Modelling Electroweak Interactions at High Energies

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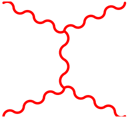
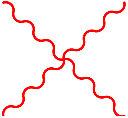
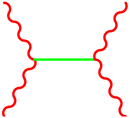
May 2015

Higgs and Vector-Boson Scattering

$O(E^4)$  $+ O(E^4)$  $+ O(E^2)$  $= O(1)$

Higgs exchange cancels the E^2 rise exactly (in the SM):
the Minimal SM Higgs Sector.

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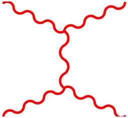
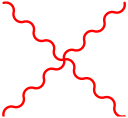
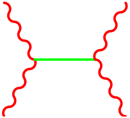
$O(E^4)$  $+ O(E^4)$  $+ O(E^2)$  $= O(1)$

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the Minimal SM Higgs Sector.

Discoveries

1. Higgs production in WW fusion: the Higgs boson exists.
2. SM confirmed in VBS: the Higgs mechanism works as expected.

Future Expectation for VBS

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If SM is true, VBS amplitude is bounded and small: m_H^2/v^2.

LHC:

Production cross section falls of with increasing effective energy, i.e., invariant mass of the *WW* pair system.

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Production cross section falls of with increasing effective energy, i.e., invariant mass of the WW pair system.

NLO: some logarithmic corrections.

No problem with unitarity, of course.

And What If Not?

Two classes of modifications to the SM (or mixture):

1. New weakly interacting particles, direct production. Example: 2HDM

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- Add higher-dimensional operators to the SM Lagrangian.
- Use only SM fields, respect SM gauge invariance
- Operator of dimension n carries prefactor $1/\Lambda^{n-4}$

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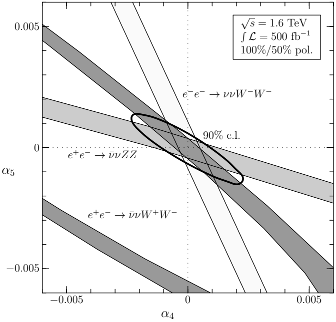
$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{d=5}^{\infty} \frac{1}{\Lambda^{n-4}} \mathcal{O}_n$$

Vector-Boson Scattering at high energies,

E. Boos, H.-J. He, WK, A. Pukhov, C.-P. Yuan, P.M. Zerwas, PRD 57 (1998)



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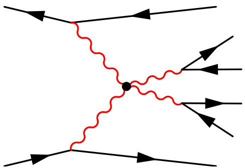

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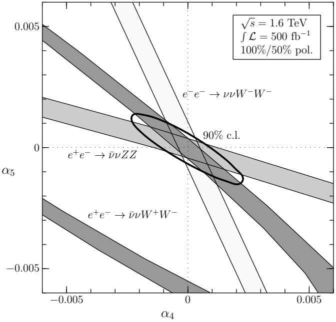

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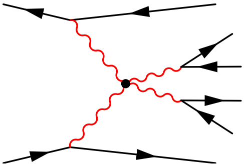

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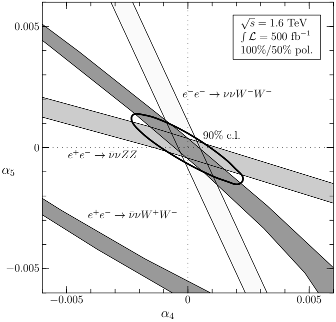

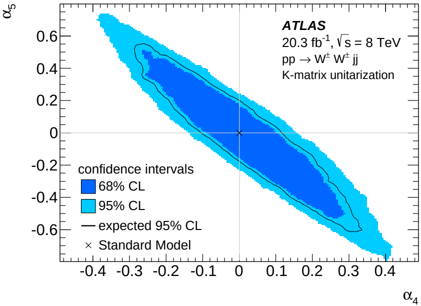

Discovery of Vector-Boson Scattering

ATLAS collaboration, PRL 113 (2014)

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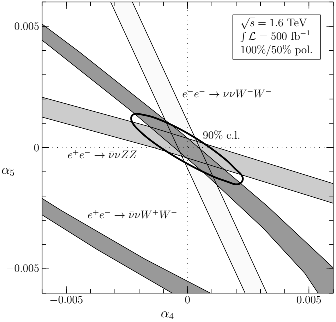
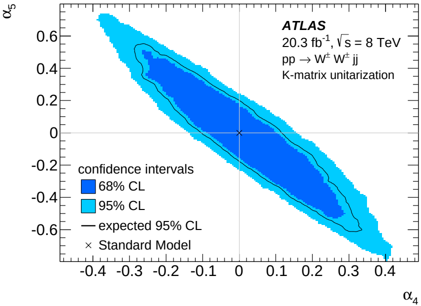




