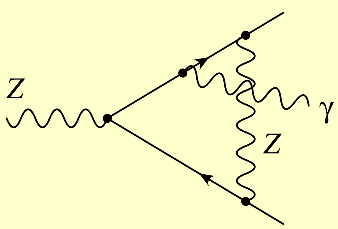


+ finite, with $\otimes$ =



Additional non-factorizable contributions, e.g.



$\longrightarrow$ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98

$\longrightarrow$ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty on Γ_Z, σ_Z, maybe larger for A_b
(improvements may be needed)

Theory challenges

Full SM corrections at $\geq$2-loop:

- Large number of diagrams and tensor integrals, $\mathcal{O}(100) - \mathcal{O}(10000)$
- Many different scales (masses and ext. momenta)

Computer algebra methods:

- Generation of diagrams with *FeynArts*, *QGraf*, ...

  Küblbeck, Eck, Mertig '92, Hahn '01

  Nogueira '93
- Dirac/Lorentz algebra with *Form*, *FeynCalc*, ...

  Vermaseren '89,00

  Mertig '93

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods must be automizable, stable, fastly converging
- Need procedure for isolating divergent pieces

Analytic calculations

- Useful for diagrams with up to two scales
  ($e.g.$ M_{W} & m_{t} or M_{W} & M_{Z})
- Reduce to master integrals with integration-by-parts and Lorentz-invariance identities — Chetyrkin, Tkachov ’81; Gehrmann, Remiddi ’00; Laporta ’00; ...
- Evaluate master integrals with differential equations or Mellin-Barnes representations — Kotikov ’91; Remiddi ’97; Smirnov ’00,01; ...

Current status:

Single-scale problems: $Zf\bar{f}$ QED/QCD vertex corrections up to 4-loop

Gorishnii, Kataev, Larin ’88,91; Chetyrkin, Kühn, Kwiatkowski ’96
Baikov, Chetyrkin, Kühn, Rittinger ’12

Two-scale problems: $Zf\bar{f}$ electroweak 2-loop vertex diagrams with $m_f = 0$

Awramik, Czakon, Freitas, Weiglein ’04

Extendability: Possible, but much work needed

Asymptotic expansions

- Exploit large mass ratios,
  $e.g.\ M_Z^2/m_t^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

Current status:

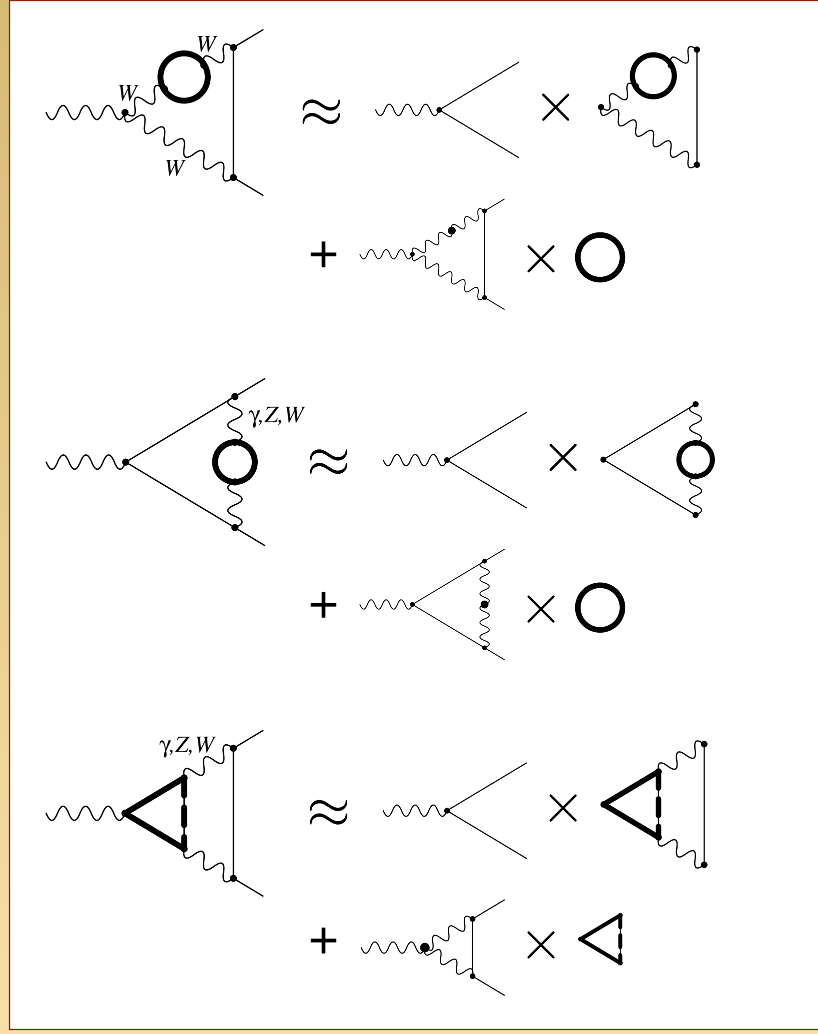
Two-scale problems: $\mathcal{O}(\alpha\alpha_s^n)$ for $\Delta\rho$, Δr

$\longrightarrow$ Several expansion terms up to 3-loop, leading term up to 4-loop

Djouadi, Verzegnassi '87; Bardin, Chizhov '88
Chetyrkin, Kühn, Steinhauser '95
Faisst, Kühn, Seidensticker, Veretin '03; ...

