Journey through the frontier of precision Higgs

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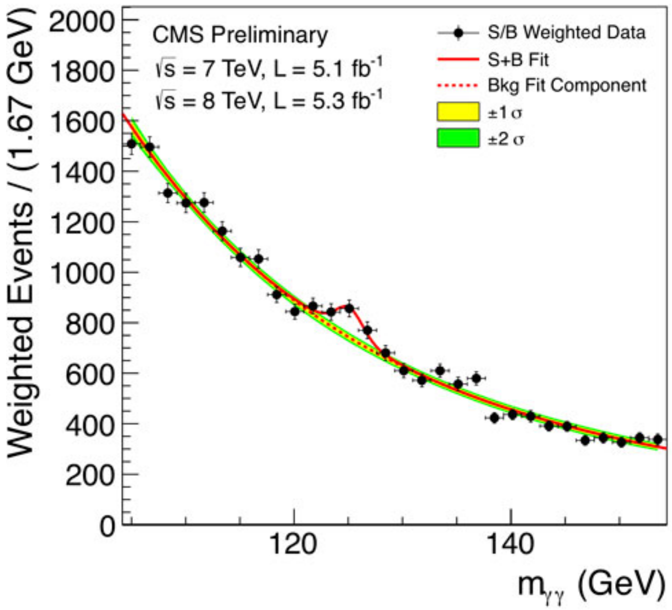
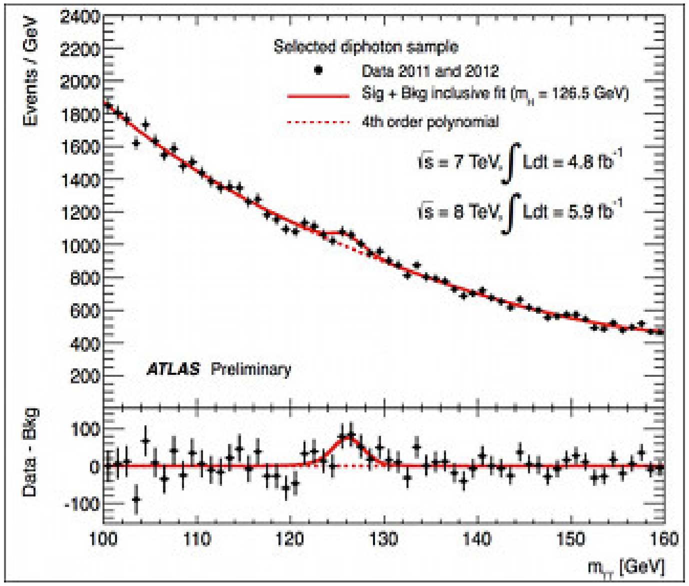
SLAC NATIONAL ACCELERATOR LABORATORY

Milestone of particle physics: discovery of Higgs Boson





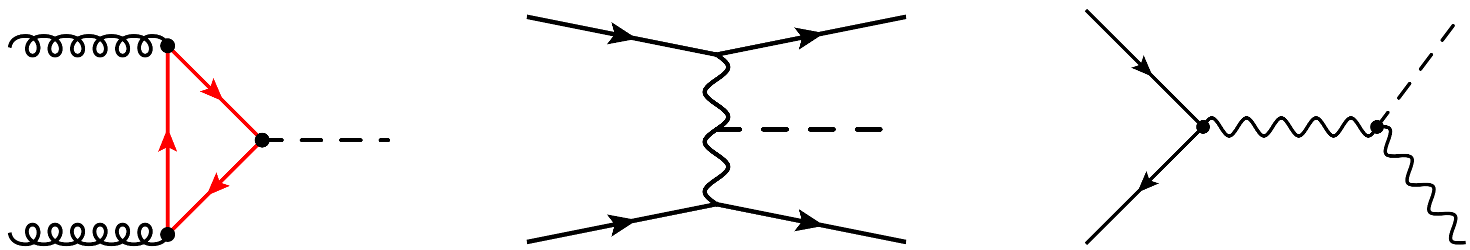
Higgs found. What's next?

- Unfortunately, new physics still yet to be found at LHC
- If new physics do exist at TeV scale, we may need to scan every corner of new physics parameter space, and dig it out from huge SM backgrouond

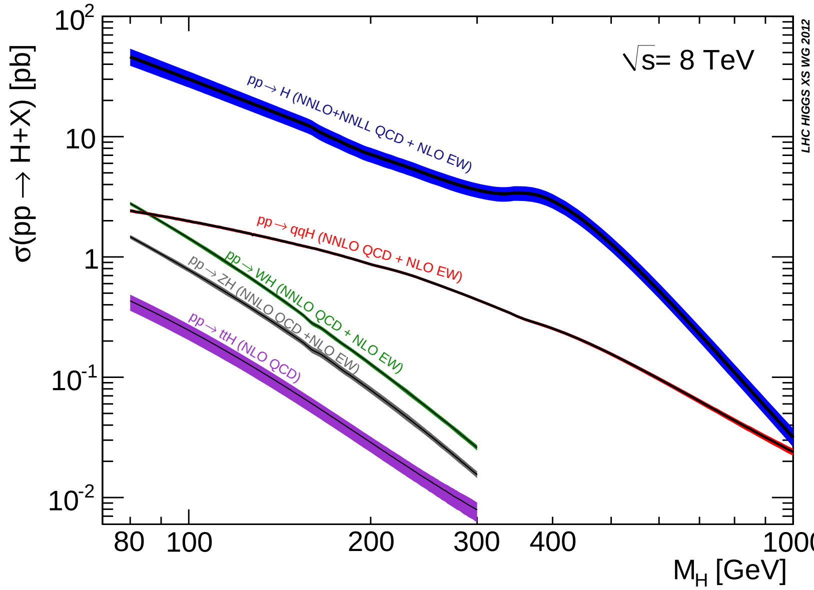
- It's likely that new physics have strong impact to the Higgs sector: **precise measurement** of the Higgs boson properties are of utmost importance!

Production of Higgs at LHC

- Three major production mechanism



gluon-gluon fusion vector-boson fusion associated production



[Higgs Working Group report]

Experimental status

- With 7 and 8 TeV data, LHC has already done a good job in precision higgs measurement.
- Current all channels combined results for total Higgs production Xsec:

$$\sigma_{\text{exp}} = (0.80 \pm 0.14)\sigma_{\text{SM}}$$

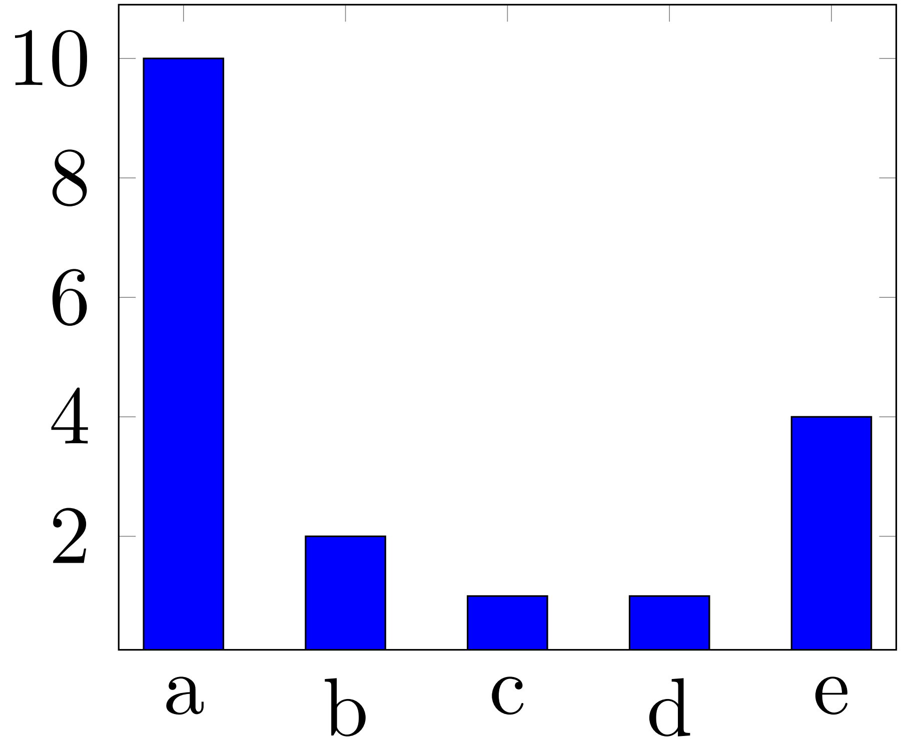
- **Percent level uncertainties** can be achieved in the 14 TeV run [CMS snowmass report 2013]