Discovery of Vector-Boson Scattering

ATLAS collaboration, PRL 113 (2014) — using WHIZARD

Tool for Calculation and Simulation

WHIZARD is a universal Monte Carlo for high-energy processes
(first version 1999, currently 2.2.6)

- tailored for lepton colliders (LEP, ILC, CEPC, CLIC)
- applied also for hadron colliders (ATLAS/CMS, CPPC)

The WHIZARD Team

- University of Siegen: Wolfgang Kilian
- DESY (Hamburg): Jürgen Reuter
- University of Würzburg: Thorsten Ohl

New Study of *WW* Scattering

WK, T. Ohl, J. Reuter, M. Sekulla, arXiv:1408.6207 (PRD, to appear)

- Investigate quartic anomalous couplings of vector bosons
- Match to EFT with Higgs (linear representation)
- Extrapolate to high energies without violating unitarity in the calculation
- Implement as non-Lagrangian model in WHIZARD
- Evaluate for full SM particle set, observables LHC processes

Concrete Examples:

Anomalous Interactions

$$\mathcal{L}_{HD} = F_{HD}\ \mathrm{tr}\left[\mathbf{H}^\dagger\mathbf{H} - \frac{v^2}{4}\right] \cdot \mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger(\mathbf{D}^\mu\mathbf{H})\right] \qquad HVV \quad D = 6$$

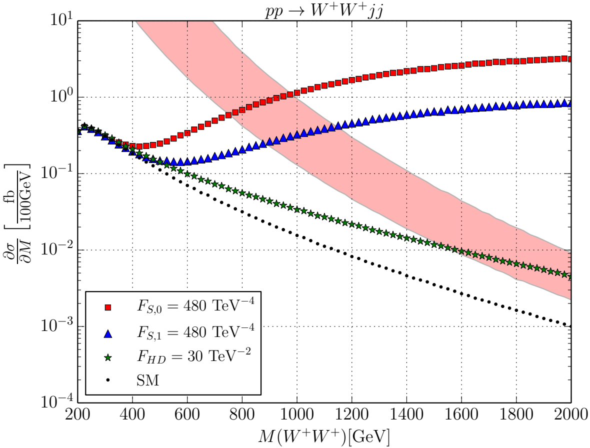
$$\mathcal{L}_{S,0} = F_{S,0}\ \mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger\mathbf{D}_\nu\mathbf{H}\right] \cdot \mathrm{tr}\left[(\mathbf{D}^\mu\mathbf{H})^\dagger\mathbf{D}^\nu\mathbf{H}\right] \qquad VVVV \quad D = 8$$

$$\mathcal{L}_{S,1} = F_{S,1}\ \mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger\mathbf{D}^\mu\mathbf{H}\right] \cdot \mathrm{tr}\left[(\mathbf{D}_\nu\mathbf{H})^\dagger\mathbf{D}^\nu\mathbf{H}\right] \qquad VVVV \quad D = 8$$

Linear Higgs/Goldstone Field Representation:

$$\mathbf{H} = \frac{1}{2}\begin{pmatrix} v + h - \mathrm{i}w^3 & -\mathrm{i}\sqrt{2}w^+ \\ -\mathrm{i}\sqrt{2}w^- & v + h + \mathrm{i}w^3 \end{pmatrix} \qquad . \qquad (1)$$

Nice, but. . .



Calculation: WHIZARD

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Gauge invariance + Higgs exchange
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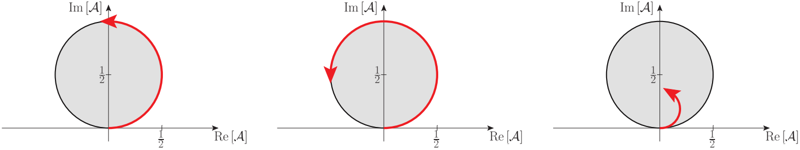
[There are perturbative models, e.g, the 2HDM. But they access only a small fraction of the conceivable Model Space.]