Three-scale problems: $Zf\bar{f}$ vertex at 2-loop

Barbieri et al. '92,93
Fleischer, Tarasov, Jegerlehner '93,95
Degrassi, Gambino, Sirlin '97
Awramik, Czakon, Freitas, Weiglein '04



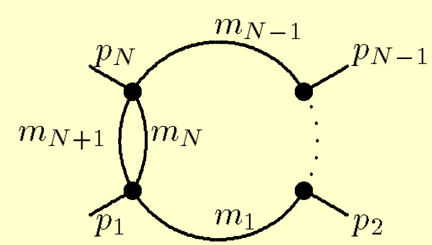
Extendability: Promising, limited by computing/algorithms

Dispersion relations

- Numerically integrate over cuts
- High precision, but no known path towards full automatization
- Subtraction of UV-divergencies by hand

Example: Topologies with **self-energy sub-loop** S. Bauberger et al. '95

$$B_0(p^2, m_1^2, m_2^2) = -\int_{(m_1+m_2)^2}^{\infty} \mathrm{d}s \, \frac{\Delta B_0(s, m_1^2, m_2^2)}{s - p^2}$$



$$T_{N+1}(p_i; m_i^2) = -\int_{s_0}^{\infty} \mathrm{d}s \, \Delta B_0(s, m_N^2, m_{N+1}^2)$$

$$\times \int \mathrm{d}^4 q \, \frac{1}{q^2 - s} \, \frac{1}{(q+p_1)^2 - m_1^2} \cdots \frac{1}{(q+p_1+\cdots+p_{N-1})^2 - m_{N-1}^2}$$

Dispersion relations

- Numerically integrate over cuts
- High precision, but no known path towards full automatization
- Subtraction of UV-divergencies by hand

Current status:

Self-energy and vertex diagrams with arbitrary number of scales

Freitas, Hollik, Walter, Weiglein ’00; Awramik, Czakon ’02; Awramik, Czakon, Freitas ’04

Extendability: Only for certain applications

Direct numerical integration

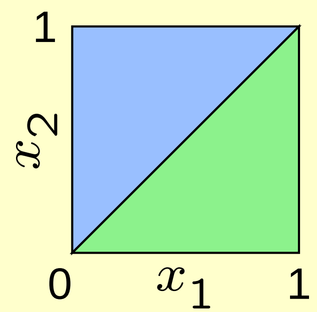
General form of Feynman integral:

$$I = \int_0^1 dx_1 \ldots dx_n \, \delta(1 - \sum_i x_i) \, \frac{N(x_i)}{D(x_i)^{r+\varepsilon}}$$

$\longrightarrow$ Can be integrated numerically (if finite)

Alternatives: Integration in momentum space, Mellin-Barnes space

Treatment of divergencies:



- **Sector decomposition:** Sub-divide integration space such that divergent terms factorize
  Binoth, Heinrich '00,03

- **Subtraction terms:** Remove divergencies with simple terms that can be integrated analytically
  Nagy, Soper '03
  Becker, Reuschle, Weinzierl '10; Freitas '12

Direct numerical integration

- Automizable, but computing intensive
- Internal thresholds reduce numerical convergence (contour deformation)

Current status:

Several 2-loop applications with many scales

Anastasiou et al., Petriello et al., Borowka et al., ...

Individual 3-loop integrals

Extendability: Likely, but more work needed

Mellin-Barnes representations

Transform Feynman integral with Mellin-Barnes representation

$$\frac{1}{(A_0 + \ldots + A_m)^Z} = \frac{1}{(2\pi i)^m} \int_{\mathcal{C}_1} dz_1 \cdots \int_{\mathcal{C}_m} dz_m$$

$$\times A_1^{z_1} \cdots A_m^{z_m} A_0^{-Z-z_1-\ldots-z_m}$$

$$\times \frac{\Gamma(-z_1) \cdots \Gamma(-z_m) \Gamma(Z + z_1 + \ldots + z_m)}{\Gamma(Z)},$$

Transform Feynman integral with Mellin-Barnes representation

$$\frac{1}{(A_0 + \ldots + A_m)^Z} = \frac{1}{(2\pi i)^m} \int_{\mathcal{C}_1} dz_1 \cdots \int_{\mathcal{C}_m} dz_m$$
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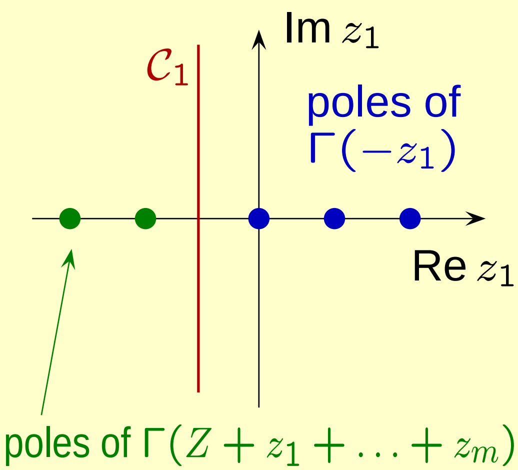
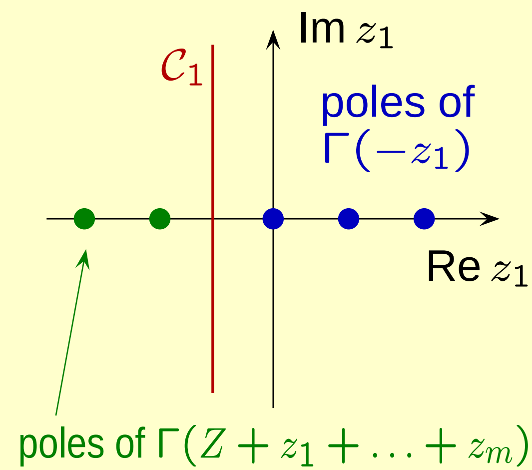