L ($\text{fb}^{-}1$)	$\gamma\gamma$	WW	ZZ	$b\bar{b}$
300	[6%, 12%]	[6%, 11%]	[7%, 11%]	[11%, 14%]
3000	[4%, 8%]	[4%, 7%]	[4%, 7%]	[5%, 7%]

- Theoretical accuracy need to match the experimental accuracy
- Great challenge to (QCD) theorist, but also great opportunities!

Theoretical status: glue-glue fusion

- Current best theory predictions



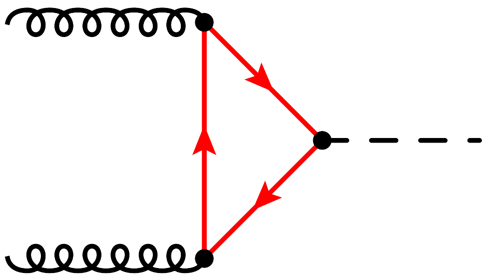
[Higgs Working Group Report]

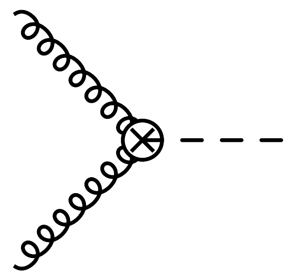
a. NNLO scale b. NLO EW c. Large m_t approx. d. quark mass input e. PDF

- Largest uncertainty from **NNLO QCD scale variation**: MUST BE REDUCED!

Higgs effective theory

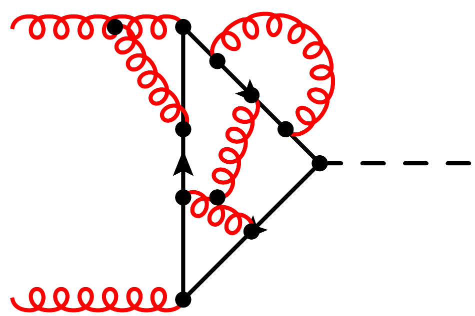
- To very good approximation ($\sim 1\%$), ggH interaction can be represented by an effective coupling [Shifman, Vainshtein, Voloshin, Zakharov, 1979]





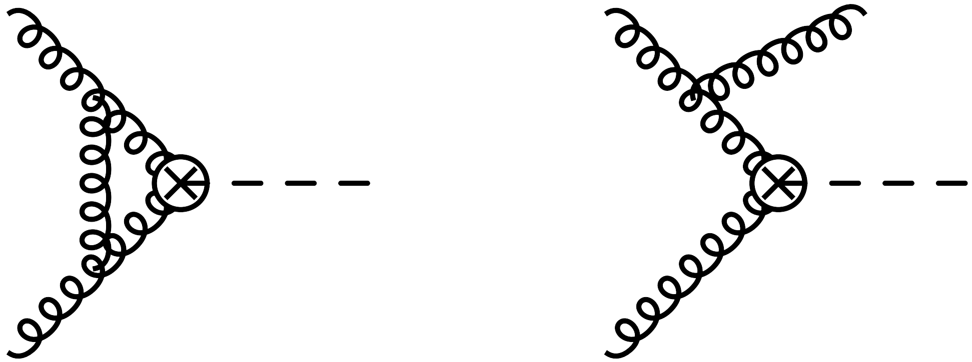
$$\mathcal{L}_{eff} = -\frac{1}{4}\lambda_t H G^{\mu\nu,a} G^a_{\mu\nu}$$

- The matching coefficient λ_t can be calculated perturbatively
- λ_t known to **five loops**!! [Schroder, Steinhauser, 2005; Chetyrkin, Kuhn, Sturm, 2005]
- Based on EFT, higher order QCD radiative corrections feasible

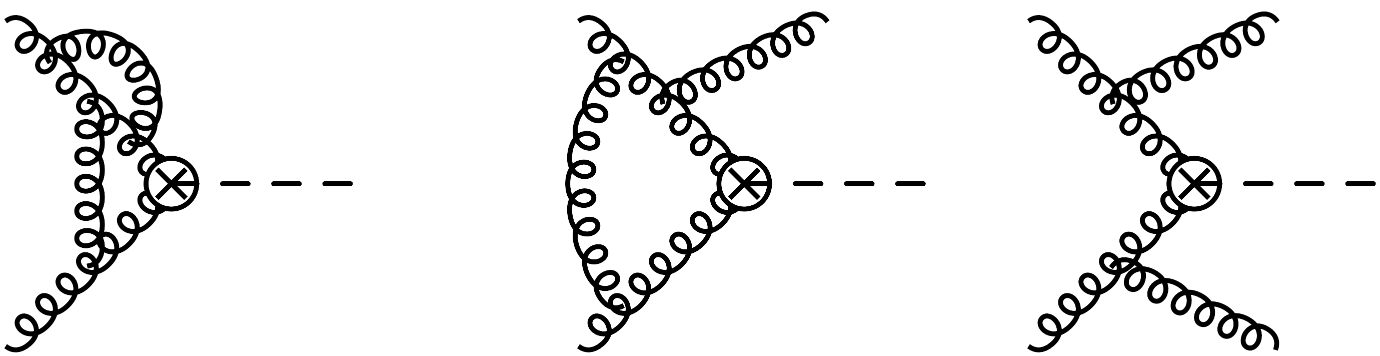


History of precision Higgs physics

- NLO [Dawson, 1991; Djouadi, Spira, Zerwas, 1993]

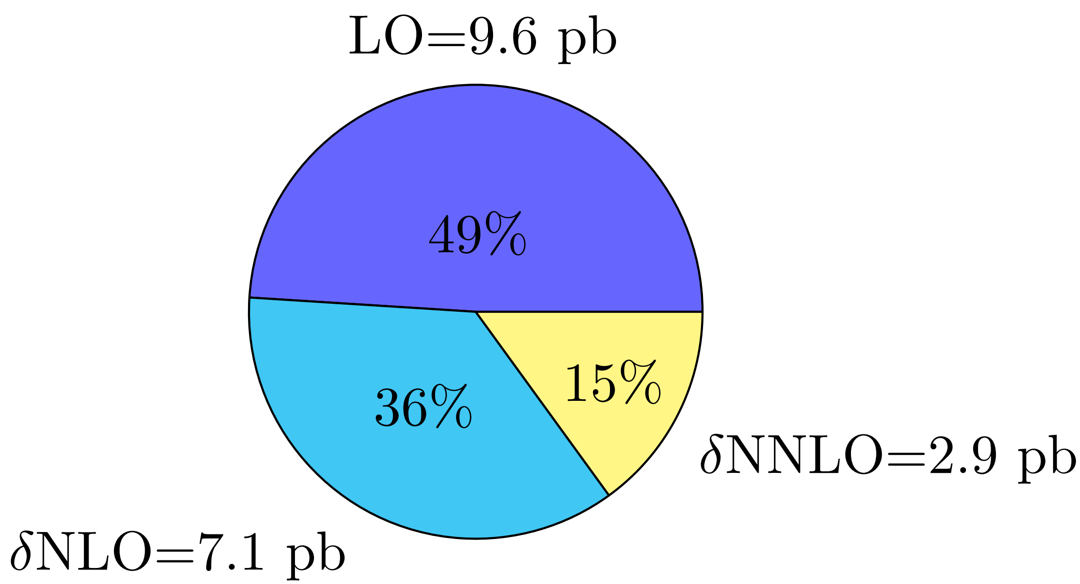


- NNLO [Harlander, Kilgore, 2002; Anastasiou, Melnikov, 2002; Ravindran, Smith, van Neerven, 2003]