Unitarity

The scattering of w, z is a (quasi-) elastic process. Properly diagonalized (isospin I, spin J) and normalized, the partial-wave amplitudes must lie on the Argand Circle.

Possibilities



A: Saturation B: Resonance C: Inelastic

Unitarization

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There are zillions of papers that investigate this problem.

- Heavy Higgs as Unitarization
- K-Matrix Unitarization
- Padé Unitarization
- Inverse Amplitude Method
- O(N) Model Unitarization
- N/D Method
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Which makes a difference.

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Do we want a prediction with assumptions? We want a framework.

K Matrix

(Heitler 1941, for QED): Cayley Transform

$$S = \frac{\mathbb{1} + \mathrm{i}K/2}{\mathbb{1} - \mathrm{i}K/2}\,, \qquad \text{where} \quad K = K^\dagger \qquad \text{and} \quad S = \mathbb{1} + \mathrm{i}\,T$$

The K Matrix, exactly:

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The K Matrix, in Perturbation Theory:

$$K = T - \frac{\mathrm{i}}{2}T^2 \pm \ldots$$

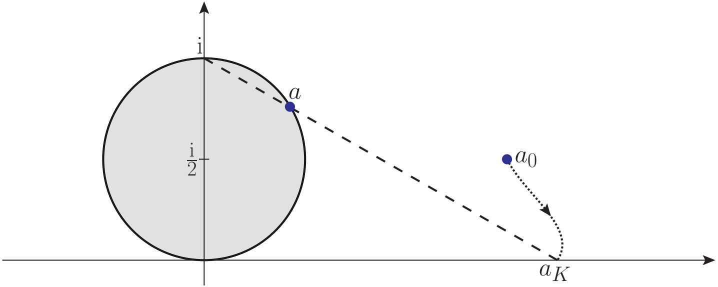
Original K Matrix algorithm (Gupta, for QCD/EW):

- Compute T matrix perturbatively
- Reconstruct K matrix order by order
- Insert into S matrix formula, without expanding again

This is elegant, but relies on perturbation theory.

Graphical Visualization: K Matrix

Start from arbitrary amplitude a_0 in perturbative expansion:



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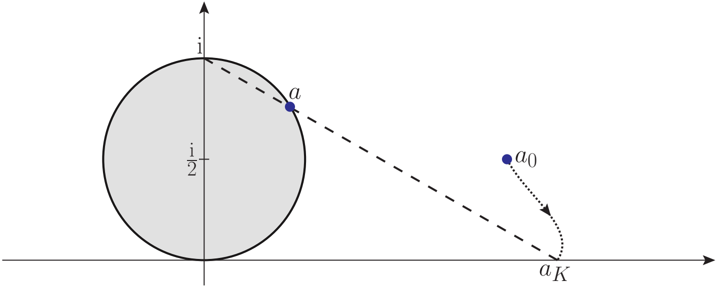
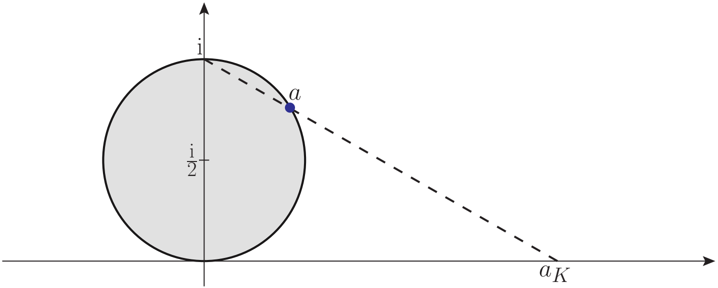