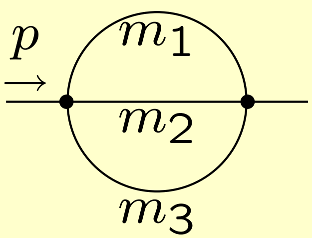
Transform Feynman integral with Mellin-Barnes representation

$$\frac{1}{(A_0+\ldots+A_m)^Z} = \frac{1}{(2\pi i)^m}\int_{\mathcal{C}_1} dz_1 \cdots \int_{\mathcal{C}_m} dz_m$$
$$\times A_1^{z_1}\cdots A_m^{z_m} A_0^{-Z-z_1-\ldots-z_m}$$
$$\times \frac{\Gamma(-z_1)\cdots\Gamma(-z_m)\Gamma(Z+z_1+\ldots+z_m)}{\Gamma(Z)},$$



After Feynman parameter integration: Γ functions and exponentials

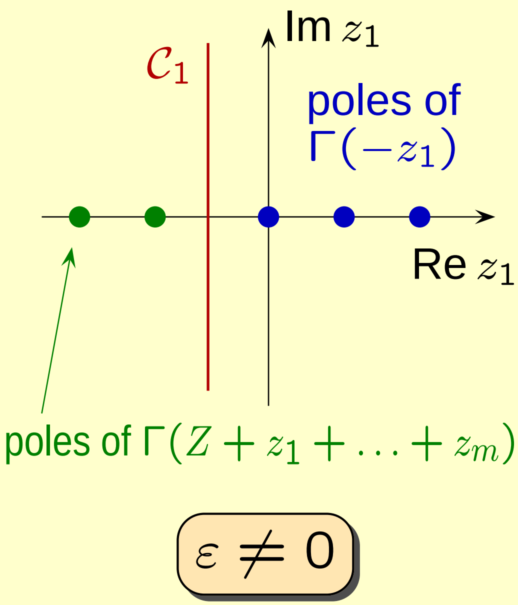
Example:



$$= \frac{-1}{(2\pi i)^3}\int dz_1 dz_2 dz_3\, (m_1^2)^{-\varepsilon-z_1-z_2}(m_2^2)^{z_2}(m_3^2)^{1-\varepsilon+z_1-z_3}(-p^2)^{z_3}$$
$$\times \Gamma(-z_2)\Gamma(-z_3)\Gamma(1+z_1+z_2)\Gamma(z_3-z_1)$$
$$\times \frac{\Gamma(1-\varepsilon-z_2)\Gamma(\varepsilon+z_1+z_2)\Gamma(\varepsilon-1-z_1+z_3)}{\Gamma(2-\varepsilon+z_3)}$$

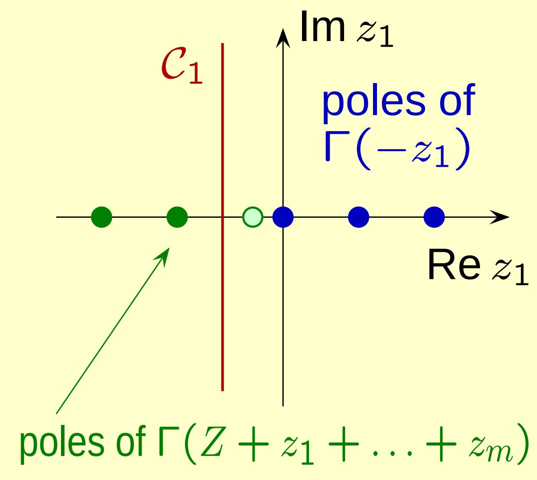
Mellin-Barnes representations

- Consistent choice of all $\mathcal{C}_i$ often requires $\varepsilon \neq 0$ $(Z = n + \epsilon)$



Mellin-Barnes representations

- Consistent choice of all $\mathcal{C}_i$ often requires $\varepsilon \neq 0$ $(Z = n + \epsilon)$



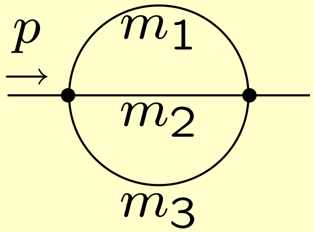
- For $\varepsilon \longrightarrow 0$: residues from pole crossings $\longrightarrow 1/\varepsilon^k$ terms

  Czakon '06
  Anastasiou, Daleo '06

- Do remaining $\mathcal{C}_i$ integrations numerically

$\varepsilon \longrightarrow 0$

Mellin-Barnes representations



$$= \frac{-1}{(2\pi i)^3} \int dz_1 dz_2 dz_3 \, (m_1^2)^{-\varepsilon - z_1 - z_2} (m_2^2)^{z_2} (m_3^2)^{1-\varepsilon+z_1-z_3} (-p^2)^{z_3}$$