Impact of NLO and NNLO

- It turns out that QCD corrections to $gg \to H$ are significant.
- Total Xsec for $gg \to H$ up to NNLO at $\sqrt{s} = 8$ TeV LHC:



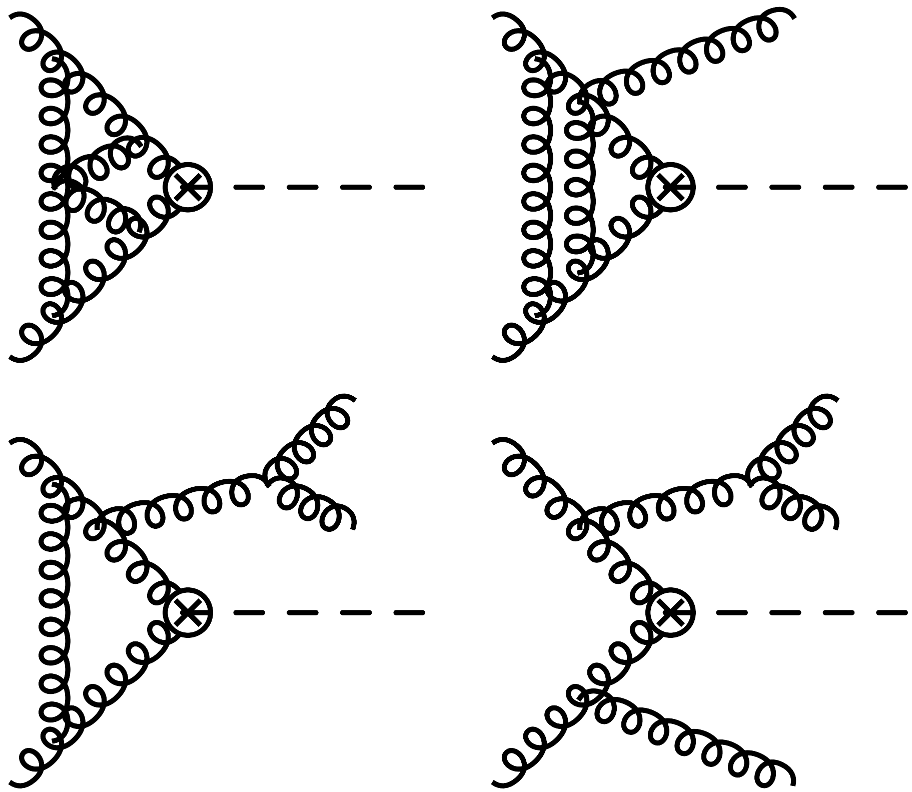
- Surprisingly large K factor at NLO and NNLO
- It's expected that N^3LO will have important impact
- Need to push calculation to N^3LO to reduce uncertainty to percent level!

N^3LO soft-virtual corrections for Higgs production

[Y. Li, H. X. Zhu, 1309.4391; Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 1404.5839; Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 1410.xxxx]

Higgs production at N^3LO

- The calculation at N^3LO looks formidable
- $\mathcal{O}(10^4)$ diagrams
- $\mathcal{O}(10^3)$ real/virtual master integrals



- Approximation needed!

Dominant part of cross section

- The Higgs production Xsec is a convolution of PDFs and partonic Xsec

$$\sigma_H = f \otimes f \otimes \hat{\sigma}_H(z)$$

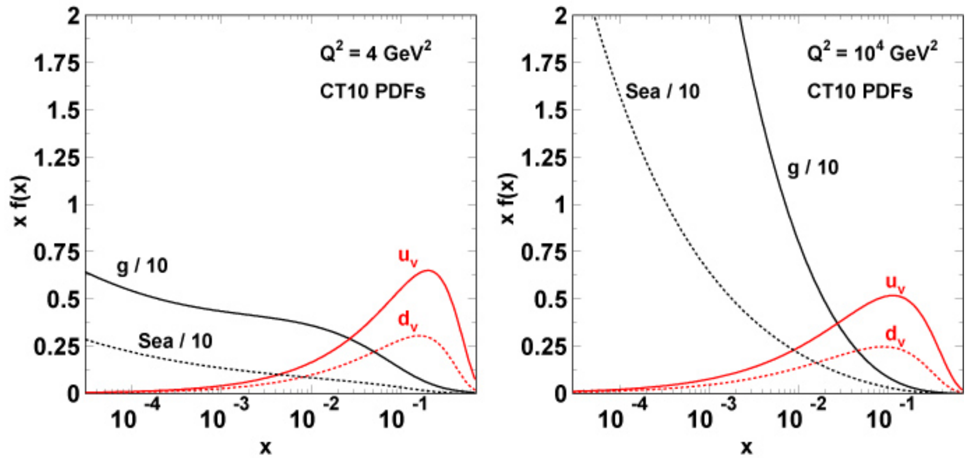
- The partonic Xsec $\hat{\sigma}(z)$ depends on threshold variable $z = \frac{M_H^2}{\hat{s}} = \frac{M_H^2}{x_1 x_2 S}$

- $z \to 1$ is the partonic threshold limit. QCD radiations are severely restricted

- $\hat{\sigma}_H(z)$ can be divided into **threshold singular part**, and regular part

$$\hat{\sigma}_H(z) = \mathrm{Sing}(1-z) + \mathrm{Reg}(1-z)$$

- **Rapid decreasing** of gluon luminosity implies that σ_H is dominated by the singular part



[CTEQ collaboration]

Soft-virtual approximation

- The **soft-virtual corrections** $\mathrm{Sing}(1-z)$ has the expansion

$$\begin{aligned}
\mathrm{Sing}(1-z) =& C_{00}\delta(1-z) \\
\text{NLO} & + C_{10}\alpha_s\delta(1-z) + C_{11}\alpha_s L_1 + C_{12}\alpha_s L_2 \\
\text{NNLO} & + C_{20}\alpha_s^2\delta(1-z) + C_{21}\alpha_s^2 L_1 + C_{22}\alpha_s^2 L_2 + C_{23}\alpha_s^2 L_3 + C_{24}\alpha_s^2 L_4 \\
\text{N}^3\text{LO} & + C_{30}\alpha_s^3\delta(1-z) + C_{31}\alpha_s^3 L_1 + C_{32}\alpha_s^3 L_2 + \cdots + C_{36}\alpha_s^3 L_6 \\
& + \cdots
\end{aligned}$$

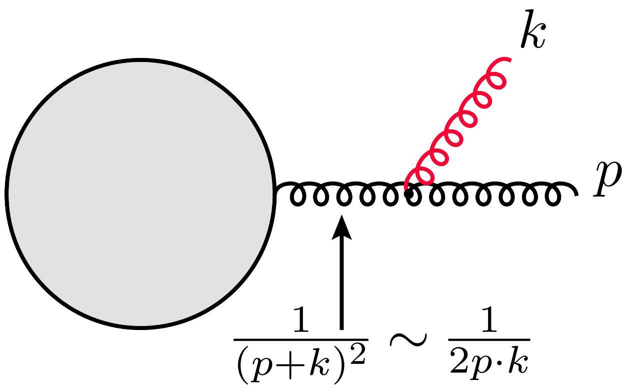
- The singular Logarithms $L_i = \left[\frac{\ln^{i-1}(1-z)}{(1-z)}\right]_+$
- The plus distribution is defined as

$$\int_0^1 \mathrm{d}z \left[\frac{\ln^{i-1}(1-z)}{(1-z)}\right]_+ g(z) = \int_0^1 \mathrm{d}z \left[\frac{\ln^{i-1}(1-z)}{(1-z)}\right] (g(z) - g(1))$$

- Where do the singular soft-virtual contributions come from?