First reconstruct a_K, then compute a

Our suggestion: compute unitarized T matrix directly, without detour

Graphical Visualization: Direct T Matrix Unitarization

Start from real amplitude $a_0 = a_K$: Inverse stereographic projection

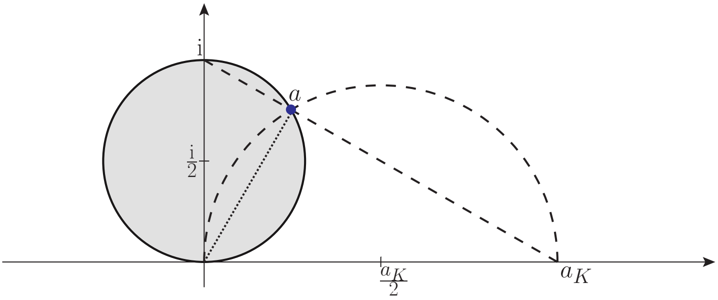


⇒ No reference to perturbative expansion

⇒ Unitary amplitude a_0 left invariant

Graphical Visualization: Direct T Matrix Unitarization

Start from real amplitude $a_0 = a_K$: Thales circle projection



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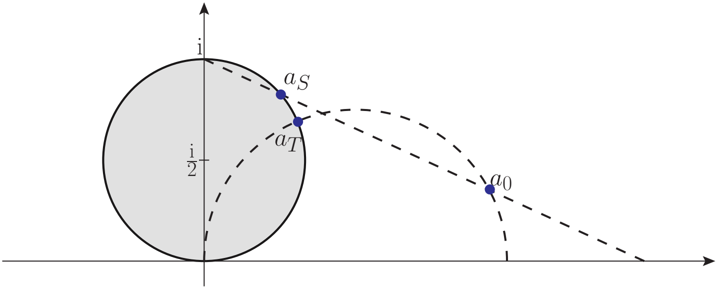
Graphical Visualization: Direct T Matrix Unitarization

Start from complex amplitude a_0:



⇒ No reference to perturbative expansion

⇒ Unitary amplitude a_0 left invariant

⇒ But scheme dependence for complex a_0

Linear Construction "Stereographic"

$$T = \frac{\mathrm{Re}\, T_0}{\mathbb{1} - \frac{\mathrm{i}}{2} T_0^\dagger}.$$

for normal matrices ($T^\dagger T = T T^\dagger$), otherwise need operator ordering

- well behaved near $T = 0$
- weird behavior for eigenvalues above $T = \mathrm{i}$

Circular Construction "Thales"

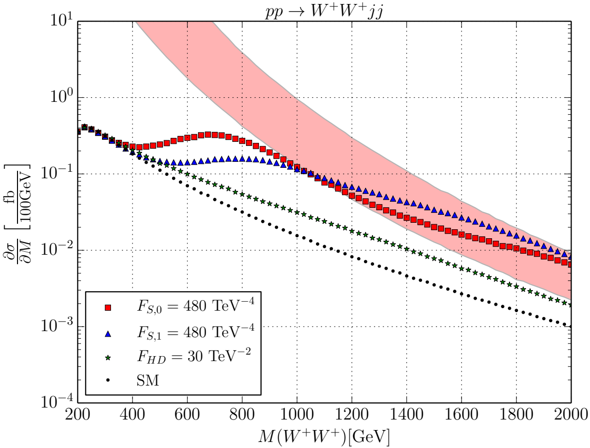
$$T = \frac{1}{\mathrm{Re}\left(\frac{1}{T_0}\right) - \frac{\mathrm{i}}{2}\mathbb{1}} .$$

- singular at $T = 0$ (but harmless)
- well behaved above $T = \mathrm{i}$

Algorithm

1. Start with input model
2. Extract strong-interaction part in Goldstone limit
3. Unitarize via T Matrix projection
4. Re-insert correction as form factor into Feynman rules
5. Extrapolate off-shell
6. Use in Monte Carlo simulation

Result: Unitarized Cross Section



Calculation: WHIZARD

And Beyond?

- Padé & Co. yield predictions: resonances
- work in QCD (vector dominance) ...?
- restricted to quasi-elastic scattering?

$\Rightarrow$ Add any additional information in T Matrix framework

Resonances and Anomalous Couplings

A resonance is a pole in the elastic scattering matrix:

$$A(s) = \frac{g^2}{s - \hat{m}^2} + \hat{A}_{\text{nonres}}(s)$$

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At low energy, the resonant amplitude has a Taylor expansion

$$A(s) = -\frac{g^2}{m^2} + \frac{g^2}{m^4}\, s + \ldots$$

The second term corresponds to an anomalous coupling (matching).

Guideline for Simplified Models

- The rise of an amplitude (anomalous coupling) may be the Taylor expansion of a resonance.
- We have no idea which resonances exist and where they come from.
- Including a resonance in the model, there still may be further sources for anomalous couplings (further resonances, $A_{\text{nonres}}(s)$, deviation from the Breit-Wigner shape, etc.)
- Beyond the resonance, the amplitude may eventually rise and need unitarization again.

Consequence:

We allow for resonances in all accessible spin/isospin channels.
We also include extra anomalous couplings.