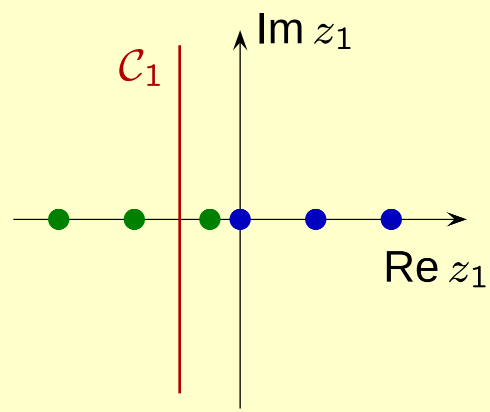
$$\times \Gamma(-z_2)\Gamma(-z_3)\Gamma(1+z_1+z_2)\Gamma(z_3-z_1)$$

$$\times \frac{\Gamma(1-\varepsilon-z_2)\Gamma(\varepsilon+z_1+z_2)\Gamma(\varepsilon-1-z_1+z_3)}{\Gamma(2-\varepsilon+z_3)}$$

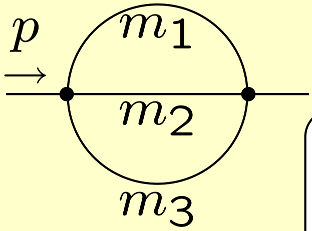
$z_3 = c_3 + iy_3, \qquad y_i \in (-\infty, \infty)$

$$(-p^2)^{z_3} = \underbrace{(p^2)^{c_3+iy_3} e^{-i\pi c_3}}_{\text{oscillating}} \underbrace{e^{\pi y_3}}$$

div. for $y_3 \to \infty$, eventually over-come by Γ funct.



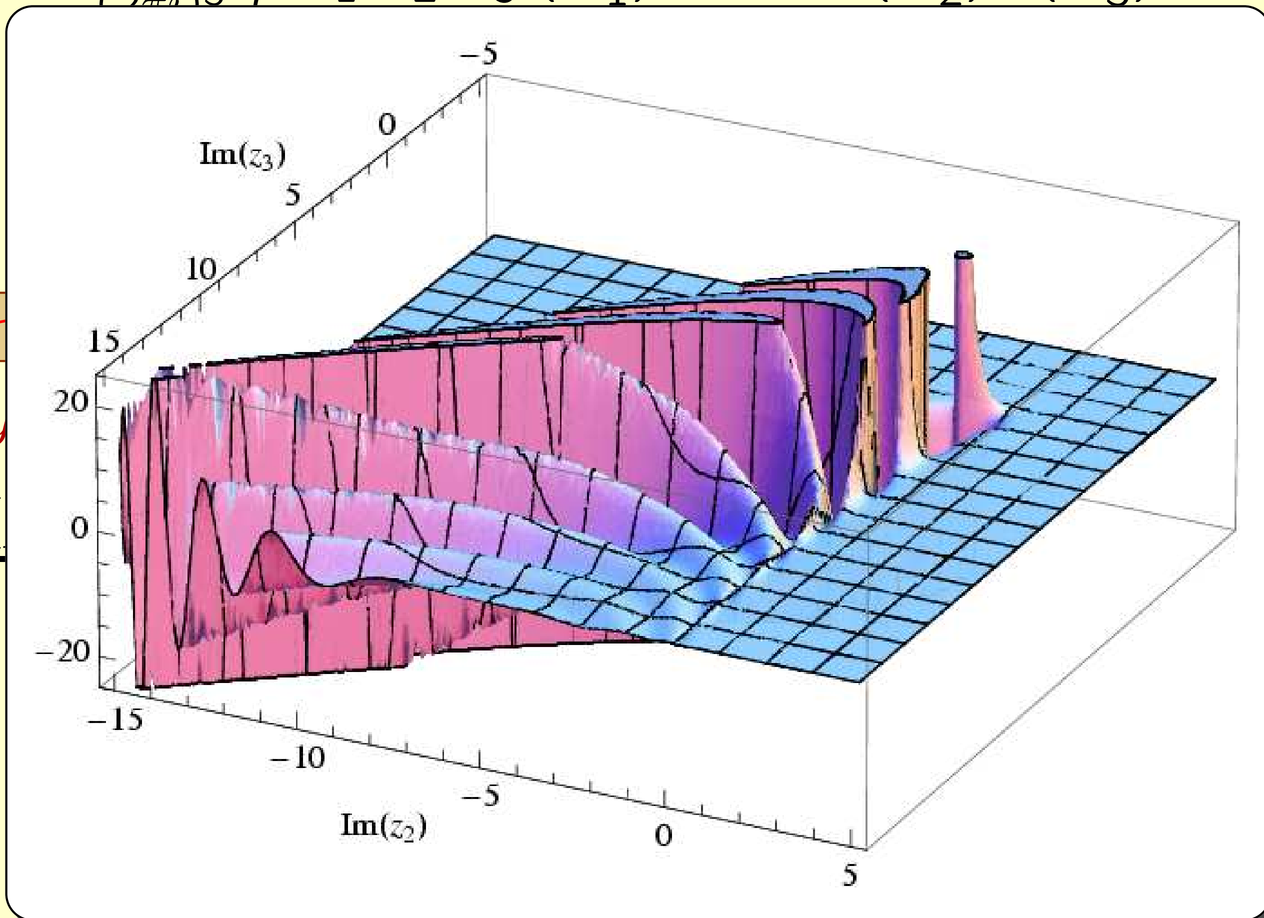
Mellin-Barnes representations



$= \frac{-1}{(2\pi i)^3} \int dz_1 dz_2 dz_3 \, (m_1^2)^{-\varepsilon - z_1 - z_2} (m_2^2)^{z_2} (m_3^2)^{1-\varepsilon+z_1-z_3} (-p^2)^{z_3}$