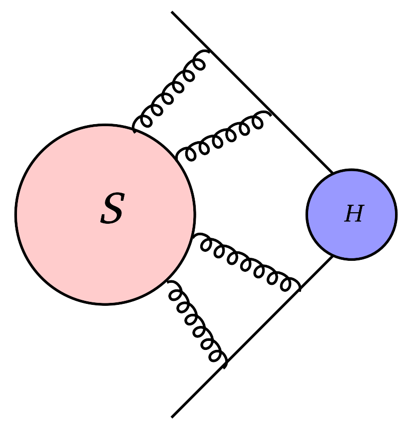
Origin of singular contributions

- Scattering amplitudes in QCD have well-known **soft** singularities



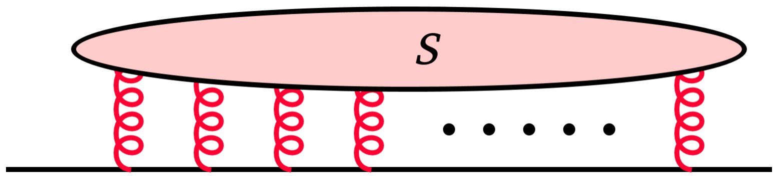
- The singular distribution L_i and $\delta(1-z)$ are due to these soft singularities!

- The QCD soft interactions factorize
- Interaction of soft gluon and gluon with large virtuality can be neglected
- Leads to the picture of two onshell hard incoming parton, a cloud of soft gluon, and a short distance hard interaction part



Factorization of singular contributions

- The interaction between soft gluon and onshell hard parton can be organized into a **Wilson line**



$$Y = 1 + \sum_{m=1}^{\infty} \sum_{\text{perm}} \frac{(-g)^m}{m!} \frac{n \cdot A_s^{a_n} \cdots n \cdot A_s^{a_1}}{n \cdot (\sum_{i=1}^{n} k_i + i\varepsilon) \cdots (n \cdot k_1 + i\varepsilon)} T_{a_n} \cdots T_1 = \sum_{\text{perm}} \exp\left[\frac{1}{n \cdot \mathcal{P} + i\varepsilon}(-gn \cdot A_s)\right]$$

- $n^\mu = p^\mu / p^0$ is a lightlike vector characterizes the direction of hard parton
- In equation, the singular contributions enjoy the factorization form

$$\text{Sing}(1-z) = \mathcal{H}(Q)\mathcal{S}(1-z)$$

- $\mathcal{H}$ is the so-called hard function. Known from three-loop gluon form factor [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser, 2009; Gehrmann, Glover, Huber, Ikizlerli, Studerus, 2010]

The soft function

- The soft function $\mathcal{S}$ encodes the effects of real or virtual soft gluon radiations
- It's defined as vacuum expectation value of **intersected semi-infinite lightlike Wilson lines**

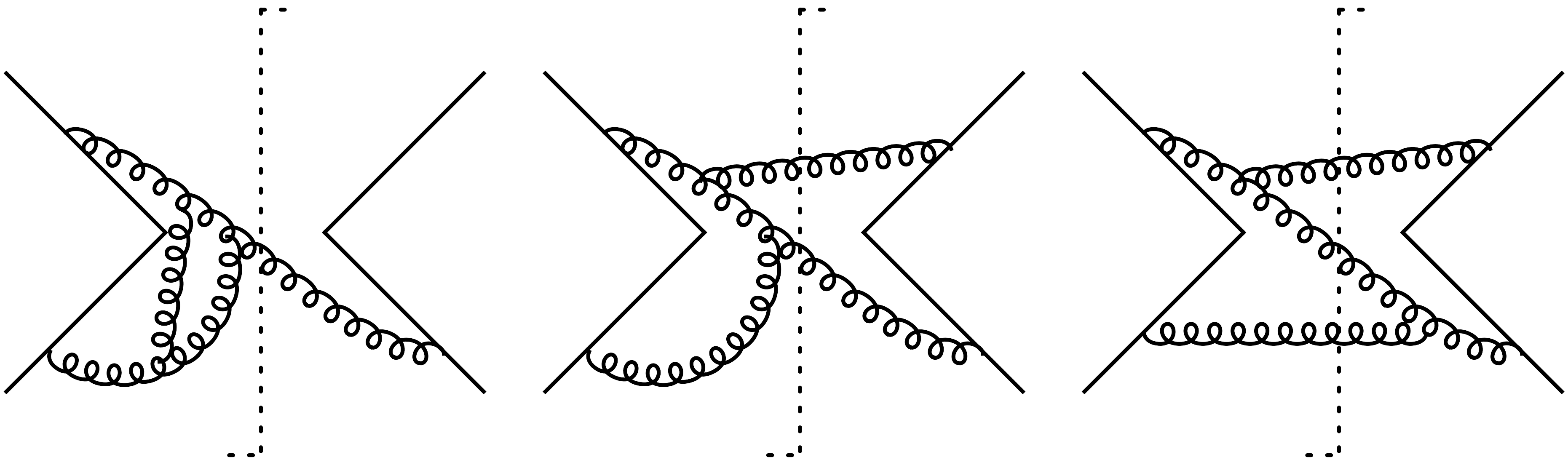
$$\mathcal{S}(1-z) \equiv \sum_{X_s} \langle 0|T\{Y_n Y_{\bar{n}}^\dagger\}|X_s\rangle \delta(2E_{X_s} - (1-z)M_H)\langle 0|T\{Y_{\bar{n}} Y_n^\dagger\}|0\rangle$$

$$Y_n(x) = \mathrm{P}\exp\left[ig_s \int_{-\infty}^{0} \mathrm{d}s\, n \cdot \mathbf{A}(x+sn)\right]$$

- To get N^{3}LO soft-virtual corrections, we need to calculate the soft function to N^{3}LO!
  - Involve phase space integrals with up to three soft gluons
  - Lots of diagrams, lots of integrals
  - Require advanced loop techniques
- I will try to explain the technique involved in a few slides

$\mathbf{N^3LO}$ Feynman diagrams for $\mathcal{S}$

- At N^3LO, we have up to three real gluon emitted from the Wilson lines



double virtual real | virtual double real | triple real

- For analytical calculation, it's often the case that diagrams with more real gluon radiations are more difficult than those with more virtual gluon radiations
- Therefore, the most difficult diagrams are those with three real gluons crossing the cut