Simplified Models: Generic Resonances

I	0	1	2
$J=0$	σ^0	.	$\phi^{--}, \phi^{-}, \phi^{0}, \phi^{+}, \phi^{++}$
1	.	$\rho^{-}, \rho^{0}, \rho^{+}$	.
2	f^0	.	$t^{--}, t^{-}, t^{0}, t^{+}, t^{++}$
...	...	...	...

- $I=0$: resonant in W^+W^- and ZZ scattering
- $I=1$: resonant in W^+Z and W^-Z scattering
- $I=2$: resonant in W^+W^+ and W^-W^- scattering

Model Parameters

VBS, total (isospin preserved, CP, higher spin ignored):

- 5 resonances with 3 parameters each (M, g_L, g_T)
- quartic anomalous couplings of longitudinal VB
- quartic anomalous couplings of transversal VB
- quartic anomalous couplings mixing T and L

Project

WK, T. Ohl, J. Reuter, M. Sekulla, 2015 (work in progress)

- Supplement SM by generic resonances in the TeV range
- Match to EFT with Higgs (linear representation)
- Extrapolate to high energies without violating unitarity in the calculation
- Implement as non-Lagrangian model in WHIZARD
- Evaluate for full SM particle set, observables LHC processes

Breakdown of Possibilities

Breakdown of Possibilities:

1. Vector resonance (ρ)
   - Parity odd, isospin 1 enhanced
   - Can mix with W, Z
   - Can thus couple to light-fermion currents
2. Scalar and/or tensor resonance ($\sigma/\phi/f/t$)
   - C,P even (or odd)
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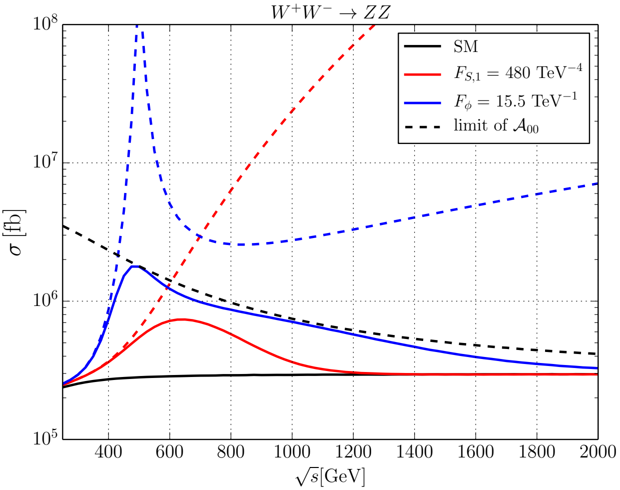
Let's consider the second case. cf. extra dimensions

Example: Scalar Resonance



The Tensor Is Special

Massive spin-2 field in relativistic quantum field theory

On-shell: $SO(3)$ rotation group in rest frame: 5 polarization components

Off-shell: $SO(3) \times SO(3)$ Lorentz group: 10 components of symmetric tensor

- $\Rightarrow$ 5 components: genuine tensor, couples to current $J_{\mu\nu}$
- $\Rightarrow$ 3 components: vector, couples to divergence $\partial^\mu J_{\mu\nu}$
- $\Rightarrow$ $1 + 1$ components: 2 scalars, couple to J^μ_μ and $\partial^\mu \partial^\nu J_{\mu\nu}$

Embedded in Standard Model: $\partial_\mu \to D_\mu$, and divergences don't vanish due to EWSB

The Tensor Is Special

Consequences for scattering amplitudes:

Beyond the resonance, amplitudes rise again

- Goldstones coupled to genuine tensor ⇒ analog to scalar, can use T matrix method for the amplitude
- Goldstones coupled to scalar ⇒ additional terms in T matrix
- EWSB contributions for scalar parts ⇒ extra terms proportional to m_W^2/M_T^2 and m_h^2/M_T^2, non-universal
- EWSB contribution for vector parts ⇒ extra terms proportional to m_W^2/M_T^2 in transversal VB interactions

Have to estimate contributions that are not covered by Goldstone-limit unitarization

Summary

- Effective theory: good for TGC, limited applicability for QGC.
- Unitarization schemes tend to introduce theoretical prejudice

$\Rightarrow$ We propose a framework how to reconcile EFT with unitarity without losing its benefits

$\Rightarrow$ Direct T-Matrix unitarization as catch-all scheme for new models

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