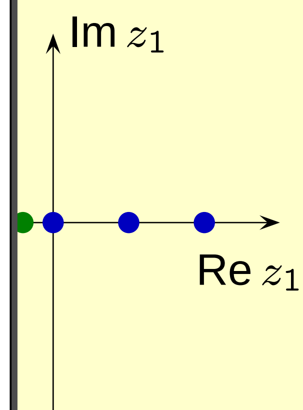
$)$

$z_1 + z_3)$

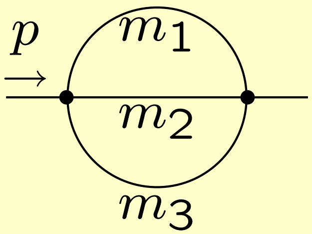


$z_3 = c_3 + iy$

$(-p^2)^{z_3} = ($



Mellin-Barnes representations



$$= \frac{-1}{(2\pi i)^3}\int dz_1 dz_2 dz_3\,(m_1^2)^{-\varepsilon-z_1-z_2}(m_2^2)^{z_2}(m_3^2)^{1-\varepsilon+z_1-z_3}(-p^2)^{z_3}$$
$$\times\,\Gamma(-z_2)\Gamma(-z_3)\Gamma(1+z_1+z_2)\Gamma(z_3-z_1)$$
$$\times\,\frac{\Gamma(1-\varepsilon-z_2)\Gamma(\varepsilon+z_1+z_2)\Gamma(\varepsilon-1-z_1+z_3)}{\Gamma(2-\varepsilon+z_3)}$$

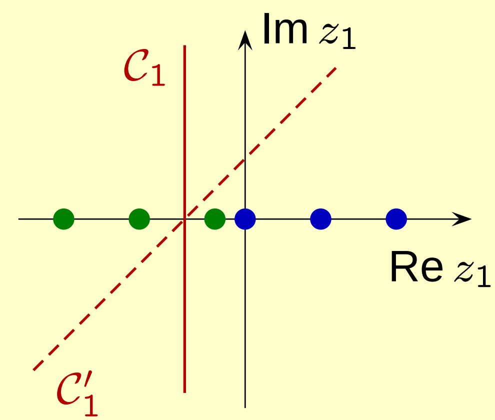
$z_3 = c_3 + iy_3, \qquad y_i \in (-\infty,\infty)$

$$(-p^2)^{z_3} = \underbrace{(p^2)^{c_3+iy_3}e^{-i\pi c_3}}_{\text{oscillating}}\;\underbrace{e^{\pi y_3}}_{\text{div. for } y_3\to\infty}$$

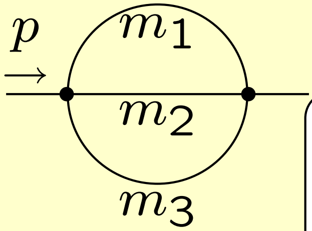
$y_i \to y_i - i\theta$

$$(-p^2)^{z_3} = (p^2)^{c_3+iy_3}e^{-i\pi(c_3+\theta y_i)}e^{(\pi+\theta\log p^2)y_3}$$