Reverse unitarity

- Calculation of real phase space integral can be simplified using the method of reverse unitarity Anastasiou, Melnikov, 2002; Anastasiou, Dixon, Melnikov, Petriello, 2003

$$\mathrm{d}^d k \delta(k^2)\theta(k^0) = \frac{1}{2\pi i}\mathrm{d}^d k \left(\frac{1}{k^2 + i\varepsilon} - \frac{1}{k^2 - i\varepsilon}\right)$$

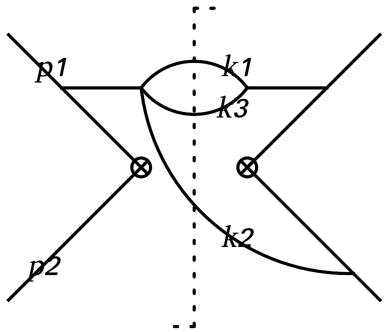
- In the sense of integral reduciton, phase space integral can be treated as loop integrals. Powerful technique of Integration-By-Parts identities can be used to reduce complicated phase space integral to simpler one
- IBP reduction is based on the relation [Tkachov, 1981]

$$\int \mathrm{d}^D k \frac{\partial}{\partial k^\mu}\Big[q^\mu I(\{k_i\}, \{p_i\})\Big] = 0$$

- Action of differential operator on the integrand leads to linear relations between different integrals
- Several public computer packages available for IBPs reduction based on Laporta algorithm [Laporta, 2000]

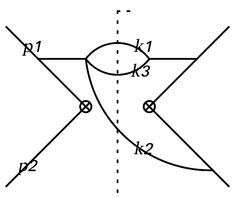
An integral reduction example

- Consider an integral with five propagators

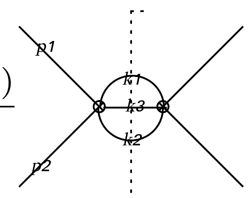


$$= \int \frac{d^dk_1 d^dk_2 d^dk_3 \delta_+(k_1^2)\delta_+(k_2^2)\delta_+(k_3^2)\delta(1-(p_1+p_2)\cdot(k_1+k_2+k_3))}{(k_1+k_2+k_3)^2(k_1+k_3)^2(k_1+k_3)\cdot p_1 \, k_2\cdot p_2}$$

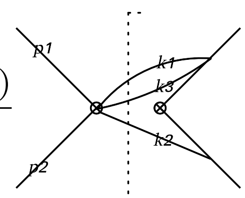
- After IBP reduction



$$= \frac{10(-13+3d)(-11+3d)(-10+3d)(-8+3d)(-7+3d)}{(-4+d)^4(-14+3d)}$$



$$- \frac{4(-13+3d)(-11+3d)}{(-4+d)(-14+3d)}$$



- The original integral is reduced to simpler (zero- and three-propagator) integrals

Reducing # of loop by scaling symmetry

- Typical master integral encountered

$$\mathrm{MI} = \int \frac{d^d k_1\, d^d k_2\, d^d k_3\, \delta_+(k_1^2)\delta_+(k_2^2)\delta_+(k_3^2)\delta(1-(p_1+p_2)\cdot(k_1+k_2+k_3))}{(k_2\cdot p_2)\,((k_1+k_3)\cdot p_1))}$$

- This is a "three-loop" integral. Omitting the delta function, The integral transform under the following scaling transformation unifromly

$$p_1 \to \lambda_1 p_1 \quad , p_2 \to \lambda_2 p_2, \quad \mathrm{MI} \to \frac{1}{\lambda_1 \lambda_2}\mathrm{MI}$$

- Insert a unit operator, $1 = \int \mathrm{d}^d Q \delta^{(d)}(Q-k_1-k_2-k_3)$, The original integral factorizes into two part

$$\mathrm{AuxI} = \int \frac{d^d k_1 d^d k_2 \mathrm{d}^d k_3 \delta_+(k_1^2)\delta_+(k_2^2)\delta_+(k_3^2)\delta^{(d)}(Q-k_1-k_2-k_3)}{(k_2\cdot p_2)\,((k_1+k_3)\cdot p_1)}$$

$$\mathrm{MI} = \int \mathrm{d}^D Q \Theta(Q^2)\frac{1}{Q\cdot p_1\, Q\cdot p_2}\mathrm{AuxI}\left(\frac{Q^2 p_1\cdot p_2}{Q\cdot p_1\, Q\cdot p_2}\right)$$

- Only need to calculate a "two-loop" integral AuxI! The Q integral is almost trivial

Dispersion integral method

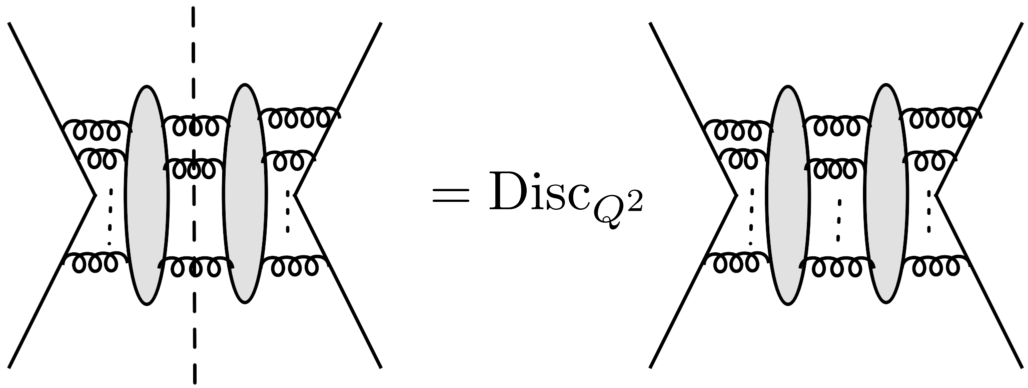
- Consider the auxiliary "two-loop" integral,

$$\text{AuxI} = \int \frac{d^d k_1 d^d k_2 \delta_+(k_1^2)\delta_+(k_2^2)\delta_+((Q-k_1-k_2)^2)}{(k_2\cdot p_2)\,((Q-k_2)\cdot p_1)}$$

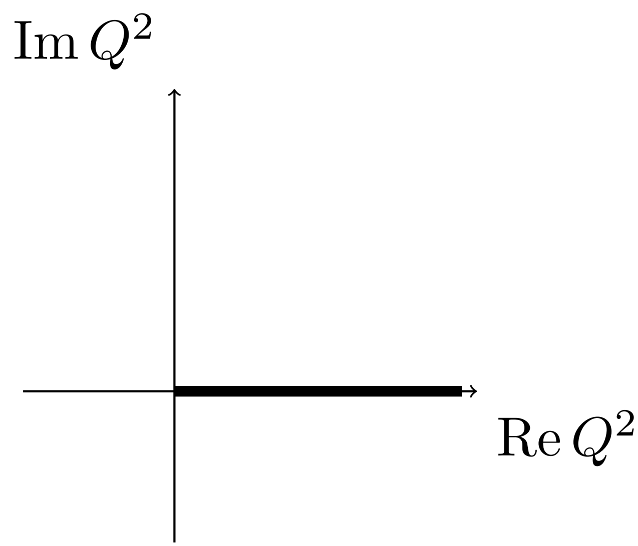
- Calculation of this class of integrals by explicit phase space parametrization is difficult, because of the delta function constraint $\delta((Q-k_1-k_2)^2)$. In particular, Lorentz invariance of the reuslt become obscure once introducing explicit phase space parameterization.
- Again the solution comes from dispersion relation

Dispersion integral method

- Cutting a feynman diagram is equivalent to taking the appropriate discontinuity of the same diagram



- $Q = \sum_i k_i$ is the sum of momentum crossing the cut. $Q^2 > 0$ open the threshold where all k_i are onshell
- Instead of computing phase space integral, we can now compute a loop integral and then take the discontinuity. This has the great advantage of being able to use all sorts of techniques developed for loop integrals



Hypergeometric function integral

- The corresponding loop integrals have Feynman parameter representation $\int [dx] P(\{x_i\})/Q(\{x_i\})$, P and Q are rational function of x_i.
- Example: the auxiliary integral defined in the previous slides have Feynman parameter representation

$$\mathrm{AutI} = \int \frac{d^d k_1 d^d k_2 \delta_+(k_1^2)\delta_+(k_2^2)\delta_+((Q-k_1-k_2)^2)}{(k_2 \cdot p_2)\,((Q-k_2)\cdot p_1)}$$

$$= \pi^2 \Gamma(5-d) \int_0^\infty \prod_{i=1}^{5} \mathrm{d}x_i \, \delta(1 - \sum_{i=1}^{5} x_i) \frac{(x_2 x_3 + x_1(x_2+x_3))^{5-3d/2}}{((-Q^2)x_1 x_2 x_3 + 2(x_3(x_2+2x_4)x_5 + x_1(x_3 x_4 + (x_2+2x_4)x_5)))^{-5+d}}$$

- Most of the integrals of this kind can be integrated in closed form into (generalized) hypergeometric function (in $d = 4 - 2\epsilon$ dimension)

$$\mathrm{AuxI} \sim \ {}_2F_1(1,1;2-2\epsilon;Q^2) \times \ {}_3F_2(1,1-2\epsilon,1-\epsilon;-2\epsilon,1+\epsilon;1)$$

- It's remarkable fact that the seemingly in the very frontier of perturbative QCD calculations, special functions like hypergeometric functions studied by Mathematians 300 years ago (Euler, Gauss, Riemann) still play an important role

soft-virtual corrections at N^{3}LO

- The final results of this lengthy calculation is the soft-virtual corrections at N^{3}LO [Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 1410.xxxx]

$$\mathrm{Sing}^{(3)}(1-z) = C_{30}\alpha_s^3\delta(1-z) + C_{31}\alpha_s^3 L_1 + C_{32}\alpha_s^3 L_2 + \cdots + C_{36}\alpha_s^3 L_6$$

- The coefficients of the logarithmic terms $C_{31}\cdots C_{36}$ are known long ago by threshold resummation [Moch, Vogt, 2005; Idilbi et.al.; Ravindran et.al.; L. L. Yang et.al., 2008] . We find complete agreement with them: non-trivial check!
- The coefficient C_{30} can not be predicted by resummation: new result from our calculation!

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$$\begin{aligned}(4\pi)^3 C_{30} =& C_A^3\left(\frac{979\zeta_2\zeta_3}{24} + \frac{16151\zeta_2}{1296} - \frac{15257\zeta_4}{864} - \frac{2003\zeta_6}{48} - \frac{7579\zeta_5}{144} + \frac{413\zeta_3^2}{6} - \frac{819\zeta_3}{16} + \frac{215131}{5184}\right)\\
&+ C_A^2 N_f\left(-\frac{125\zeta_2\zeta_3}{12} - \frac{70\zeta_2}{81} + \frac{2629\zeta_4}{432} + \frac{869\zeta_5}{72} + \frac{1231\zeta_3}{216} - \frac{98059}{5184}\right)\\
&+ C_A C_F N_f\left(3\zeta_2\zeta_3 - \frac{71\zeta_2}{36} + \frac{11\zeta_4}{72} + \frac{5\zeta_5}{2} + \frac{13\zeta_3}{2} - \frac{63991}{5184}\right)\\
&+ C_A N_f^2\left(-\frac{133\zeta_2}{324} - \frac{19\zeta_4}{36} + \frac{43\zeta_3}{108} + \frac{2515}{1728}\right) + C_F^2 N_f\left(\frac{37\zeta_3}{12} - 5\zeta_5 + \frac{19}{18}\right)\\
&+ C_F N_f^2\left(-\frac{23\zeta_2}{72} - \frac{\zeta_4}{36} - \frac{7\zeta_3}{6} + \frac{4481}{2592}\right)\end{aligned}$$

- Results expressed in terms of rational numbers and Riemann's zeta function, $\zeta_s = \sum_{n=1}^{\infty}\frac{1}{n^s}$

Compare with Durham-Zurich collaboration

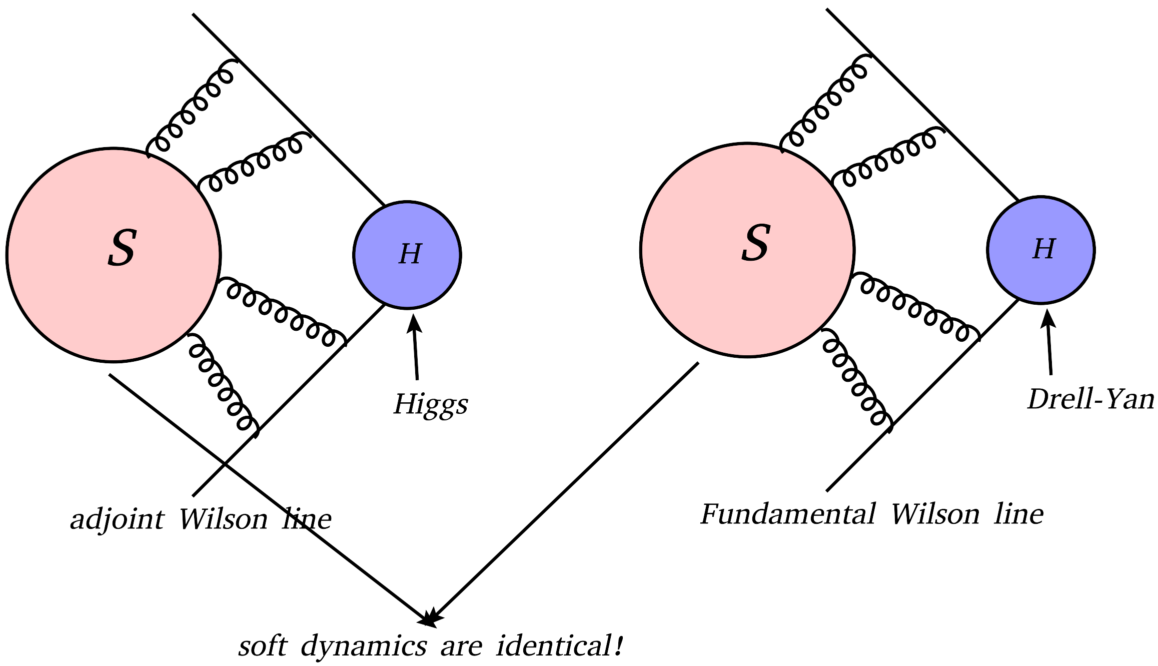
- Recently, an independent calculation of the soft-virtual corrections for Higgs production is reported by a Durham-Zurich collaboration [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger, 1302.4379, 1309.4393,1311.1425,1403.4616]
- In their calculations, they use energy and angular parameterization of phase space integral, Mellin-Barnes, symbols, and coproduct techniques for evaluating the integrals
- In constrast, we use **dispersion method, reduction # of loops by scaling symmetry, and hypergeometric function** for the integral evaluation
- The calculations by the two groups are in full agreement!
- Furthermore, we are able to get closed from expression for all the integrals to all orders in the dimensional regulation parameter ϵ, which will be important for future N^4LO calculation.
- In addition, we also compute the soft-virtual corrections to Drell-Yan and $\mathcal{N} = 4$ Supersymmetric Yang-Mills theory, which they don't have.