Huang, Freitas '10



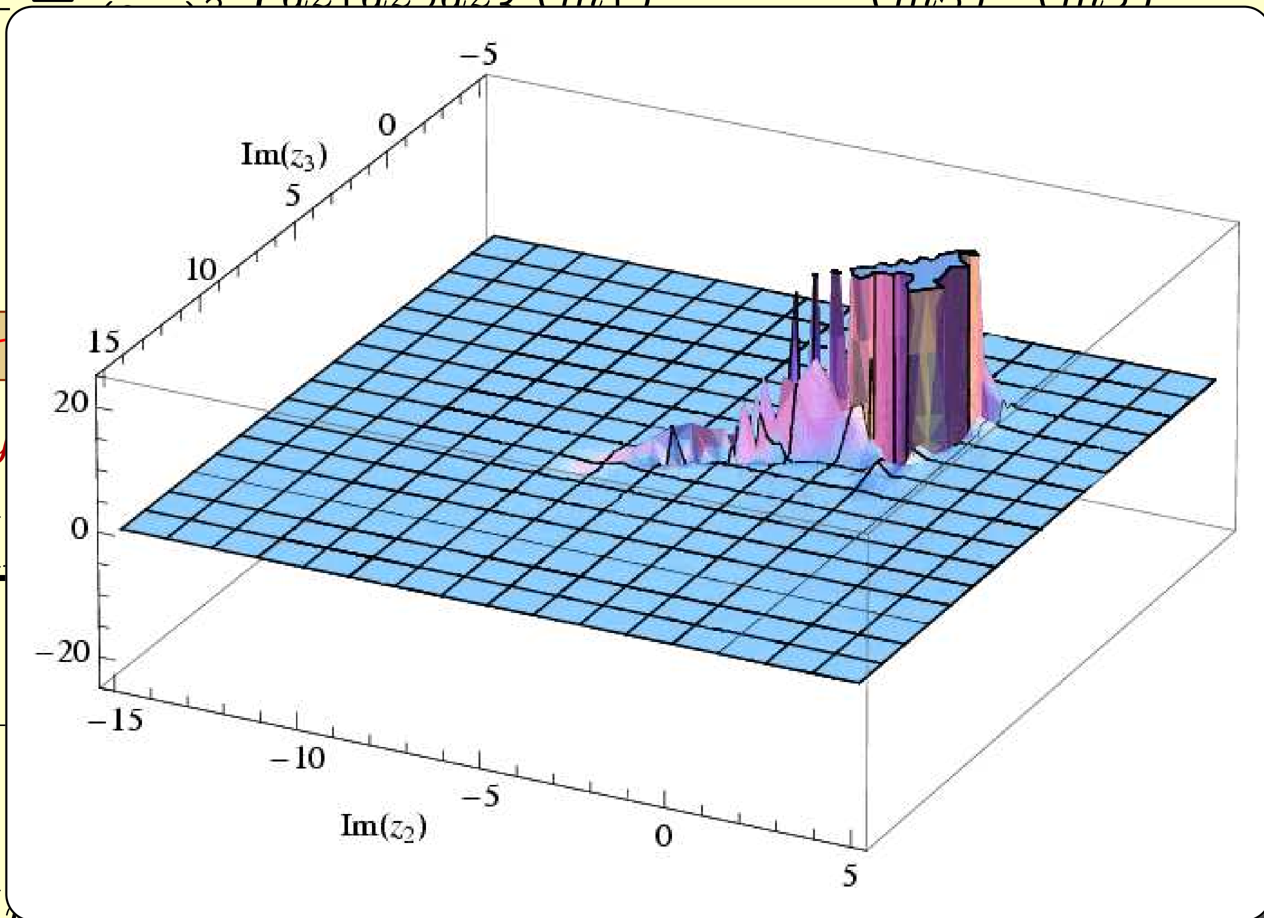
Mellin-Barnes representations



$= \frac{-1}{\ldots} \int dz_1 dz_2 dz_3 \, (m_1^2)^{-\varepsilon - z_1 - z_2} (m_2^2)^{z_2} (m_3^2)^{1-\varepsilon+z_1-z_3} (-p^2)^{z_3}$

$\ldots)$

$\ldots z_1 + z_3)$



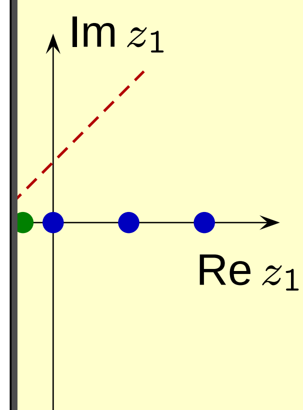
$z_3 = c_3 + iy \ldots$

$(-p^2)^{z_3} = (\ldots$

$y_i \to y_i - i\theta$

$(-p^2)^{z_3} = (p \ldots$

Freitas, Huang '10



Mellin-Barnes representations

Counter rotations not always successful:

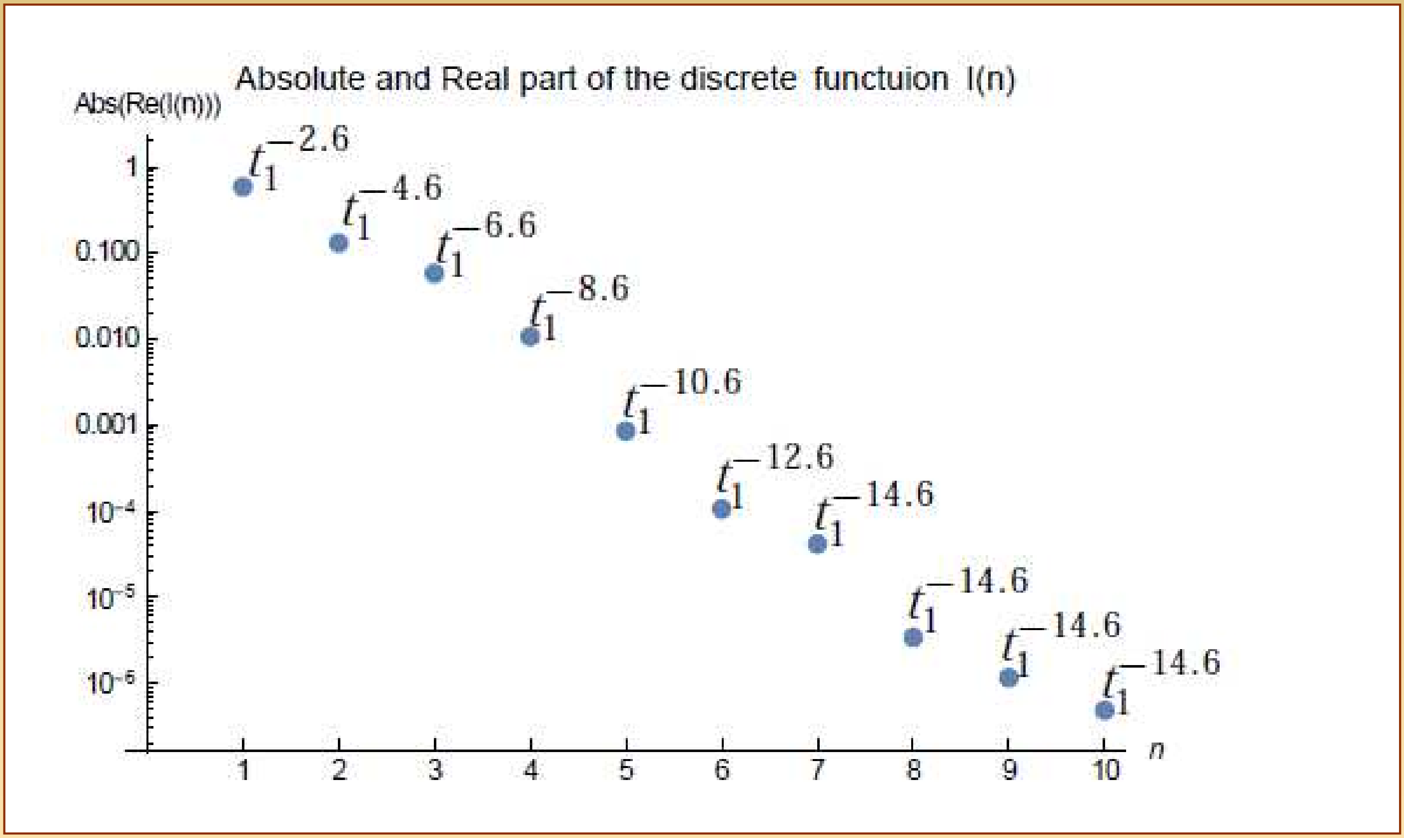
$$\frac{1}{(2\pi i)^2}\int dz_1 dz_2\; 2(m^2)^{-2}\Big(-\frac{p^2}{m^2}\Big)^{-z_1-z_2}$$
$$\times\frac{\Gamma(-z_2)\Gamma^3(1+z_2)\Gamma(-z_1-z_2)\Gamma(1+z_1+z_2)\Gamma(-1-z_1-2z_2)}{\Gamma(1-z_1)}$$