N^{3}LO soft-virtual corrections for Drell-Yan lepton pair production

[Y. Li, H. X. Zhu, 1309.4391; Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 1404.5839; Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 1410.xxxx]

Soft-virtual corrections to Drell-Yan production

- QCD factorization implies that the soft-gluon radiations doesn't know too much about the underlying hard process
- Soft-virtual corrections calculated for Higgs production can be easily translated into Drell-Yan production



Soft-virtual corrections to Drell-Yan production

- We find that the soft-virtual corrections for Drell-Yan production has the form

$$
\begin{aligned}
(4\pi)^3 C_{30}^{DY} = & \left(\frac{208\zeta_3\zeta_2}{3} - \frac{28132\zeta_2}{81} - \frac{6016\zeta_3}{81} - \frac{2878\zeta_4}{27} - 8\zeta_5 + \frac{110651}{243}\right) C_A C_F N_f \\
& + \left(\frac{3280\zeta_3^2}{3} + \frac{28736\zeta_2\zeta_3}{9} - \frac{20156\zeta_3}{9} - \frac{13186\zeta_2}{27} - \frac{832\zeta_4}{27} - \frac{39304\zeta_5}{9} - \frac{2602\zeta_6}{9} + \frac{74321}{36}\right) C_A C_F^2 \\
& + \left(-\frac{400\zeta_3^2}{3} - \frac{884\zeta_2\zeta_3}{3} + \frac{82385\zeta_3}{81} + 843\zeta_2 + \frac{14611\zeta_4}{54} - 204\zeta_5 + \frac{1658\zeta_6}{9} - \frac{1505881}{972}\right) C_A^2 C_F \\
& + C_F \Bigg(20\zeta_2 \mathrm{NFV} C_A - \frac{80\zeta_2 \mathrm{NFV}}{C_A} + \frac{28}{3}\zeta_3 \mathrm{NFV} C_A - \frac{112\zeta_3 \mathrm{NFV}}{3C_A} - 2\zeta_4 \mathrm{NFV} C_A + \frac{8\zeta_4 \mathrm{NFV}}{C_A} - \frac{160}{3}\zeta_5 \mathrm{NFV} C_A + \frac{640\zeta_5 \mathrm{NFV}}{3C_A} \\
& + 8 \mathrm{NFV} C_A - \frac{32 \mathrm{NFV}}{C_A}\Bigg) + \left(-\frac{5504}{9}\zeta_3\zeta_2 + \frac{2632\zeta_2}{27} + \frac{3512\zeta_3}{9} + \frac{136\zeta_4}{27} + \frac{5536\zeta_5}{9} - \frac{421}{3}\right) C_F^2 N_f \\
& + \left(\frac{2416\zeta_2}{81} - \frac{1264\zeta_3}{81} + \frac{320\zeta_4}{27} - \frac{7081}{243}\right) C_F N_f^2 \\
& + \left(\frac{10336\zeta_3^2}{3} + 80\zeta_2\zeta_3 - 460\zeta_3 - \frac{130\zeta_2}{3} + 206\zeta_4 + 1328\zeta_5 - \frac{23092\zeta_6}{9} - \frac{5599}{6}\right) C_F^3
\end{aligned}
\tag{1}
$$

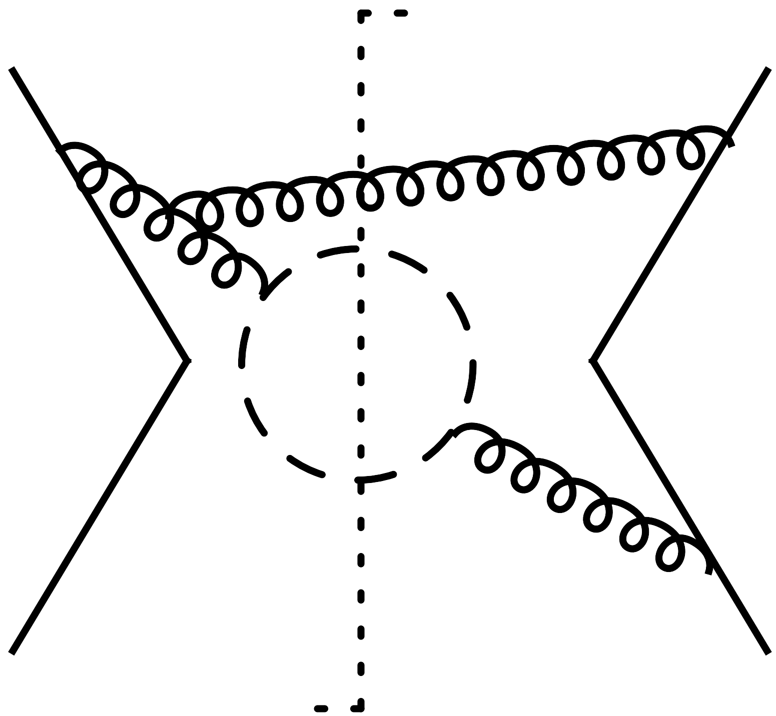
- Again, results are expressed in terms of zeta values.

N^3LO soft-virtual corrections in $\mathcal{N} = 4$ Supersymmetric Yang-Mills theory

[Y. Li, H. X. Zhu, 1309.4391; Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 1404.5839; Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 1410.xxxx]

$\mathcal{N} = 4$ SYM

- $\mathcal{N} = 4$ SYM is a close cousin of QCD
- It has one adjoint gluon, four adjoint fermion and six adjoint scalar
- The radiations of gluon in $\mathcal{N} = 4$ SYM exhibite the same soft singularities as in QCD
- Our calculations are general enough to generalize to $\mathcal{N} = 4$ SYM



- But you might ask: why $\mathcal{N} = 4$ SYM?

$\mathcal{N} = 4$ **SYM**

- $\mathcal{N} = 4$ SYM is believed to be the "simplest" QFT by many people [Arkani-Hamed et.al.]
- It's a supersymmetrc conformal field theory
- At large N_c it has a gravity duality
- It's an integrable QFT at large N_c
- A perfect model for studying perturbative radiation corrections
- If there is pattern in perturbative QFT, it will be easier to identify in $\mathcal{N} = 4$ SYM
- A particularly interesting property is the maximal transcendentality principle [Kotikov, Lipatov, Onishchenko, Velizhanin, 2004] .

Maximal transcendentality principle

- The maximal transcendentality principle states that, for anomalous dimension of Wilson twist two operator in $\mathcal{N}=4$ (Mellin moment of DGLAP splitting kernel), $\gamma_{ab}(j) = -\int_0^1 \mathrm{d}x\, x^{j-1} P_{b\to a}(x)$, it has the propertiy of **unifrom transcendentality**, and coincides with the leading transcendental part of the QCD result [Kotikov, Lipatov, Onishchenko, Velizhanin, 2004] . *E.g.*, in the limit of $j\to\infty$, the dominant behaviour is controlled by cusp anomalous dimension

$$\gamma(j) = \frac{1}{2}\gamma_K(\alpha_s)\ln(j) + \mathcal{O}(j^0)$$

- At four loops, the cusp anomalous in large N_c limit is given by [Bern, Czakon, Dixon, Kosower, Smirnov, 2006]

$$\gamma_K(\alpha_s) = \hat{a} - \frac{\pi^2}{6}\hat{a}^2 + \frac{11}{180}\pi^4\hat{a}^3 - \left(\frac{73}{2520}\pi^6 + \zeta_3^2\right)\hat{a}^4 + \cdots \qquad \hat{a} = \frac{\alpha_s N_c}{2\pi}$$

The **transcendental weight** of ζ_n is n, and the transcendentality is additive for products of zeta value. The cusp anomalous dim. has weight $2L-2$ at L loops.

maximal transcendentality principle

- The consideration maximal transcendentality principle has led Beisert, Eden and Staudacher to find the the correct integral equation satisfied by the cusp anomalous dim. in $N = 4$ SYM.
- Remarkably, the maximal transcendentality principle applies not for anomalous dim., but also for $\mathcal{N} = 4$ SYM scattering amplitudes at leading color and subleading color level, light-like Wilson loops, form factor, and correlation function.
- However, it's also known that in some cases, the principle doesn't hold, *e.g.*, in angular distribution of dijet production at NLO.
- Given our ablitity to compute soft-virtual corrections in $\mathcal{N} = 4$ SYM, it would be interesting for us to test this principle.

soft-virtual corrections and maximall transcendentality principle

- We find that the soft-virtual corrections for the production of a component of stress-tensor operator is surprisingly simple [Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 1410.xxxx]

$$(4\pi)^3 C_{30}^{\mathcal{N}=4} = N_c^3 \left(-\frac{2003\zeta_6}{48} + \frac{413\zeta_3^2}{6} \right)$$

- to be contrasted with the results for Higgs production

$$\begin{aligned}
(4\pi)^3 C_{30}^H =& C_A^3 \left(\frac{979\zeta_2\zeta_3}{24} + \frac{16151\zeta_2}{1296} - \frac{15257\zeta_4}{864} - \frac{2003\zeta_6}{48} - \frac{7579\zeta_5}{144} + \frac{413\zeta_3^2}{6} - \frac{819\zeta_3}{16} + \frac{215131}{5184} \right) \\
&+ C_A^2 N_f \left(-\frac{125\zeta_2\zeta_3}{12} - \frac{70\zeta_2}{81} + \frac{2629\zeta_4}{432} + \frac{869\zeta_5}{72} + \frac{1231\zeta_3}{216} - \frac{98059}{5184} \right) \\
&+ C_A C_F N_f \left(3\zeta_2\zeta_3 - \frac{71\zeta_2}{36} + \frac{11\zeta_4}{72} + \frac{5\zeta_5}{2} + \frac{13\zeta_3}{2} - \frac{63991}{5184} \right) \\
&+ C_A N_f^2 \left(-\frac{133\zeta_2}{324} - \frac{19\zeta_4}{36} + \frac{43\zeta_3}{108} + \frac{2515}{1728} \right) + C_F^2 N_f \left(\frac{37\zeta_3}{12} - 5\zeta_5 + \frac{19}{18} \right) \\
&+ C_F N_f^2 \left(-\frac{23\zeta_2}{72} - \frac{\zeta_4}{36} - \frac{7\zeta_3}{6} + \frac{4481}{2592} \right)
\end{aligned}$$

- Remarkably, **all** the lower weight terms in the QCD results all **cancel** in the combination of all contributions!

soft-virtual corrections and maximall transcendentality principle

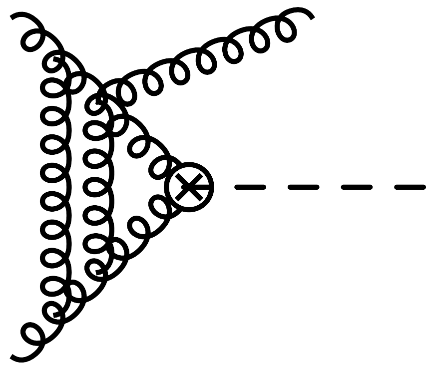
- We therefore confirm that the principle of maximal transcendentality holds also in cross section like physical observable.
- The simplicity of the results in $\mathcal{N} = 4$ SYM hints at a better organization of the calculations, in which the cancellation of lower weight terms become explicit.
- Example is in the calculation of scattering amplitudes, where a conformally invariant basis for the integrals exist [Drummond, Henn, Smirnov, Sokatchev, 2006]
- The agreement of $\mathcal{N} = 4$ SYM and QCD for the leading transcendental term for soft-virtual corrections open the possibility that **QCD results** can be derived from $\mathcal{N} = 4$ **results**, in the sense that gluonic contributions can be extracted once the fermion and scalar contributions are known. And for the latter case, the calculation is usually easier than the gluonic contributions.

Conclusion

- Precision Higgs coupling measurement requires N^3LO calculation.
- N^3LO soft-virtual corrections finished, a key step towards full N^3LO [Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 2013,2014; Anastasiou et.al., 2013,2014]
- Calculations based on **reverse unitarity, IBP reduction, reduction of # of loops by scaling symmetry, dispersion reliation, and hypergeometric function representation**.
- We also present Soft-virtual corrections for Drell-Yan and $\mathcal{N} = 4$ SYM [Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 1410.xxxx].
- Remarkably, **principle of maximal transcendentality** also works for soft-virtual corrections
- So far, the transcendentality principle is only an **observational fact**. Explaining it in terms of symmetry principle would be very interesting. A better understanding of this transcendentality principle might lead to better calculational technique for this kind of problem.

Outlook

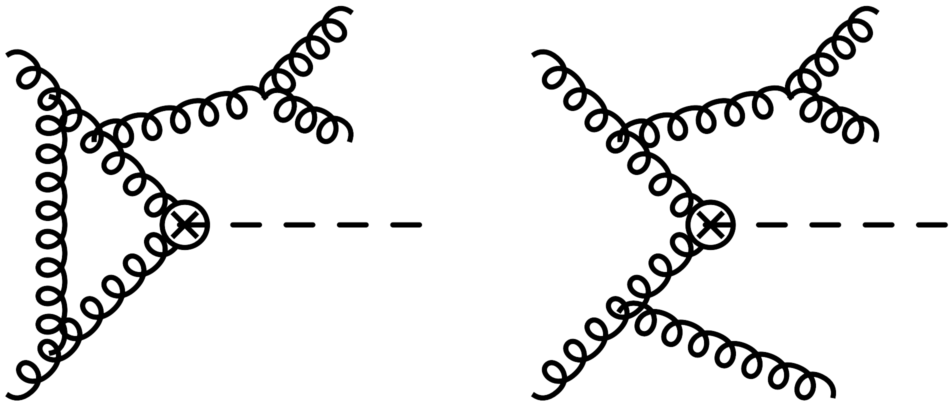
- In the future, a full N^{3}LO calculation goes beyond soft-virtual approximation highly demanded
- All the matrix elements are known. The obstacle is the phase space integrals
- *E.g.*, For the case of double virtual real contributions



- the integration over final phase space is divergent.
- It's possible to do the phase space integral directly, with the two-loop QCD splitting amplitudes to $\mathcal{O}(\epsilon^2)$ as input [L. Dixon, H. X. Zhu, in preparation]

Outlook

- For the virtual double real and triple contributions direct integration is difficult



- However, the partonic cross section, and therefore all the integrals, are functions of $z = \frac{M_H^2}{x_1 x_2 S}$
- It's possible to derive a symstem of **first order ordinary differential equations** satisfied by these integrals
- The differential equations will only have singularities at $z = 0$, $z = 1$, $z = \infty$
- The solution of the differential equation has the form of iterative integral, solution given by **harmonic polylogarithms**
- The soft-virtual corrections [Y. Li, A. von Manteuffel, R. Schabinger, H. X. Zhu, 2013,2014; Anastasiou et.al., 2013,2014] will be essential for the determination of integration constant for the differential equation!
- We're looking forward to the full calculation of $\mathrm{N^3LO}$ QCD corrections, which plays an important role in precision Higgs phenomenology


Thank you !