For $p^2 = m^2$ contour rotation has no effect

Shift countour: $z_1 = c_1 + iy_1,\; z_2 = c_2 + n + iy_2$

- Worst asymtotic behaviour of integrand for $y_1 \to -\infty$, $y_2 = 0$:
  $\sim y_1^{-2-2(c_2+n)}$ (for $n = 0$ and $c_2 = -0.7$: $\sim y_1^{-0.6}$)
- Pick up (finite number of) pole residues from contour shift

- Shifts improve asymptotic behaviour and size of numerical integral
- Automatic algorithms for finding suitable shifts in development (`MBnumerics`) Usovitsch '16



Conclusions

- **Electroweak precision tests** probe physics at the TeV scale
- Experimental precision from LEP/SLC/Tevatron/LHC demands SM prediction with **complete 2-loop corrections** and **partial 3-loop corrections**
- **ILC/CEPC/FCC-ee** with $\sqrt{s} \sim M_Z$ will reduce exp. error by $\mathcal{O}(10)$
  $\longrightarrow$ 3-loop (and maybe some 4-loop) corrections needed!
- **Numerical techniques** are promising but need to be improved substantially

Backup slides

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{\text{bos}}^2) \sim \mathcal{O}(\alpha_{\text{bos}})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors: $\mathcal{O}(\alpha_{\text{bos}}^2) \sim \Gamma_Z \alpha^2 \sim 0.1 \text{ MeV}$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{\text{lq}}}{\pi} \alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.5$ MeV

Example: Error estimation for M_W

- Renormalization scheme dependence:

a) Uncertainty of $\mathcal{O}(\alpha^2)$ corrections beyond leading $\alpha^2 m_\mathrm{t}^4$ and $\alpha^2 m_\mathrm{t}^2$ from comparison of $\overline{\mathrm{MS}}$ and OS schemes: Degrassi, Gambino, Sirlin '96

$\delta M_\mathrm{W} \sim 2$ MeV (for $M_\mathrm{H} \sim 100$ GeV)

Actual remaining $\mathcal{O}(\alpha^2)$ corrections: Freitas, Hollik, Walter, Weiglein '00

$\delta M_\mathrm{W} \sim 3$ MeV (for $M_\mathrm{H} \sim 100$ GeV)

b) Estimate of missing $\mathcal{O}(\alpha^3)$ corrections from comparison of $\overline{\mathrm{MS}}$ and OS results: Awramik, Czakon, Freitas, Weiglein '03
Degrassi, Gambino, Giardino '14

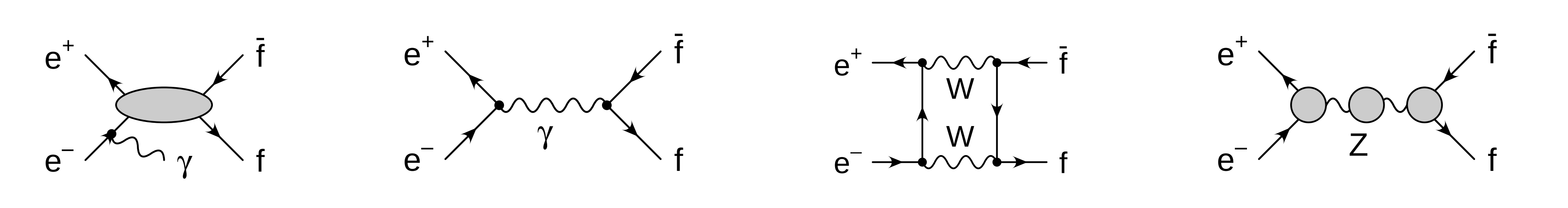
$\delta M_\mathrm{W} \sim 4...5$ MeV (after accounting for $\mathcal{O}(\alpha_\mathrm{t}\alpha_\mathrm{s}^3)$ corrections)

$\longrightarrow$ Saturates previous δM_W estimate!

Note: Differences in (implicitly) resummed higher-order contributions

“Analytical” tools for $e^+e^- \to f\bar{f}$

- State of the art: `Zfitter 6.42` — Bardin et al. ’99, Arbuzov et al. ’05
  Older code: `TOPAZ0` — Montagna, Nicrosini, Passarino, Piccinini ’98,01
- Describes true observables ($\sigma_{e^+e^-\to f\bar{f}}(s)$, etc.)
  and pseudo-observables (Γ_Z, σ^0_{had}, $\mathcal{A}_f$, etc.)
- Final-state QED and QCD corrections at $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$, $\mathcal{O}(\alpha_s^3)$
- Deconvolution of initial-state and initial-final QED radiation
  at $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2 L)$ and $\mathcal{O}(\alpha^3 L^2)$ ($L \equiv \log(s/m_e^2)$)
- Full NLO electroweak corrections for $e^+e^- \to f\bar{f}$
- Partial $\mathcal{O}(\alpha^2)$ and higher-order electroweak corrections



“Analytical” tools for $e^+e^- \to f\bar{f}$

Drawbacks:

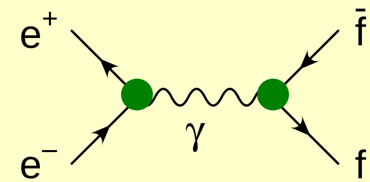
- Not all available NNLO and higher-order corrections implemented (code structure makes implementation difficult)
- For consistent treatment beyond NLO, need expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_\mathrm{Z}^2 - iM_\mathrm{Z}\Gamma_\mathrm{Z}$:

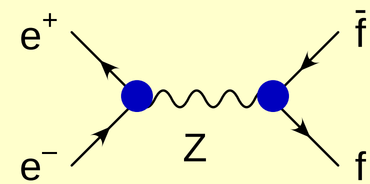
$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)T + \dots$$

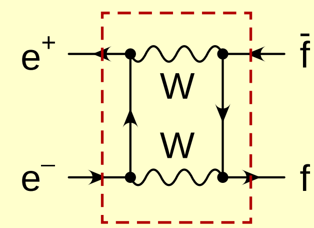
$$R = g_\mathrm{Z}^e(s_0)\,g_\mathrm{Z}^f(s_0)$$

$$S = \left[\frac{1}{M_\mathrm{Z}^2} g_\gamma^e g_\gamma^f + g_\mathrm{Z}^e g_\mathrm{Z}^{f\,\prime} + g_\mathrm{Z}^{e\,\prime} g_\mathrm{Z}^f + S_\mathrm{box}\right]_{s=s_0}$$

$g_\mathrm{V}^f(s)$: effective $Vf\bar{f}$ couplings







At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

Monte-Carlo tools for $e^+e^- \to f\bar{f}$

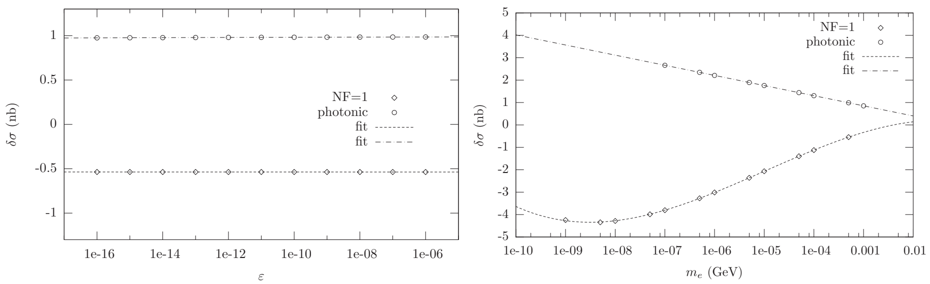
- State of the art: `KKMC, BabaYaga` — Jadach, Ward, Wąs '13; Carloni Calame et al. '12
- YFS exponentiation for QED radiation, approximate NNLO QED
- currently $\mathcal{O}(0.1\%)$ precision, $\mathcal{O}(0.01\%)$ feasible in (near) future, but more may be needed for FCC-ee

Comparison with (a subset of) NNLO

G. Balossini et al., NPB758 (2006) 227

Comparison of $\sigma_{\rm SV}^{\alpha^2}$ calculation of BabaYaga@NLO with

- Penin (photonic): switching off the vacuum polarisation contribution in BabaYaga@NLO, as a function of the logarithm of the soft photon cut–off (left plot) and of a fictitious electron mass (right plot)



★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\rm SV}^{\alpha^2}(\text{Penin}) - \sigma_{\rm SV}^{\alpha^2}(\text{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

F. Piccinini (INFN Pavia) | RC for QED processes | FCC-ee | 12 / 17

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